Time Dependent Temperature Distribution in Pulsed RF Waveguide CO\textsubscript{2} Laser

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Abstract: - The Temperature variation for RF-excited waveguide CO\textsubscript{2} laser in the pulse regime is studied theoretically by considering one-dimensional model for heat conduction. The temperature increases exponentially with time whereas it increases, reaches maximum and then decreases with the increase of position.

Key-Words: - Waveguide CO\textsubscript{2} laser, Pulsed operation, RF excitation, Temperature variation

1 Introduction
The relatively long time effects of heating of an optical medium by beam passing through it has been given considerable attention in recent years. Thermodynamic processes created in the laser cavity by pulsed plasma, influence the shape and spectral contents of the output laser pulse. Since the distance between the electrodes is relatively small (about 2 mm) and changes of the temperature in this area can be dramatic when the power is delivered to the electrodes [1].

The waveguide RF excited laser cavity is not totally closed structure. Thus not only a temperature and pressure can change in the cavity. It means that an isochoric processes occurs or can occur in the cavity only at the very beginning of the gas excitation [2,3]. An important mechanism which deteriorates the uniformity of the low pressure laser discharges is the thermal instability. A thermal instability causes changes in gas composition. It increases the electron temperature from the optimum for different excitation, causes local gas overheating and discharge constriction, reducing the laser gain [4]. Furthermore, the addition of heat also leads to a density variation in the gas and therefore to a variation of the intracavity refractive index; this can degrade the laser beam quality [5-12].

In this paper, the temperature variation with time in the laser cavity is studied theoretically considering the energy balance equation when the energy is applied at the input in the pulsed form (for the three kinds of pulses [13]) as well as the energy generated in the cavity through stimulated emission.

2 Theory
In pulsed lasers, the major cause of a change in the output power is due to the change of the temperature inside the laser cavity and the discharge instabilities arise [6]. In particular the $\alpha-\gamma$ transition may modify or remove the current limiting properties of the ion sheeths, resulting in severe loss of uniformity in the discharge often followed by damage to the electrodes.

Considering the heat balance equation in one-dimensional co-ordinate system [14], as shown in fig. 1:

\begin{equation}
\frac{\rho c_p}{\partial t} \frac{dT}{\partial t} = k \frac{d^2T}{dx^2} + Q
\end{equation}

After some mathematical rearrangements we get:

\begin{equation}
\frac{dT}{\partial t} = \alpha \frac{d^2T}{dx^2} + \frac{Q}{\rho c_p}
\end{equation}

Where the derivatives are the partial derivatives, as the time $t$ and the position $x$ are both independent variables. In above equation the term on left side describes the rate of change of temperature with time. The first term on right side of the equation describes the rate of change of temperature with position and the second term describes the...
generation of heat in laser cavity. In this equation 
\[ \alpha = \sqrt{\frac{k}{\rho c}} \]
and is called the thermal diffusivity [15], \( k \) is the thermal conductivity of the laser gas mixture, \( \rho \) is the mass density of the medium and \( c_v \) is the specific heat capacity at constant volume. Usually thermal conductivity ‘\( k \)’ is a function of temperature. However we usually simplify the heat conduction analysis by taking ‘\( k \)’ to be independent of temperature, so ‘\( k \)’ is then independent of position also. Defining the temperature in Fourier series as:
\[ T = \sum_{n=1}^{\infty} a_n(t) \sin \left( \frac{n\pi}{d} x \right) \]  
\[ (3) \]
Then
\[ T_{xx} = \sum_{n=1}^{\infty} c_{-n} a_n(t) \sin \left( \frac{n\pi}{d} x \right) \]  
\[ (4) \]
and
\[ Q(t) = Q(t) \sum_{n=1}^{\infty} \frac{2}{n\pi} \left[ 1 - (-1)^n \right] \sin \left( \frac{n\pi}{d} x \right) \]  
\[ (5) \]
The value of heat generation term \( Q(t) \) in equation (5) is given by:
\[ Q(t) = \frac{w_0^3 (1 - \eta)}{\eta sd} \]  
\[ (6) \]
Where \( w_0^3 \) is the applied energy, \( \eta \) is the efficiency of the laser system, \( s \) is the surface area of the electrode system, \( d \) is the gap between the electrodes and \( t \) is the applied pulse width. The equation for temperature is then given by:
\[ T(t) = T_0 + \sum_{n=1}^{\infty} \frac{2 w_0^3 (1 - \eta)}{n\pi\rho c d} \sin \left( \frac{n\pi}{d} x \right) \exp \left( -\frac{n^2 \pi^2}{d^2} t \right) \]  
\[ (7) \]

3 Experimental Conditions
The discharge structure considered for these theoretical calculations consists of all metal-slab waveguide with length of 38.6 cm, width of 2 cm and the gap between the two electrodes is 0.2 cm. The laser gas mixture consists of CO2:N2:He:Xe with mixture ratios 1:1:3:0.25 respectively. The pressure of the gas is considered to be 100 torr. The three pulses namely the square, sine and the triangular ones with pulse width of 10 µs are applied for excitations [13]. The laser efficiency is considered to be 10%. The value of thermal conductivity ‘\( k \)’ is taken from the ref. [16].

4 Results and Discussions:
In this paper the variation of the temperature in the laser cavity with time in pulse regime is studied theoretically. Fig.2 represents the variation of temperature with time at the distance 0.02 cm from the top electrode.

**Fig.2: Temperature Vs Time at x=0.02 cm**

It is clear that the temperature increases exponentially with the increase of time. Figs.3 & 4 show the variation of temperature with time at a distance of 0.06 and 0.1 cm respectively from the top electrode. The temperature variation behavior is similar to that in fig.2 except the

**Fig.3: Temperature Vs Time at x=0.06 cm**

**Fig.4: Temperature Vs Time at x=0.1 cm**

magnitude of the temperature is increased. It is interesting to note that as we move towards the centre of the laser cavity the magnitude of temperature increases and at the centre of the cavity it is maximum. As we move away from the centre of the laser cavity (figs.5 & 6) the magnitude of temperature decreases till we reach the bottom electrode but the behavior of the temperature with time remains the same i.e., it increases with the increase of time.
If the temperature variation is compared with the variation of the position from the top to the bottom electrode as shown in fig.7, it can be seen that the temperature increases with the increase of the distance, reaches the maximum value at the centre of the laser cavity and then starts decreasing with further increase of the distance till we reach the lower electrode and the temperature reaches the room temperature value as the cavity walls are diffusively cooled.

This increase of the gas temperature in the laser cavity can be explained by considering the relaxation rates of the populations of the upper and lower laser levels. As in [17] it is stated that before the laser action the population inversion reaches its maximum value (storage of energy in the upper level). At the beginning of the laser action the stimulated emission induces a population increase of the lower laser level. This process indirectly affects the gas temperature because a fast transition of the molecules from the lower laser level to the ground state produces a transient heating of the gas in the cavity.

In this paper the results of the square excitation pulse are presented. The behavior of the other two excitation pulses i.e., sine and triangular, are same except that of the little decrease in the magnitude of temperature and are not presented here.

5 Conclusions

The temperature of the gas increases exponentially with the increase of time. As we move from top electrode to the centre of the laser cavity the temperature of the gas increases. At the centre of the laser cavity the temperature is maximum. As we move away from the centre towards the bottom electrode the magnitude of the gas temperature decreases. The behavior of the other two pulses namely the sine and the triangular ones are similar to that of the square pulse except that of a little decrease in the magnitude of the temperature.

References:
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