Research and Application for Arithmetic of Image Compression Based on Wavelet Transform and Clustering

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Abstract: This paper mainly introduces a basic method for image compression using wavelet transform. After transformation clustering is used to quantize the wavelet coefficient of high frequency band. Digital image compressing using traditional Wavelet Transform discards high-frequency information of source image which includes edge information and texture information; some luminance information will lose too. So the reconstructing image will take the form of indention and the luminance is lower than source image. Our method keeps the information of image edge and texture, hence the quality of reconstructing image is improved..

Key-Words: Wavelet Transform, Clustering, Image Compression, Image Transaction, blur effort, luminance

1 Introduction

Image Compressing Coding is always one of most active areas in the research of information processing, in which the technology of Image Compressing Coding based on Wavelet Transform (WT) is a very important kind of method. JPEG 2000 is the latest Image Compressing Standard at present. It gives up using the Fractal Block Coding (FBC) which based on Discrete Cosine Transform (DCT) and adopted by JPEG, uses state-of-the-art compression techniques on the basis of wavelet technology. Image Compressing based on WT is Multi-Resolution image decomposing by using WT. It decomposes source image into different spaces, different frequencies, and subbands of different directions. In which, low-frequency subband is an approach to the source image and high-frequency subband reflects the details of the source image. The traditional WT compressing discards the high-frequency information of source image and reserves low-frequency information only. Although, this method has kept most information of image, the image often takes the form of indention after reduced owing to lack of high-frequency information. At the same time, to the colored image, because luminance information is kept in high-frequency information mostly, the reduced image is obviously partial to dark. The instance in the article can relatively verify these. So high-frequency information is essential to the image too. This article puts forward a method which combines WT and Clustering to compress image. Low-frequency subband is used after WT as a compressing approach to the source image, the algorithms of Clustering is utilized to carry high-frequency subband after WT Quantization to get certain keeping of the details of the picture. According to that, through WT and clustering, under high ratio compress, a small amount of quantization information is used to show more high-quality image.

2 The basic processes of digital image compressing

At first, let’s discuss the basic processes of using WT in image compressing and in order to make it convenient to compare traditional WT compressing with WT compressing which added high-frequency clustering and discuss the differences of the basic process between them.
2.1 The basic processes of traditional WT compressing

FIG. 1 The basic processes of traditional WT image compressing and reducing, and they are basic processes of JPEG 2000 too.

2.2 The basic processes of WT compressing added clustering

FIG. 2. shows the basic processes of WT compressing added clustering which this article discussed.
There are two very important parameters in WT compressing added clustering; the meanings of them are follows:
- Compress demand
  It is used to compare the results of compressing which used different wavelet coefficients in WT to compress different types of images.
- Quality demand
  Quality demand is the number of transformation times of low-frequency information set by the client and the number of high-frequency information clusters. This parameter used to adjust the Compressing Quality and Compressing Ratio.
  The concrete set methods of above mentioned parameters will be introduced in detail later.
Two dotted lines in the FIG.2 represent respectively the processes dealing with low-frequency information when adopting WT several times. In the transform compressing, WT used for low-frequency information can be applied more than one time to enhance compressing ratio. Then in the reducing, some methods should be adopted to combine the low-frequency information which is through several times transformed and corresponding high-frequency information to get final image.

The main difference between the processes of WT compressing added clustering and the processes of traditional WT compressing is as follows:

The WT compressing added clustering makes the high-frequency subband on the clusters, then encode high-frequency information according to the result of the clustering. Generally, high-frequency information contains border information of image. Because of which, certain filling algorithm can be adopted to reduce border information in the process of image reconstruction. And traditional WT compressing adopts the method that discards the high-frequency directly to realize the compressing of image. So it always causes the blur result of reducing image border. To make compressing image have better qualify after reducing, make high-frequency clustering be adopted to resolve blur effect of image border.

3 Relevant algorithms of digital image compressing
3.1 Base theory of WT
WT includes three types mainly: continuous wavelet transform (CWT), wavelet progression launching and discrete wavelet transform (DWT). Let’s introduce the relative theories first.

\[ W_c (a,b) = \int_{-\infty}^{\infty} f(x) \psi_{(a,b)}(x) dx = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} f(x) \psi(\frac{x-b}{a}) dx \]

\[ \psi(x) \] is the wavelet function, \( f(x) \) is the function to be transformed. Because the information of image is discrete, DWT should be adopted. Now provide the wavelet progression launching and DWT.

Given the condition of orthogonal unitary wavelet set, wavelet progression launching of a limited bandwidth continuous function \( f(t) \) is:

\[ C_{j,k} = \int_{-\infty}^{\infty} f(t) \psi_{j,k}(t) dt \]

\[ f(t) = \sum_{j,k} C_{j,k} \psi_{j,k}(t) \]

Coefficient sum and accumulate sum can get through an index \( n=0, 1, \ldots, N-1 \),

\[ h(n) = \sum_{k=0}^{\log2(N)-1} h(k-2n) s_{m-1}(k) \]

\[ g(n) = \sum_{k=0}^{\log2(N)-1} g(k-2n) s_{m-1}(k) \]

\( h(n) \) is low-pass filter; \( g(n) \) is high-pass filter; \( m \) is clays of multi-scale decomposition; \( Sm(n) \) is the Sm-1(n) approximation at 2-m; \( Cm(n) \) is discarded information of signal \( S \) when its approximation is from 2-m+1 to 2-m.

The accurate reconstruction algorithm of above mentioned decomposition algorithm is:

\[ S_{m-1}(k) = \sum_{n} h(k-2n) S_{m}(n) + \sum_{n} g(k-2n) C_{m}(n) \]

3.2 Basic algorithms of Clustering
In order to make the reducing of compressing image look more real and eliminate the blur effect better, the first step is to cluster high-frequency information which contains the information of image border.

Then, use center point to represent the other elements of same kind cluster setting, at last use \( \log2n \) bit information to represent all information in the high-frequency matrix and the compressing of high-frequency information is realized. For example, if high-frequency information was divided into 4 types, 2 bits can be used to represent at least 8 bits information of source image, so compressing of high frequency information could be realized.

In the process of clustering, what is utilized mainly is k-means algorithm, the basic process of this algorithm are:
1) Choose \( k \) objects as initial clusters center randomly;
2) According to average value of each cluster, distribute each object to most similar cluster newly;
3) Update average value of clusters, count average value of all objects in each cluster;
4) If any variety happens, turn to step 2, else algorithm end.
4 Concrete realizing of digital image compressing

The concrete realizing processes of WT used in digital image compressing as follows:

1) Choose a group of average-value wavelet coefficients and a group of variance wavelet coefficients. These two groups wavelet coefficients both are odd column vectors, and the average-value wavelet coefficients are \((a_1, a_2, a_3, a_4, a_5)\), variance wavelet coefficients are \((b_1, b_2, b_3, b_4, b_5)\).

2) Transfer image to corresponding gray-level matrix, and fill \(n/2\) ‘0’ at the beginning and end of each line, ‘\(n\)’ represents the number of wavelet coefficient. For example, to a \(5 \times 5\) image, after transforming, the matrix is:

\[
\begin{bmatrix}
0 & 0 & c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & 0 & 0 \\
0 & 0 & c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & 0 & 0 \\
0 & 0 & c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & 0 & 0 \\
0 & 0 & c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & 0 & 0 \\
0 & 0 & c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & 0 & 0
\end{bmatrix}
\]

3) Do transformation for the matrix. The transformation process is that chooses respective number of column vectors then uses them to make convolution with wavelet coefficients. The results of convolution save at respective position in the matrix, so above-mentioned matrix will be transformed into following one:

\[
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{14} & d_{15} \\
d_{21} & d_{22} & d_{23} & d_{24} & d_{25} \\
d_{31} & d_{32} & d_{33} & d_{34} & d_{35} \\
d_{41} & d_{42} & d_{43} & d_{44} & d_{45} \\
d_{51} & d_{52} & d_{53} & d_{54} & d_{55}
\end{bmatrix}
\]

\(d_y\) is the convolution result by \((c_{i(j-2)}, c_{i(j-1)}, c_y, c_{i(j+1)}, c_{i(j+2)}\)) and wavelet coefficients.

4) The matrix which is formatted by the convolution result of image matrix with average-value wavelet coefficients is low-frequency information of source image; and the matrix which is formatted by the convolution result of image matrix with variance wavelet coefficients is high-frequency information of source image.

5) For low-frequency information, it is reasonable to fetch odd rows as new average-value matrix of image. Because according to the theory of low-pass filters, the new information which is formatted by discontinuous sampling from the transformed source image information also can resume the source low-frequency information completely.

6) For high-frequency information, because the result matrix after convolution wavelet transformation has a lot of ‘0’, so it is reasonable to cluster the elements which valuable is not 0, then compress coding to them.

The reducing processes of WT can be divided into two parts, the reducing processes of low-frequency information and the reducing processes of high-frequency information. The main steps are presented as follows:

1) In processes of reducing low-frequency information, fill the low-frequency transformed information to the same type matrix of source image with 0 firstly, and then do reverse WT.

2) In reducing high-frequency information, do reverse coding firstly, then use the result do contradictory WT.

3) Add low-frequency information and high-frequency information to form final reducing image.

As to the compressing processes with multi-compresses at low-frequency information, up-level low-frequency information should be reduced iteratively until getting final reducing image. As above mentioned, the compressing used WT is realized by filtering out the high-frequency information and make low-frequency information simple, so it is not a kind of lossless compressing. Following wavelet coefficients of WT and reverse WT can be utilized in practical application.

Low frequency WT coefficients:
\[
\{0.0267487, -0.016864, -0.0782234, 0.266864, 0.602949, 0.266864, -0.0782234, -0.016864, 0.0267487\}
\]

Low frequency contradictory WT coefficients:
\[
\{-0.0267487, -0.016864, 0.0782234, 0.266864, -0.602949, 0.266864, 0.0782234, 0.016864, -0.0267487\}
\]

High frequency WT coefficients:
\[
\{0. -0.0456358, -0.0287716, 0.295636, 0.557543, 0.295636, -0.0287716, -0.0456358, 0\}
\]

High frequency contradictory WT coefficients:
\[
\{0, -0.0456358, 0.0287716, 0.295636, -0.557543, 0.295636, 0.0287716, -0.0456358, 0\}
\]

High frequency contradictory WT coefficients:
\[
\{-0.0267487, -0.016864, 0.0782234, 0.266864, -0.602949, 0.266864, 0.0782234, -0.016864, -0.0267487\}
\]
5 Result of experiment

5.1 Basic interface and realized function of digital image compressing software introduce

According FIG.3, this software mainly realizes traditional WT compressing and WT compressing added clustering; exhibits source image, compressed image by traditional WT, compressed image by WT added clustering and compares compress ratio and compress quality between them. As for compressing image by traditional WT, this software just compresses the low-frequency information and discards high-frequency information. As for compressing image by WT compressing added clustering, this software compresses the low-frequency information and high-frequency information, the compressing ratio of high-frequency information is set in menu ‘Compress’. And this software can both compress gray-image and colored image.

5.2 Image compressing result and analysis

From FIG.4 it can be found that to a 61953 pixels image, if using traditional WT only, the reducing image will take the form of indention and the luminance is lower than source image owning to lacks of high-frequency information. According the result of adding clustering high-frequency information, although compressing ratio increased, the quality of compressing is much better than traditional WT compressing, the blur effect is vanished and luminance information is saved, the reducing image will be almost as same as source image. So adding high-frequency information clustering is essential and effective.

6 Conclusion

Digital image compressing based on WT and clustering, saves high-frequency information of source image through making the high-frequency subband on the clusters then encodes compressing to high-frequency information according to the result of the cluster. And this part of information can be reduced by adopting certain filling algorithm, so the blur effect of image border is resolved and image luminance is enhanced. Experiment result indicates that, this method, in condition of guaranteed compressing ratio, enhances the compressing quality of image, solves the blur effect of image border basically, so it is essential and effective.

To make the algorithm performs better; we can do more in the following areas:

1) Consider the difference between Gray-scale information and Colored information of the image. Design some methods to get better compressing ratio by it.
2) Find the relationship between the times of clustering and compressing ratio. Because users always set compressing ratio when they use some image compressing algorithms.

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