Dynamics of Two Resistively Coupled Electric Circuits of 4th Order

M. S. PAPADOPOULOU, I.M. KYPRIANIDIS, and I. N. STOUBOULOS
Physics Department
Aristotle University of Thessaloniki
Thessaloniki, 54124
GREECE

Abstract: - We have studied the case of chaotic synchronization of two identical nonlinear autonomous electric circuits, which are mutually (bidirectionally) coupled via a linear resistor \( R_C \). The two circuits can be either \( \upsilon_C^1 \)-coupled or \( \upsilon_C^2 \)-coupled. Both, experimental and simulation results, have shown that chaotic synchronization is possible only in the case of \( \upsilon_C^2 \)-coupling.

Key-Words: - Nonlinear circuit, Chaos, Attractor, Mutual coupling, Reverse period-doubling.

1 Introduction
Since the discovery by Pecora and Carroll [1], that chaotic systems can be synchronized, the topic of synchronization of coupled chaotic circuits and systems has been studied intensely [2] and some interesting applications, as in broadband communications systems, have come out of this research [3-5]. Generally, there are two methods of chaos synchronization available in the literature. In the first method, due to Pecora and Carroll [1], a stable subsystem of a chaotic system could be synchronized with a separate chaotic system, under certain suitable conditions. The second method to achieve chaos synchronization between two identical nonlinear systems is due to the effect of resistive coupling without requiring to construct any stable subsystem [6-8]. According to Carroll and Pecora [9], periodically forced synchronized chaotic circuits are much more noise-resistant than autonomous synchronized chaotic circuits.

In this paper we have studied the case of two identical fourth-order autonomous nonlinear electric circuits (Fig.1) with two active elements, one linear negative conductance and one nonlinear resistor with a symmetrical piecewise linear \( v - i \) characteristic (Fig.2). Using the capacitances \( C_1 \) and \( C_2 \) as the control parameters, we have observed a reverse period-doubling sequence, as well as a crisis phenomenon, when the spiral attractor suddenly widens to a double-scroll attractor [10].

2 The Nonlinear Circuit
The circuit, we have studied, is shown in Fig.1 and has two active elements, a nonlinear resistor \( R_N \) and a linear negative conductance \( G \), which were implemented using the circuits shown in Figs.3 and 4 respectively. The real \( v - i \) characteristic of the conductance \( G \) is shown in Fig.5.
The real v-i characteristic of the negative conductance $G$. The state equations of the circuit are:

$$
\frac{dv_{C1}}{dt} = \frac{1}{C_1} (i_{L1} - i) \tag{1}
$$

$$
\frac{dv_{C2}}{dt} = -\frac{1}{C_2} (G \cdot v_{C2} + i_{L1} + i_{L2}) \tag{2}
$$

$$
\frac{di_{L1}}{dt} = \frac{1}{L_1} (v_{C2} - v_{C1} - R_i i_{L1}) \tag{3}
$$

$$
\frac{di_{L2}}{dt} = \frac{1}{L_2} (v_{C2} - R_i i_{L2}) \tag{4}
$$

where

$$
i_{L1} = g(v_{C1}) = G_a v_{C1} + 0.5(G_a - G_b)(|v_{C1} + B_{p1}| - |v_{C1} - B_{p1}|) + 0.5(G_b - G_c)(|v_{C1} + B_{p2}| - |v_{C1} - B_{p2}|) \tag{5}
$$

The values of the circuit parameters are: $L_1 = 10.2\, \text{mH}$, $L_2 = 21.5\, \text{mH}$, $R_1 = 2.02\, \Omega$, $R_2 = 108\, \Omega$, $G_a = 0.5\, \text{mS}$, $G_b = 0.5\, \text{mS}$, $L_R = 7.5\, \text{V}$, $G_c = -0.833\, \text{mS}$, $G_b = -0.514\, \text{mS}$, $G_c = 2.9\, \text{mS}$, $B_{p1} = 1.46\, \text{V}$ and $B_{p2} = 10.1\, \text{V}$.

The Bifurcation diagram of the dynamics of the circuit is shown in Fig. 6. We can observe forward and reverse period-doubling cascades, i.e., antimonotonocity [10].

![Autonomous 4th-Order C2 = 5.30nF](image)

The bifurcation diagram $(i_{L2})_p$ vs. $C_1$ for $C_2 = 5.30\, \text{nF}$.

3 Mutual Resistive Coupling

If two identical circuits are resistively coupled, complex dynamics can be observed, as well as chaotic synchronization, as the coupling parameter $R_C$ is varied. The two circuits can be either $v_{C1}$-coupled or $v_{C2}$-coupled, as we can see in Figs. 7 and 8 respectively.

3.1 $v_{C1}$-coupling

The system of the two identical 4th order nonlinear circuits $v_{C1}$-coupled via the linear resistor $R_C$ is shown in Fig. 7. The state equations of the system are:

$$
\frac{dv_{C11}}{dt} = \frac{1}{C_1} \left\{ -\frac{v_{C11} - v_{C12}}{R_C} + i_{L11} - g(v_{C11}) \right\} \tag{6}
$$

$$
\frac{dv_{C21}}{dt} = \frac{1}{C_2} \left\{ -G_v v_{C12} - i_{L11} - i_{L21} \right\} \tag{7}
$$

$$
\frac{di_{L11}}{dt} = \frac{1}{L_1} \left\{ -v_{C11} + v_{C21} - R_i i_{L11} \right\} \tag{8}
$$

$$
\frac{di_{L21}}{dt} = \frac{1}{L_2} \left\{ v_{C21} - R_i i_{L21} \right\} \tag{9}
$$

$$
\frac{dv_{C12}}{dt} = \frac{1}{C_1} \left\{ -\frac{v_{C11} - v_{C12}}{R_C} + i_{L12} - g(v_{C12}) \right\} \tag{10}
$$

$$
\frac{dv_{C22}}{dt} = \frac{1}{C_2} \left\{ -G_v v_{C12} - i_{L12} - i_{L22} \right\} \tag{11}
$$

Fig. 4. The negative conductance implementation

![Fig.4. The negative conductance implementation](image)

Fig. 5. The real v-i characteristic of the negative conductance $G$. The values of the circuit parameters are:

- $L_1 = 10.2\, \text{mH}$
- $L_2 = 21.5\, \text{mH}$
- $R_1 = 2.02\, \Omega$
- $R_2 = 108\, \Omega$
- $G_n = -0.5\, \text{mS}$
- $G_p = -0.5\, \text{mS}$
- $L_R = 7.5\, \text{V}$
- $G_a = -0.833\, \text{mS}$
- $G_b = -0.514\, \text{mS}$
- $G_c = 2.9\, \text{mS}$
- $B_{p1} = 1.46\, \text{V}$
- $B_{p2} = 10.1\, \text{V}$

Fig. 6. The bifurcation diagram $(i_{L2})_p$ vs. $C_1$ for $C_2 = 5.30\, \text{nF}$.

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\begin{align}
\frac{di_{12}}{dt} &= \frac{1}{L_1} \left\{ -v_{c12} + v_{c22} - R_i i_{12} \right\} \quad (12) \\
\frac{di_{22}}{dt} &= \frac{1}{L_2} \left\{ v_{c22} - R_i i_{12} \right\} \quad (13) \\
\end{align}

where \( g(v_{c1j}) \) is given by Eq. (14)

\( g(v_{c1j}) = G(v_{c1j} + 0.5(G_a - G_b)(|v_{c1j} + B| - |v_{c1j} - B|)) + 0.5(G_b - G_a)(|v_{c1j} + B| - |v_{c1j} - B|) \)

and \( j=1, 2 \).

Fig. 7. The \( v_{c1} \)-coupled system of the two identical 4\(^{th}\) order nonlinear circuits.

### 3.2 \( v_{c2} \)-coupling

The system of the two identical 4\(^{th}\) order nonlinear circuits \( v_{c2} \)-coupled via the linear resistor \( R_c \) is shown in Fig. 8. The state equations of the system are:

\begin{align}
\frac{dv_{c11}}{dt} &= \frac{1}{C_1} \left\{ i_{c11} - g(v_{c11}) \right\} \quad (15) \\
\frac{dv_{c21}}{dt} &= \frac{1}{C_2} \left\{ \frac{v_{c21} - v_{c22}}{R_c} - G_0 v_{c21} - i_{111} - i_{121} \right\} \quad (16) \\
\frac{di_{111}}{dt} &= \frac{1}{L_1} \left\{ -v_{c11} + v_{c21} - R_i i_{111} \right\} \quad (17) \\
\frac{di_{121}}{dt} &= \frac{1}{L_2} \left\{ v_{c21} - R_i i_{121} \right\} \quad (18) \\
\frac{dv_{c12}}{dt} &= \frac{1}{C_1} \left\{ i_{c12} - g(v_{c12}) \right\} \quad (19) \\
\frac{dv_{c22}}{dt} &= \frac{1}{C_2} \left\{ \frac{v_{c21} - v_{c22}}{R_c} - G_0 v_{c22} - i_{112} - i_{122} \right\} \quad (20) \\
\frac{di_{122}}{dt} &= \frac{1}{L_1} \left\{ -v_{c12} + v_{c22} - R_i i_{122} \right\} \quad (21) \\
\end{align}

\begin{align}
\frac{di_{122}}{dt} &= \frac{1}{L_2} \left\{ v_{c22} - R_i i_{122} \right\} \quad (22)
\end{align}

Fig. 8. The \( v_{c2} \)-coupled system of the two identical 4\(^{th}\) order nonlinear circuits.

### 4 Dynamics of the Coupled System

We have studied the evolution of the two identical resistively coupled autonomous and nonlinear 4\(^{th}\) order electric circuits from nonsynchronized to synchronized oscillations, when they exhibit chaotic behavior. By "synchronization of chaos" we mean, that the chaotic oscillations of individual circuits are identical to each other.

Using the bifurcation diagram of Fig. 6, we have chosen the values of \( C_1 \) where chaotic behavior is observed and have studied the possibility of chaotic synchronization by plotting the diagrams of \( v_{c22} - v_{c21} \) vs. \( R_1 / R_c \), the coupling factor.

Chaotic synchronization was never observed in the case of \( v_{c1} \)-coupling.

#### 4.1 Chaotic attractors

The chaotic attractors of the two circuits, when they are uncoupled, are shown in the following figures.

In Figs. 9 and 10 we can see the theoretical phase portraits for \( C_2=5.3nF \) and \( C_1=8.0nF \) and \( C_1=8.4nF \) respectively, while in Figs. 11-14 the experimental chaotic attractors are shown, for the two identical circuits.
Fig. 9. Chaotic attractor for $C_2=5.30\text{nF}$ and $C_1=8.00\text{nF}$.

Fig. 10. Chaotic attractor for $C_2=5.30\text{nF}$ and $C_1=8.40\text{nF}$.

Fig. 11. Chaotic attractor for $C_2=5.30\text{nF}$ and $C_1=8.00\text{nF}$ for the first circuit ($i_{L2}$ vs. $v_{C2}$).

Fig. 12. Chaotic attractor for $C_2=5.30\text{nF}$ and $C_1=8.40\text{nF}$ for the first circuit ($i_{L2}$ vs. $v_{C2}$).

Fig. 13. Chaotic attractor for $C_2=5.30\text{nF}$ and $C_1=8.00\text{nF}$ for the second circuit ($i_{L2}$ vs. $v_{C2}$).

Fig. 14. Chaotic attractor for $C_2=5.30\text{nF}$ and $C_1=8.40\text{nF}$ for the second circuit ($i_{L2}$ vs. $v_{C2}$).

4.2 The case of $v_{C2}$-coupling

We have studied the possibility of chaotic synchronization when the initial conditions of the coupled circuits belong to the same basin of attraction.

The diagrams of $v_{C22}$-$v_{C21}$ vs. $R_1/R_C$ are shown in Figs. 15 and 16 correspond to the case, when the initial conditions of the coupled circuits belong in the same basin of attraction.

In Figs. 17-19 we can see the experimental results from the coupled system for $C_2=5.30\text{nF}$, $C_1=8.00\text{nF}$ and various values of the coupling resistor $R_C$.

In Figs. 20-22 we can see the experimental results for $C_2=5.30\text{nF}$ and $C_1=8.40\text{nF}$, as the coupling resistance $R_C$ is varied.

We observe how the system passes from nonsynchronized states to synchronized ones, depending on the coupling resistance $R_C$. 
Fig. 15. The diagram of $v_{C22}$ vs. $R_1/R_C$, for $C_2=5.30\text{nF}$ and $C_1=8.00\text{nF}$.

Fig. 16. The diagram of $v_{C22}$ vs. $R_1/R_C$, for $C_2=5.30\text{nF}$ and $C_1=8.40\text{nF}$.

Fig. 17. $v_{C2}$-coupling for $C_2=5.30\text{nF}$, $C_1=8.00\text{nF}$ and $R_C=\infty$ ($v_{C22}$ vs. $v_{C21}$).

Fig. 18. $v_{C2}$-coupling for $C_2=5.30\text{nF}$, $C_1=8.00\text{nF}$ and $R_C=49\text{K}\Omega$ ($v_{C22}$ vs. $v_{C21}$).

Fig. 19. $v_{C2}$-coupling for $C_2=5.30\text{nF}$, $C_1=8.00\text{nF}$ and $R_C=1.3\text{K}\Omega$ ($v_{C22}$ vs. $v_{C21}$). Chaotic synchronization.

Fig. 20. $v_{C2}$-coupling for $C_2=5.30\text{nF}$, $C_1=8.40\text{nF}$ and $R_C=\infty$ ($v_{C22}$ vs. $v_{C21}$).

Fig. 21. $v_{C2}$-coupling for $C_2=5.30\text{nF}$, $C_1=8.40\text{nF}$ and $R_C=40\text{K}\Omega$ ($v_{C22}$ vs. $v_{C21}$).

Fig. 22. $v_{C2}$-coupling for $C_2=5.30\text{nF}$, $C_1=8.40\text{nF}$ and $R_C=1.5\text{K}\Omega$ ($v_{C22}$ vs. $v_{C21}$). Chaotic synchronization.
5 Conclusion

In this paper we have shown the process of chaotic synchronization using computer simulation and experiment.

We have studied the dynamics of the individual nonlinear autonomous electric circuit and determined the parameter’s region, where the circuit exhibits chaotic behavior. Furthermore, we observed the chaotic synchronization of two identical circuits, which are mutually coupled via a linear resistor $R_C$, when the initials conditions of the coupled circuits belong to the same basin of attraction. Both, experimental and simulation results have shown that chaotic synchronization is possible in the case of $\upsilon_{C2}$-coupling and not possible in the case of $\upsilon_{C1}$-coupling. In the first case, the coupling is established at the node of the linear part of the autonomous circuit, while in the second case the coupling is established at the node of the nonlinear part. The transferred energy from the coupling branch is not enough to synchronize the coupled system in the second case, as it has been verified by simulation and experiment. This is also enhanced in the case when conductance $G_n$ becomes nonlinear. The increased complexity of the system does not allow synchronization, even in the case of $\upsilon_{C2}$-coupling configuration [11].

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References:
