Proceedings of the 7th WSEAS International Conference on Automation & Information, Cavtat, Croatia, June 13-15, 2006 (pp83-88)

Adaptive Nonlinear Control Algorithms for Robotic Manipulators

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Abstract: - In this paper some adaptive nonlinear multivariable techniques used in the control of robotic manipulators are presented. The nonlinear control law and state feedback are used in achieving a linear input-output behavior for the controlled system. For the design of the adaptive nonlinear control, the exact feedback input-output linearization and the method of gradient are used. The nonlinear control law achieves also decoupling. Computer simulations are included to demonstrate some theoretical aspects and the performances of these controllers for a typical structure of robotic manipulator.

Key-Words: - Robotic arm, Nonlinear control, Linearizing control, Adaptive control

1 Introduction

The control of the robotic manipulators is an important area for research, development, and manufacturing. If we consider some approximations on the robot dynamical model we can do a linear analysis of the manipulator control problem. Without these approximations we have a nonlinear model. The field of nonlinear control theory is large (a lot of methods of control): the computed torque method, the robust control method, the adaptive control method [1], the force control method etc. (see [4], [6], [7]). The control requires the knowledge of a mathematical model and of some sort of intelligence to act on the model. The model of a robot is obtained from the basic physical laws governing its movement. There are many methods to obtain the dynamical model (see [5], [7], [9]): Lagrange method, Euler method, d'Alembert method, Kane method etc. Here is used the Lagrange method to obtain the dynamical model for a robot which works in cylindrical coordinates.

In the last years, significant advances have been made in the development of ideas such as feedback linearizing and input-output decoupling techniques ([3], [6]). In this paper, by using the feedback linearizing techniques, a multivariable nonlinear control law is obtained for a robotic manipulator - widely discussed in [2] for both monovariable and multivariable cases. In many practical situations, some robotic manipulator parameters are unknown; therefore an adaptive control strategy is required in order to maintain the performances of the controlled system. In this paper, an adaptive control law based on reference model for the exactly linearized model is also designed.

The paper is organized as follows: in Section 2, some basics of the exact linearization theory are presented. In Section 3, mathematical models of robotic manipulators are analysed, while in Section 4 the adaptive nonlinear controllers are developed and Section 5 include computer simulation. Finally, Section 6 collects the conclusions.

2 The Statement of the Exact Linearization Problem

A multivariable nonlinear system can be described in state space by equations of the following kind:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} g_i(x) u_i \\ y_j &= h_j(x) \qquad j = 1...m \end{aligned} \tag{1}$$

in which f(x), g_1(x), g_2(x),...., g_m(x) are smooth vector fields.

The problem of exact linearization via feedback and diffeomorphism consists in transforming a nonlinear system (1) into a linear one using a state feedback and a coordinate transformation of the systems state. The exact feedback linearization theory is widely presented by [6]. Next, some basic results of this theory are presented. These results are applied in Section 4, where adaptive nonlinear control laws are developed for robotic manipulators.

Consider the Lie derivative of a function

$h(x): R^n \to R$ along a vector field $f(x)$:

$$L_f h(x) = \sum_{i=1}^{n} \frac{\partial h(x)}{\partial x_i} f_i(x) \tag{2}$$

Definition. A multivariable nonlinear system of the form (1) has a relative degree $\{r_1, ..., r_m\}$ at a point x^0 if:

1) $L_{g_j} L_f^k h_i(x) = 0$ (3)

for all $1 \le j \le m$, for all $1 \le i \le m$ for all $k < r_i - 1$, and for x in a neighborhood of x^0,
2) the $m \times m$ matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_2(x) & . \quad . & L_{g_2} L_f^{r_1-1} h_1(x) \\ L_{g_1} L_f^{r_2-1} h_2(x) & . \quad . & L_{g_2} L_f^{r_2-1} h_2(x) \\ . & . \quad . & . \\ L_{g_1} L_f^{r_m-1} h_m(x) & . \quad . & L_{g_m} L_f^{r_m-1} h_m(x) \end{bmatrix} \tag{4}$$

is nonsingular at $x = x^0$.

Theorem. Let be the nonlinear system of the form (1). Suppose the matrix $g(x^0)$ has rank m. Then, the State Space Exact Linearization Problem is solvable if and only if:
1) for each $0 \le i \le n-1$, the distribution G_i has constant dimension near x^0;
2) the distribution G_{n-1} has dimension n;
3) for each $0 \le i \le n-2$, the distribution G_i is involutive.

3 Mathematical Model of Robotic Manipulators

We consider the robot manipulator with three axes described in Fig. 1, which is driven by a d.c. motor controlled in current. For this robot arm, which works in cylindrical coordinates, the kinetic energy is:

$$K = \frac{1}{2}\left(I_1 + I_2 + I_3 + m_3 q_3^2\right)\dot{q}_1^2 + \frac{1}{2}(m_2 + m_3)\dot{q}_2^2 + \frac{1}{2} m_3 \dot{q}_3^2 \tag{5}$$

The potential energy is:

$$P = (m_2 + m_3) g \tag{6}$$

Lagrange's equations of motion for a conservative system are given by:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \left(\frac{\partial L}{\partial q}\right) = \tau \tag{7}$$

where q is an n-vector of generalized coordinates q_i, τ is an n-vector of generalized forces τ_i, and the Lagrangian (L) is the difference between the kinetic (K) and potential (P) energies.

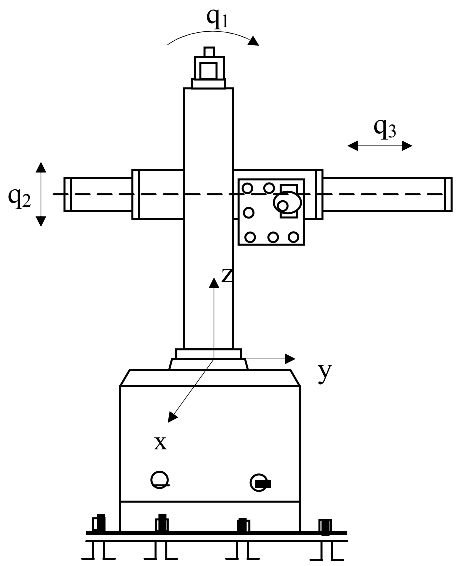

Fig. 1. Structure of a robotic manipulator

Now, we shall use Lagrange's equation to derive the general robot arm dynamics. The system is characterized by a set of three first order differential equations:

$$\begin{aligned} &\left(I_1 + I_2 + I_3 + m_3 q_3^2\right)\ddot{q}_1 + 2 m_3 q_3 \dot{q}_1 \dot{q}_3 = \tau_1 \\ &(m_2 + m_3)\ddot{q}_2 = \tau_2 - (m_2 + m_3) g \\ &m_3 \ddot{q}_3 - m_3 q_3 \dot{q}_1^2 = \tau_3 \end{aligned} \tag{8}$$

where I_1, I_2, I_3 represent the moments of inertia of the solids with respect to the axis *z;* m_2, m_3 are the solids' masses; τ_1, τ_2, τ_3 are the generalized forces.

(i). For the beginning we consider $q_2 = 0$ and we note $I = I_1 + I_2 + I_3$. The state equations are the following:

$$\dot{x} = f(x) + \sum_{i=1}^{2} g_i(x) u_i \tag{9}$$

where

$$f(x) = \begin{pmatrix} x_3 \\ x_4 \\ -\dfrac{2 m_3 x_2 x_3 x_4}{I + m_3 x_2^2} \\ x_2 x_3^2 \end{pmatrix}; \; g(x) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ \dfrac{1}{I + m_3 x_2^2} & 0 \\ 0 & \dfrac{1}{m_3} \end{pmatrix} \tag{10}$$

$x^T = [q_1, q_3, \dot{q}_1, \dot{q}_3]$ and $u^T = [u_1, u_2] = [\tau_1, \tau_3]$

For the system (9), we consider as output variables the generalized coordinates q_1 and q_3:

$$\begin{aligned} y_1 &= h_1(x) = q_1(t) = x_1(t) \\ y_2 &= h_2(x) = q_3(t) = x_2(t) \end{aligned} \tag{11}$$

In this situation, the mathematical model is multivariable and it has two inputs and two outputs.

(ii). If $q_2 \neq 0$, the state equations are the following:

$$\begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{3} g_i(x) u_i \\ y_i &= h_i(x) = x_i, \; i = 1,2,3 \end{aligned} \tag{12}$$

$x^T = [q_1, q_2, q_3, \dot{q}_1, \dot{q}_2, \dot{q}_3]$ and $u^T = [\tau_1, \tau_2, \tau_3]$

$$f(x) = \begin{pmatrix} x_4 \\ x_5 \\ x_6 \\ \dfrac{-2m_3 x_3 x_4 x_6}{I + m_3 x_3^2} \\ -g \\ x_3 x_4^2 \end{pmatrix}$$

$$g(x) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \dfrac{1}{I + m_3 x_3^2} & 0 & 0 \\ 0 & \dfrac{1}{m_2 + m_3} & 0 \\ 0 & 0 & \dfrac{1}{m_3} \end{pmatrix} \tag{13}$$

In this situation, the mathematical model is multivariable and it has three inputs and three outputs.

4 Adaptive Nonlinear Control Laws

The mathematical model in the multivariable case (i) is of the form (9), but where the inputs are the generalized coordinates τ_1 and τ_3. In this situation, we consider as output variables the generalized coordinates q_1 and q_3:

$$\begin{aligned} y_1 &= h_1(x) = x_1(t) \\ y_2 &= h_2(x) = x_2(t) \end{aligned} \tag{14}$$

For this system we have decoupling matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^1 h_1(x) & L_{g_2} L_f^1 h_1(x) \\ L_{g_1} L_f^1 h_2(x) & L_{g_2} L_f^1 h_2(x) \end{bmatrix} = \begin{bmatrix} \dfrac{1}{I + m_3 x_2^2} & 0 \\ 0 & \dfrac{1}{m_3} \end{bmatrix} \tag{15}$$

and the nonlinearities canceling vector is

$$b(x) = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \end{bmatrix} = \begin{bmatrix} -\dfrac{2m_3 x_2 x_3 x_4}{I + m_3 x_2^2} \\ x_2 x_3^2 \end{bmatrix} \tag{16}$$

Using relations (15) and (16), the input-output system can be written in the form:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{bmatrix} = b(x) + A(x) \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \tag{17}$$

An easy calculus shows that the matrix for mathematical model of the robot is nonsingular and the (vector) relative degree is *{r₁, r₂} = {2, 2}*. Because the decoupling matrix (15) is not singular, it is possible to design a nonlinear input:

$$\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = A^{-1}(x) \cdot \begin{bmatrix} -L_f^2 h_1(x) + v_1 \\ -L_f^2 h_2(x) + v_2 \end{bmatrix} \tag{18}$$

such that the obtained linear system has the transfer matrix:

$$H(s) = \begin{bmatrix} \dfrac{1}{s^2} & 0 \\ 0 & \dfrac{1}{s^2} \end{bmatrix} \tag{19}$$

Imposing on the linear system an additional feedback of the form:

$$\begin{aligned} v_1 &= c_1^0 \cdot (q_{1ref} - x_1) - c_1^1 \cdot x_3 \\ v_2 &= c_2^0 \cdot (q_{3ref} - x_2) - c_2^1 \cdot x_4 \end{aligned} \tag{20}$$

then, the obtained system has a linear input-output behavior, described by the following diagonal transfer function matrix

$$H(s) = \begin{bmatrix} \dfrac{c_1^0}{s^2 + c_1^1 s + c_1^0} & 0 \\ 0 & \dfrac{c_2^0}{s^2 + c_2^1 s + c_2^0} \end{bmatrix} \tag{21}$$

In the multivariable case (ii), for the system (12), we consider as output variables the generalized coordinates q_1, q_2 and q_3:

$$y_1 = h_1(x) = x_1(t); \; y_2 = h_2(x) = x_2(t)$$
$$y_3 = h_3(x) = x_3(t) \tag{22}$$

For this system we have decoupling matrix

$$A(x) = \begin{bmatrix} L_{g_1} L_f^1 h_1(x) & L_{g_2} L_f^1 h_1(x) & L_{g_3} L_f^1 h_1(x) \\ L_{g_1} L_f^1 h_2(x) & L_{g_2} L_f^1 h_2(x) & L_{g_3} L_f^1 h_2(x) \\ L_{g_1} L_f^1 h_3(x) & L_{g_2} L_f^1 h_3(x) & L_{g_3} L_f^1 h_3(x) \end{bmatrix} =$$
$$= \begin{bmatrix} \frac{1}{I + m_3 x_3^2} & 0 & 0 \\ 0 & \frac{1}{m_2 + m_3} & 0 \\ 0 & 0 & \frac{1}{m_3} \end{bmatrix} \tag{23}$$

and the nonlinearities canceling vector:

$$b(x) = \begin{bmatrix} L_f^2 h_1(x) \\ L_f^2 h_2(x) \\ L_f^2 h_3(x) \end{bmatrix} = \begin{bmatrix} \frac{-2m_3}{I + m_3 x_3^2} x_3 x_4 x_6 \\ -g \\ x_3 x_4^2 \end{bmatrix} \tag{24}$$

Using relations (23) and (24), the input-output system can be written in the form:

$$\begin{bmatrix} \ddot{y}_1 \\ \ddot{y}_2 \\ \ddot{y}_3 \end{bmatrix} = b(x) + A(x) \cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{25}$$

An easy calculus shows that the matrix for mathematical model of the robot is nonsingular and the (vector) relative degree is $\{r_1, r_2, r_3\} = \{2, 2, 2\}$. Because the decoupling matrix (23) is not singular, it is possible to design a nonlinear input:

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = A^{-1}(x) \cdot \begin{bmatrix} -L_f^2 h_1(x) + v_1 \\ -L_f^2 h_2(x) + v_2 \\ -L_f^2 h_3(x) + v_3 \end{bmatrix} \tag{26}$$

such that the obtained linear system has the transfer matrix:

$$H(s) = \begin{bmatrix} \frac{1}{s^2} & 0 & 0 \\ 0 & \frac{1}{s^2} & 0 \\ 0 & 0 & \frac{1}{s^2} \end{bmatrix} \tag{27}$$

Imposing on the linear system an additional feedback of the form:

$$v_1 = c_1^0 \cdot (q_{1ref} - x_1) - c_1^1 \cdot x_4$$
$$v_2 = c_2^0 \cdot (q_{2ref} - x_2) - c_2^1 \cdot x_5 \tag{28}$$
$$v_3 = c_3^0 \cdot (q_{3ref} - x_3) - c_3^1 \cdot x_6$$

the obtained decoupled closed-loop system has a desired behavior.

The implementation of the obtained nonlinear control laws (i.e. (18), (20) for the first case and (26), (28) for the second case) is hampered if some of robot parameters are unknown or variable in time (slowly). In order to overcome this disadvantage, an adaptive control law, based on reference model approach, can be designed. For the synthesis of the adaptive algorithm, the method of the gradient is used, choosing the following criterion ([8]):

$$Q_t = \frac{1}{2} e^T He \tag{29}$$

where $e(t) = x(t) - x_m(t)$ and matrix H > 0 is the solution of the Lyapunov equation

$$HA_m + A_m^T H = -G \tag{30}$$

where G is a symmetric positive definite matrix and A_m is reference model matrix. The adaptive algorithm will be:

$$\frac{dc}{dt} = -Dg_c \Psi(x, c, t) \tag{31}$$

where D is a positive definite matrix and $g_c^T \Psi = \left[\ldots\ldots \frac{\partial \Psi}{\partial c_i} \ldots\ldots \right]$ is the gradient of Ψ in rapport with c_i parameters.

The adaptation law for the controller parameters is of the form

$$\frac{dc_i^0}{dt} = -\gamma_i^0 \left[g_i^0 \left(y_i - y_i^m \right) + g_i^1 \left(\dot{y}_i - \dot{y}_i^m \right) \right] y_i$$
$$\frac{dc_i^1}{dt} = -\gamma_i^1 \left[g_i^0 \left(y_i - y_i^m \right) + g_i^1 \left(\dot{y}_i - \dot{y}_i^m \right) \right] \dot{y}_i \qquad i=1,2,3 \tag{32}$$

For the mathematical models (9), (18), (20) and (32), respectively (12), (26), (28) and (32) of controlled robotic manipulators, we choose as a reference model a transfer function of order two associated with the Integral of Time – Multiplied Absolute Value of Error (ITAE) criterion.

5 Simulation Results

Two simulation cases were considered in order to test the performances of the proposed adaptive nonlinear controllers.

i) The simulation was done for the model equations (9), (10), the nonlinear control law (18), (20) and the adaptation law (32). The performance of the controlled system is presented in Fig. 2–Fig.6. The evolution of angular position is presented in Fig. 2 and the position in Fig. 3. The reference model output versus the real output and the control input are presented only for the first input-output channel, in Fig. 4 and Fig. 5 respectively. In Fig. 6 the plane trajectory of the robot arm is presented.

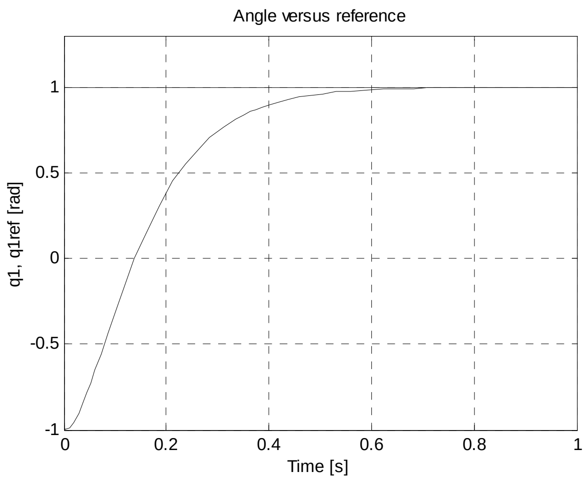

Fig. 2. Angular position – case (i)

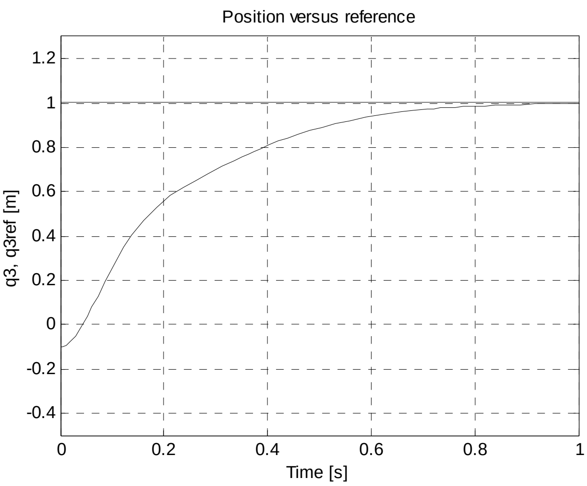

Fig. 3. Evolution of the position – case (i)

(ii) The simulation was done for the model equations (12), (13), the nonlinear control law (26), (28) and the adaptation law (32). The performance of the controlled system is presented in Fig. 7 – Fig. 9. Fig. 7 shows the time evolution of the generalized coordinate q_1 – the angle - versus reference. In Fig. 8 the reference model output versus the real output is depicted for the first channel. Finally, Fig. 9 shows the control input u_1.

We studied in both simulation cases the convergence of the controller parameters c_i for the situations when one or more parameters of the process are varying in time. It was resulted a quickly convergence of these parameters.

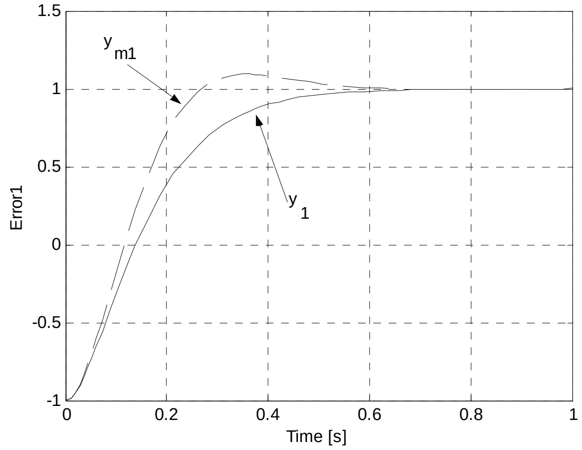

Fig. 4. Reference model output versus the real output

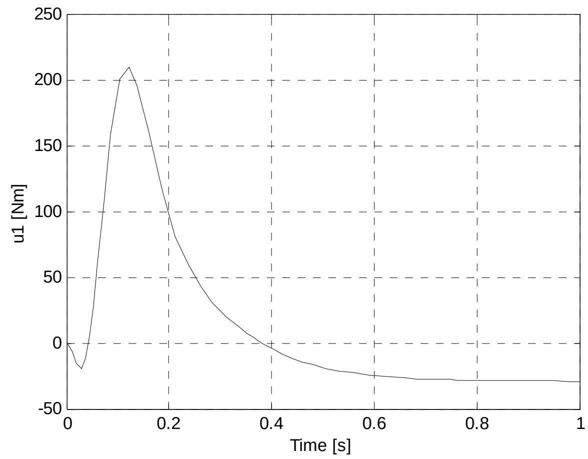

Fig. 5. Control input 1 for the case (i)

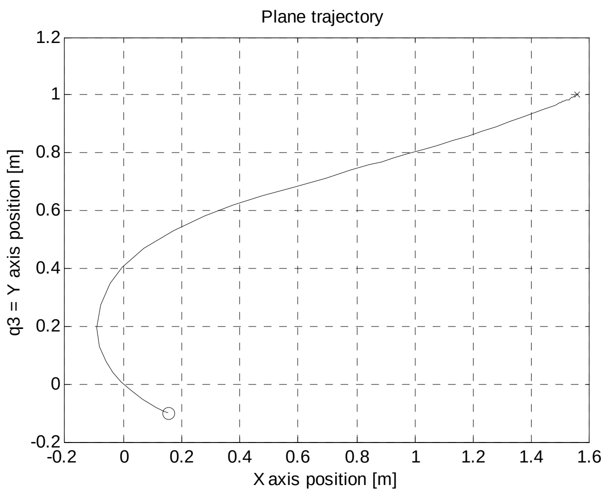

Fig. 6. Plane trajectory – case (i)

The both simulation cases show that the obtained performance is good, we have very small overshoots, the settling times are small and the evolutions of the commands are acceptable.

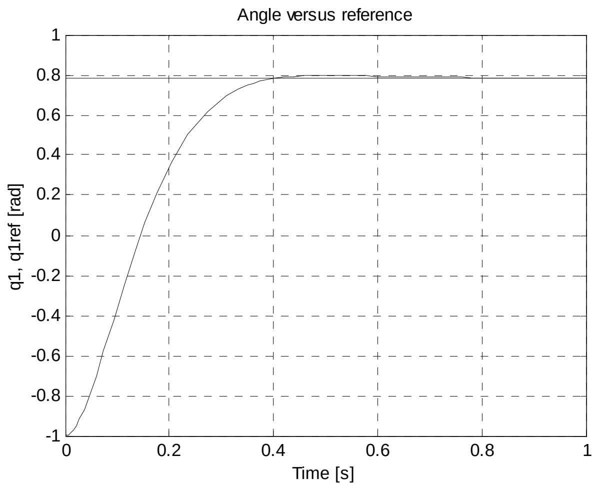

Fig. 7. Evolution of the angle – case (ii)

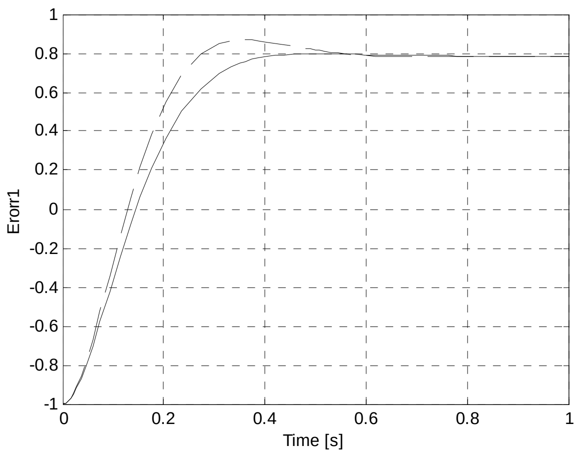

Fig. 8. Reference model output versus the real output

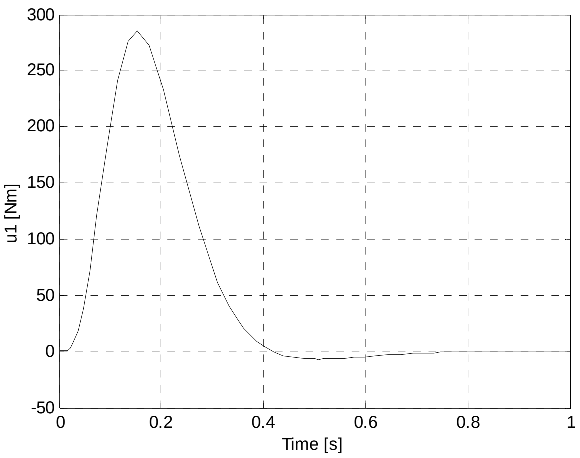

Fig. 9. Control action u_1 – case (ii)

6 Concluding Remarks

In this paper an adaptive nonlinear linearizing control technique for robotic manipulators was presented. The design of the nonlinear control law uses the exact feedback input-output linearization. The models of robotic manipulators are studied in order to implement of the control laws. Using multivariable modelling and control design, exact linearizing controllers are obtained. An adaptive control law, based on reference model approach is designed in order to overcome the disadvantage of parametric uncertainties. For the synthesis of the adaptive algorithm, the method of the gradient is used. Computer simulation is performed in order to test and validate the proposed adaptive nonlinear controllers. From the simulation results it can be seen a good behavior of the systems.

7 Acknowledgements

This work was partially supported from the grant no. 279, CNCSIS, 2006.

References:

[1] K.J. Astrom, Adaptive Feedback Control, *IEEE Transactions on Automatic Control*, Vol.75, No.2, 1987, pp. 185-217.

[2] E. Bobaşu, Nonlinear Algorithms for the Robot Control, *Annals of University of Craiova, Electrical Eng. Series*, no. 26, 2002, pp. 33-44.

[3] A.J. Fossard, D. Normand-Cyrot, *Systemes nonlineaires*, Masson, Paris, 1993.

[4] E. Freund, The Structure of Decoupled Non Linear Systems, *International Journal of Control*, Vol. 21, no. 3, 1975, pp. 443-450.

[5] B. Gorla, M. Renaud, *Modeles des Robots manipulateurs: Aplication a leur commande.* Cepadues- Editions, Toulouse, France, 1984.

[6] A. Isidori, *Nonlinear Control Systems*, 3rd edition, Springer-Verlag, Berlin, 1995.

[7] M. Ivănescu, *Industrial Robots – Control Systems and Algorithms*, Universitaria, Craiova, 1994.

[8] I.D. Landau, R. Lozano, M. M'Saad, *Direct Adaptive Control Algorithm: Theory and Applications*, Springer-Verlag, 1988.

[9] J. R. Schilling, *Fundamentals of Robotics. Analysis and Control,* Prentice Hall, 1990.

[10] *** *MATLAB – Reference Book*, The Mathworks Inc., Natick, MA, 2000.