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Modelling and Robust Control of an Electrohydraulic System

DAN POPESCU, EUGEN BOBASU
Department of Automatic Control
University of Craiova
13 A.I. Cuza, 200585, Craiova
ROMANIA
http://www.ace.ucv.ro

Abstract: - The problem of the robust control of an electrohydraulic system is considered. First, the nonlinear model of an electrohydraulic system is deduced and then a series of linearized models are obtained. In order to obtain a low order compensator, a reduced order model is used in the controller design. The approximation error of the initial model by this reduced order model is treated as an uncertainty. The two-degree of freedom compensator technique is used. Some simulation results are presented.

Key-Words: - **E**lectrohydraulic systems, Linearizing, Robust stability, Optimal control

1 Introduction

Most of the real dynamic systems lead to nonlinear mathematical models. The study of all possible evolutions of these systems represents a very complicated problem. The simple integration of evolution equations using the numeric methods is not the right solution thanks to the dependence of the movement's character to the parameters and to the initial conditions. In order to benefit of the understanding of the phenomena that participate and correlate in a dynamic process - by interpretations which are offered only by analytic studies of the mathematical model - where developed a series of methods of linearization of nonlinear models.

The electrohydraulic system that is analyzed in this paper consists of a two-stage flow control servovalve and a double-ended actuator. The servovalve has a symmetrical double-nozzle and a torque-motor driven flapper for the first stage, and a closed center fourway sliding spool for the second stage.

The organization of the paper is as follows. The nonlinear model is presented and is linearized around its equilibrium state, resulting in the cascaded servovalve linear model and actuator linear model, respectively. Various features of the derived model are discussed, and this improved model is compared with existing linear models. Robust controllers are synthesized based on the derived models and, also, for reduced order models.

The simulation results demonstrate that the control system designed based on the reduced order models assure stability robustness and corresponding performance.

2 Mathematical Model of the Electrohydraulic System

The electrohydraulic system shown in Fig. 1 consists of a two-stage flow control servovalve and a double-ended actuator. The servovalve has a symmetrical double-nozzle and a torque-motor driven flapper for the first stage, and a closed center four-way sliding spool for the second stage.

The differential equations governing the dynamical behavior of this electrohydraulic system are given in the following subsections.

2.1 Torque Motor Dynamics

$$M_a - M_r = J\frac{d^2\theta}{dt^2} + B_v\frac{d\theta}{dt} \tag{1}$$

where:

$$M_a = K_1 \cdot i + K_2 \cdot \theta \tag{2}$$

$$M_r = K_a \cdot \theta + F_{Rj} \cdot l \tag{3}$$

$$F_{Rj} = \Big\{ p_{c1} - p_{c2} + \frac{16{c_d}^2}{d^2}\Big[(u_0 - u)^2 p_{c1} - (u_0 + u)^2 p_{c2}\Big]\Big\} A_c \tag{4}$$

$u = l \cdot \theta$

J - moment of inertia of torque motor;

$A_c = A_{c1} = A_{c2}$ - cross-sectional area of orifice;

K_1 - torque-motor gain;

K_2 - electromagnetic rotational stiffness;

K_a - rotational stiffness of flexure tube;

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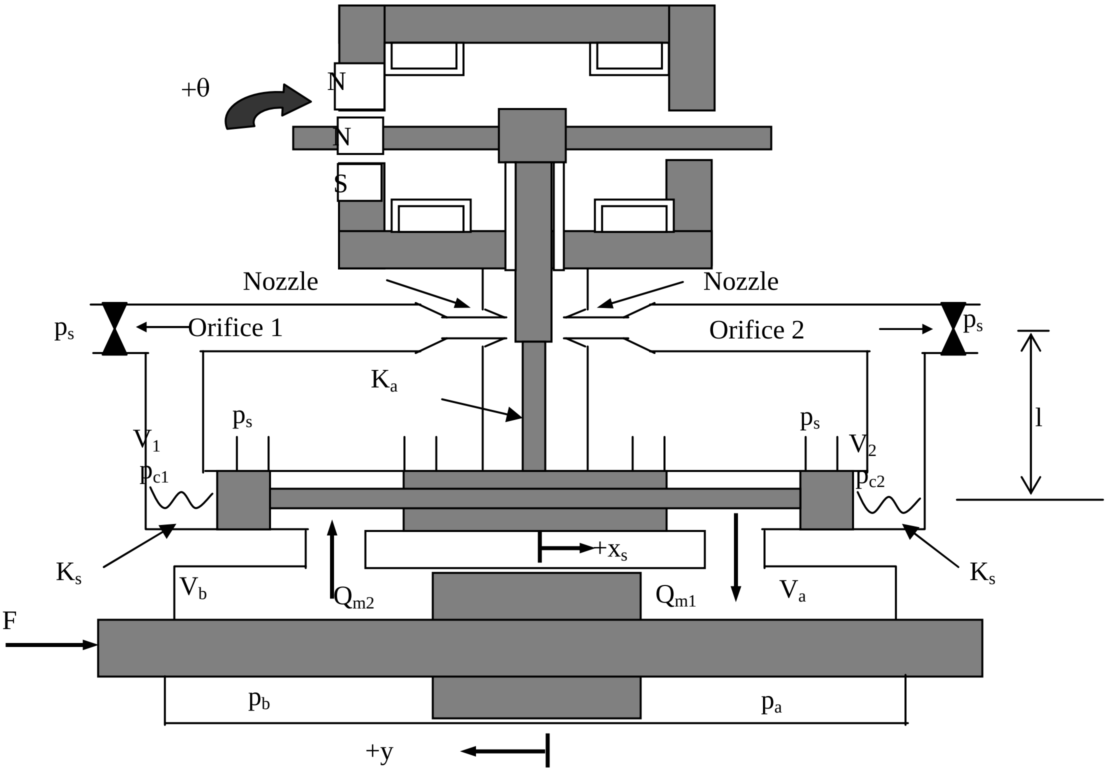

Fig. 1. Structure of the electrohydraulic system

B_v - damping coefficient of torque-motor;
θ - angular position of armature/flapper;
p_{c1}, p_{c2} - presures on left and right side of spool, respectively;
l - distance from nozzle to the pivot point of the flexure tube;
u_0 - maximum flapper displacement;
i - input current to torque-motor;
u - spool position;
c_d - nozzle flow coefficient;
d - diameter of nozzle.

2.2 Flapper-Nozzle Stage Dynamics

$$\dot{p}_{c1} = \frac{E}{V_1}\left[c_0 A_c \sqrt{\frac{2}{\rho}(p_s - p_{c1})} - \right.$$
$$\left. - c_d \pi d (u_0 - u)\sqrt{\frac{2}{\rho} p_{c1}} - S\dot{x}_s \right] \tag{5}$$

$$\dot{p}_{c2} = \frac{E}{V_2}\left[c_0 A_c \sqrt{\frac{2}{\rho}(p_s - p_{c2})} - \right.$$
$$\left. - c_d \pi d (u_0 + u)\sqrt{\frac{2}{\rho} p_{c2}} + S\dot{x}_s \right] \tag{6}$$

where:

E - bulk-modulus of fluid;
c_0 - orifice flow coefficient;
p_s - supply pressure;
S - area of spool valve;
x_s - spool position ($\dot{x}_s = v_s$ spool velocity);
ρ - density of fluid;

$$V_1 = V_{10} + S \cdot x_s \tag{7}$$

$$V_2 = V_{20} - S \cdot x_s \tag{8}$$

$V_{10} = V_{20}$ - enclosed volume on each side of spool when $x_s = 0$.

2.3 Spool Dynamics

$$\dot{v}_s = \frac{1}{m}\left[S(p_{c1} - p_{c2}) - B_s \cdot v_s - 2K_s \cdot x_s \right] \tag{9}$$

where:

m - spool mass;
B_s - damping coefficient of spool;
K_s - stiffness each direct feedback spring at the spool.

2.4 Continuity in Cylinder Chambers

$$\dot{p}_a = \frac{E}{V_a(y)}\left[Q_{m1} - S_a \cdot \dot{y}(t)\right] \quad (10)$$

$$\dot{p}_b = \frac{E}{V_b(y)}\left[S_b \cdot \dot{y}(t) - Q_{m2}\right] \quad (11)$$

$$Q_{m1} = c_{dc} a x_s \sqrt{\frac{2}{\rho}\left(p_s - p_a\right)}\,; Q_{m2} = c_{dc} a x_s \sqrt{\frac{2}{\rho} p_b} \quad (12)$$

where:

p_a, p_b - pressure in left and right cylinder chambers, respectively;

$$V_a(y) = V_{a0} + S_a y\,; V_b(y) = V_{b0} - S_b y \quad (13)$$

$V_{a0} = V_{bo}$ - enclosed volumes on each side of actuator when $y = 0$;

y - actuator position;

$S_a = S_b$ - effective areas of double-ended piston;

$v = \dot{y}$ - actuator velocity;

a - area gradient (flow area/spool displacement);

c_{dc} - orifice flow coefficient.

2.5 Piston Dynamics

$$\dot{v} = \frac{1}{M}\left[\left(p_a - p_b\right)S_a - B_a v - F\right] \quad (14)$$

where:

M - piston mass;

B_a - damping coefficient of actuator;

F - disturbance force input on actuator.

2.6 The Linearized Model

The derivation of the linearized model with respect to an equilibrium state from the above nonlinear model is tedious but straigh forward.

The equilibrum states are derived for zero inputs, $i = 0$, $F = 0$: $\theta = 0$; $u = 0$; $x_s = 0$, $y = 0$, $v = 0$.

We obtain

$$p_{c10} = p_{c20} = \frac{c_0^2 A_c^2}{c_0^2 A_c^2 + c_d^2 \pi^2 d^2 u_0^2} \cdot p_s = \frac{1}{1 + \frac{c_d^2 \pi^2 d^2 u_0^2}{c_0^2 A_c^2}} \cdot p_s \quad (15)$$

From design reasons the following condition it is imposed

$$c_0^2 A_c^2 = c_d^2 \pi^2 d^2 u_0^2, \quad (16)$$

so, we obtain

$$p_{c10} = p_{c20} = \frac{p_s}{2}$$

Linearizing the mathematic model of electrohydraulic system around the equilibrum position, we obtain the linearised mathematical model.

First, the equations (5) and (6) are linearized and the Laplace transform is applied:

$$(s + A_2)\Delta P_c(s) = 2A_1\Theta(s) - 2A_3 sU(s) \quad (17)$$

Then, the equations (1)...(4) are linearized, and the Laplace transform is applied:

$$(s^2 + A_5 s + A_4)\Theta(s) = A_7 I(s) - A_6 \Delta P_c(s) \quad (18)$$

Now, the equation (9) is linearized, and the Laplace transform is applied:

$$(s^2 + A_{10} s + A_9) X_s(s) = A_8 \Delta P_c(s) \quad (19)$$

where:

$$A_1 = c_d \pi d l \frac{E}{V_{10}} \sqrt{\frac{2 p_{c0}}{\rho}}$$

$$A_2 = \frac{E c_0 A_c}{V_{10}\sqrt{2\rho(p_s - p_{c0})}} + \frac{E c_d \pi d u_0}{V_{10}\sqrt{2\rho p_{c0}}}$$

$$A_3 = \frac{E}{V_{10}} A_c\,;\ A_4 = \frac{K_a - K_2 + 16\pi c_d^2 l^2 u_0^2 p_{c0}}{J}$$

$$A_5 = \frac{B_v}{J} \quad (20)$$

$$A_6 = \frac{A_c l + 4\pi c_d^2 u_0^2 l}{J}\,;\ A_7 = \frac{K_1}{J}$$

$$A_8 = \frac{S}{m};\ A_9 = \frac{2K_s}{m};\ A_{10} = \frac{B_s}{m}.$$

From equations of continuity (10) - (13), after linearization and applying the Laplace transform, one obtain:

$$s\Delta P(s) = 2\frac{E}{V_{a0}}\sqrt{\frac{p_s}{\rho}} c_{dc} a X_s(s) - 2\frac{E}{V_{ao}} S_a s Y(s) \quad (21)$$

Finally, the equation (14) is linearized and the Laplace transform is applied:

$$\left[s^2 + \frac{B_a}{M}s\right]Y(s) = \Delta P(s)\frac{S_a}{M} - \frac{F(s)}{M} \tag{22}$$

The linearized actuator model can be obtain as follows:

$$\frac{Y(s)}{X_s(s)} = \frac{d_0}{s^3 + c_2 s^2 + c_1 s} \tag{23}$$

and the linearized model of the two-stage servovalve it is of the form:

$$\frac{X_s(s)}{I(s)} = \frac{-b_1 s + b_0}{s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{24}$$

So, the transfer function of the resulting model of the electrohydraulic system is:

$G(s) = \dfrac{Y(s)}{I(s)}$ (of order 8).

3 Two-Degree of Freedom Controller

Now, consider the overall control system represented by the configuration of Fig. 1, with a two-degree of freedom controller (R, S, T). The design objective is to specify the two-degree of freedom controller to achieve the following two aims:

a) The controller robustly stabilizes the nominal model $G_0(s)$, with robust stability-degree assignment, against the neglected dynamics $\Delta G(s)$, by specifying *R*(*s*) and *S*(*s*).

b) The transfer function from *r* to *y* is as close to the desired model *M*(*s*) as possible via an adequately chosen *T*(*s*).

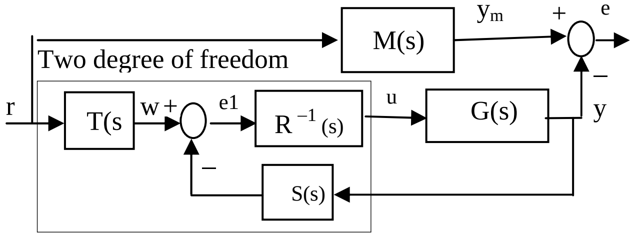

Fig. 1. The overall control system

3.1 Robust Stability Degree Assignment

We consider o stable function, with minimum phase, $\alpha(s)$, so that:

$$\left|\Delta G(j\omega - \beta)\right| < \left|\alpha(j\omega)\right|, \quad (\forall)\omega \geq 0 \tag{25}$$

In order to robustly stabilize the nominal model $\overline{G}_0(s) = G_0(s-\beta)$ against the unmodeled dynamics ΔG that satisfies (25), we use a coprime factorization of $\overline{G}_0$ [3]:

$$\begin{gathered}\overline{G}_0 = ND^{-1} = \tilde{D}^{-1}\tilde{N} \\ \begin{bmatrix} \tilde{X} & -\tilde{Y} \\ -\tilde{N} & \tilde{D} \end{bmatrix}\begin{bmatrix} D & Y \\ N & X \end{bmatrix} = I\end{gathered} \tag{26}$$

If the following compensators are considered

$$\begin{gathered}\overline{R}(s) = \tilde{X}(s) - Q(s)\tilde{N}(s) \\ \overline{S}(s) = -\tilde{Y}(s) + Q(s)\tilde{D}(s)\end{gathered} \tag{27}$$

where *Q*(*s*) is any stable proper rational function, the system from *w* to *y*, in the left moved plane, is internally stable. The sufficient condition for robust stabilization of $\overline{G}_0$(s) against $\Delta G(s)$ that satisfies (25), can be obtained as follows:

$$\left|\alpha(j\omega)D(j\omega)[-\tilde{Y}(j\omega) + Q(j\omega)\tilde{D}(j\omega)\right| \leq 1, \ (\forall)\omega \geq 0 \tag{28}$$

Now, using the inverse transformation ($s \to s + \beta$), $S(s) = \overline{S}(s+\beta)$ and $R(s) = \overline{R}(s+\beta)$ stabilize the nominal model $G_0(s)$ with robust stability degree β.

3.2. Optimal Model Matching

Based on rise time, overshoot, settling time, and so on, can be selected a desired model *M*(*s*). Consider the performance of nominal system, i.e., for $\Delta G(s) = 0$. For optimal model matching the following integration error

$$J(T) = \int_0^\infty e^2(t)dt \tag{29}$$

must be minimized.

Here, the following two cases are analyzed:

1) The nominal transfer function $G_0(s)$ is minimum phase. Then,

$$T_{opt}(s) = N^{-1}(s)M(s) \tag{30}$$

achieves the perfect model matching, i.e., $J(T) = 0$.

2) The nominal transfer function is non-minimum phase. Then, $N(s)$ can be represented as

$$N(s) = N_-(s)N_+(s) \tag{31}$$

where $N_-(s)$ is a Blaschke product and $N_+(s)$ is free of poles and zeros in $\mathrm{Re}[s] \geq 0$. If the desired model $M(s)$ is selected as the same zeros as $N(s)$ in $\mathrm{Re}[s] \geq 0$, $M(s)$ can be represented as

$$M(s) = N_-(s)M_1(s) \tag{32}$$

where $M_1(s)$ is stable. Then, the perfect model matching is achieved by

$$T_{opt}(s) = N_+^{-1}(s)M_1(s) \tag{33}$$

4 Simulation Results

In this section we shall use, for the linearized models (23), (24), the numerical values of the parameters of the electrohydraulic system as given in [4]. In order to apply the two-degree of freedom controller technique, we consider a reduced order model described by

$$G_0(s) = \frac{1.116e^5}{s^2 + 6.367e^2 s} \tag{34}$$

The magnitude of the approximation error of the 8 order model by this 3-order model is represented in Fig. 2 and this approximation error is treated as an uncertainty.

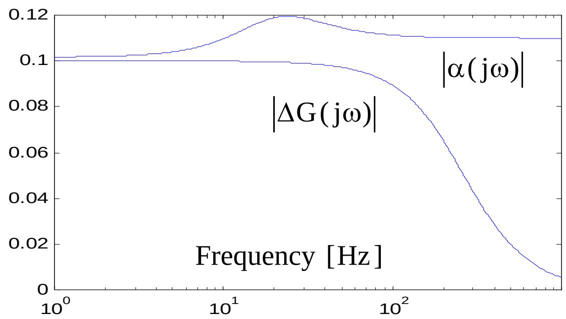

Fig. 2. The uncertainty bound

Then, we select a rational stable minimum phase function $\alpha(s)$ so that the condition (25) is satisfied for the above uncertainty (see, Fig. 2)

$$\alpha(s) = 0.11\frac{s^2 + 2\times 0.9\times 125s + 125^2}{s^2 + 2\times 0.8\times 130s + 130^2} \tag{35}$$

The sufficient condition for robust stabilization is illustrated in Fig. 3.

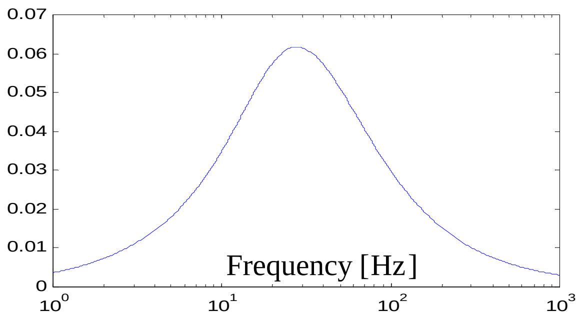

Fig. 3. Illustration of robust stability condition

The controller parameters $R(s)$ and $S(s)$ are obtained as

$$R(s) = \frac{s^2 + 9.49e^2 s + 2.47e^5}{s^2 + 7.92e^2 s + 1.11e^5}$$

$$S(s) = \frac{1.73e^2 s + 1.11e^5}{s^2 + 7.92e^2 s + 1.11e^5} \tag{36}$$

Choosing the desired model $M(s)$ as

$$M(s) = \frac{4e^6}{s^2 + 2\times 0.9\times 2e^3 s + 4e^6} \tag{37}$$

the perfect model matching is achieved for

$$T(s) = \frac{3.58e^1 s^2 + 2.84e^4 s + 4e^6}{s^2 + 3.8e^3 s + 4e^6} \tag{38}$$

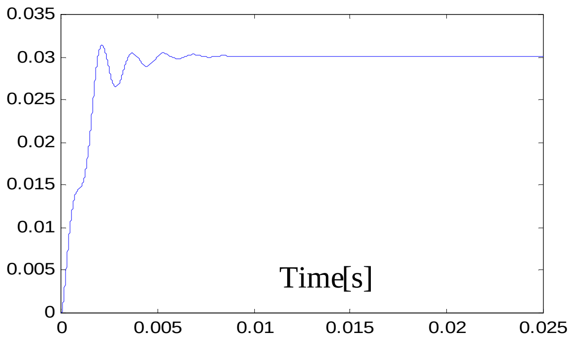

Fig. 4. Step response

The step response of the 8-order model with the designed controller and the input control are presented in Fig. 4 and, respectively, Fig. 5.

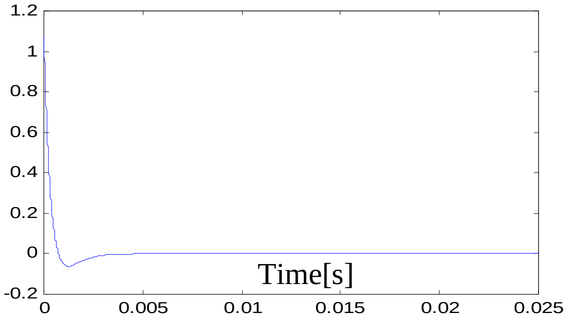

Fig. 5. Input control

5 Conclusion

In this paper, some nonlinear and linear models for an electrohydraulic system are derived and analyzed. To control this system a reduced order model is used and the error approximation is viewed as an uncertainty. The two-degree of freedom compensator technique allows to design a robust controller and the simulation results demonstrate good properties.

6 Acknowledgements

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