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A Direction Based Novel Edge Detector

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Abstract: - This paper presents a novel direction based edge detector which not only can detect the whole edges in an image like other edge detectors, but also more effectively detect edges in any directions that are pre-selected by users. It provides users with an additional function to extract pieces of edges of the directions of particular interest. The algorithm is described in details. Effectiveness of the proposed edge detector is evaluated by comparing it with the Canny edge detector. The results are found to be promising.

Key-Words: - Computer Vision, Signal Processing, Edge Detection, Wavelet, Image Processing

1 Introduction

Edge detection is one of the most commonly used operations in image analysis. Edge is the boundary between an object and the background. This means that if the edges in an image can be identified, then the objects can be located and basic properties such as area, perimeter, and shape can be determined [1].

There have been a number of edge detection algorithms reported by researchers [1,2]. Those algorithms can be broadly classified into two groups. The first is the template matching and the second is the differential gradient approach. In both cases the aim is to find where the intensity gradient magnitude is large, which stands as a reliable indicator of the edge of an object [2].

Almost all the Edge detection algorithms are designed to pick up all the possible edges in a given image region. In some applications, however, users are just interested in the edges in a certain direction. To our best knowledge, there is no sophisticated edge detection algorithm which is designed to find the edges in a pre-selected direction.

Among the reported edge detectors, the Canny edge detector is one of the optimal edge detectors [3]. It is always among the best performers in various edge operation evaluation experiments. It has become part of the standard against which the performance of a newly developed edge detector is compared [5]. In this paper, the performance between the proposed edge detector and the Canny edge detector is compared. The results are found to be promising.

This paper is organized as follows. The mathematical formulation of the proposed edge detector is introduced in section 2. The parameters determination and the algorithm efficiency are also discussed in this section. Section 3 presents the experimental results. Final conclusions are drawn in section 4.

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2 The Mathematical Formulation of the Direction Based Edge Detector

Morlet wavelet is used in the direction based edge detector. Morlet wavelet is a 2-dimentional (2D) continuous wavelet with rotational effect. The rotational effect enables to analyze the directional features of image signals. Users can detect the edges in their pre-selected directions. Edge images detected in certain pre-selected directions can complement to each other, and thus the whole edges can be composed by using these edge pieces. So the proposed direction based edge detector can be used not only to detect the edges in any selected directions, but also to detect the whole edges in the image.

Some literatures have mentioned the possible application of rotational wavelet transform to the edge detection [6,7]. However, no actual applications and algorithms were given, and no mathematical analysis based on explicit computations was formulated in order to support the application. In our study, a complete mathematical mode that use the Morlet wavelet to detect edges of an image in any directions was carried out. The results obtained provide insights of why and how rotational wavelet can be effective in edge detection. Detailed mathematical deduction was developed in [8]. But the application related analysis, such as detailed discretization, parameters determination and the algorithm efficiency analysis, was not described in the above paper due to the page limitation. In this paper, the practical direction based edge detector algorithm is presented. It can be used to detect the edges in any selected directions as well as whole edges in an image.

2.1 2D Morlet Wavelet

The 2D Morlet wavelet is

$$\phi(\mathbf{x}) = e^{i<\mathbf{K},\mathbf{X}>} e^{-(1/2)|\mathbf{X}|} - e^{-(1/2)|\mathbf{K}|^2} e^{-(1/2)|\mathbf{X}|^2}$$

Denote $\phi_M(\mathbf{x}) = e^{i<\mathbf{K},\mathbf{X}>} e^{-(1/2)|\mathbf{X}|}$,

and $\phi_E(\mathbf{x}) = e^{-(1/2)|\mathbf{K}|^2} e^{-(1/2)|\mathbf{X}|^2}$,

where $\mathbf{x}$ is the coordinate of the image. $\mathbf{k}$ is a fixed vector. It is not radial, but sensitive to rotation. The rotation only has effect on the factor $e^{i<\mathbf{K},\mathbf{X}>}$. So ϕ_E is independent of the rotation parameter, and ϕ_M is significantly dependent if $|\mathbf{k}|$ is chosen large on the rotation. In practice, ϕ_E is negligible.

Note that ϕ_M is no longer a wavelet. In fact,

$$\int \phi_M(\mathbf{x}) d\mathbf{x} = \hat{\phi}_M(0) = 2\pi^{-(1/2)|\mathbf{K}|^2} \neq 0,$$

where $\hat{\phi}_M$ is the Fourier transform of ϕ_M.

This means that the admissibility condition of the wavelet transform does not hold. If we choose $|\mathbf{k}|$ large, then $\hat{\phi}_M(0) \approx 0$ and so ϕ_M behaves like a wavelet.

Consider $\phi_{a,\theta,\mathbf{b}}(x)$ as a function on the Euclidean group with dilation, where $a > 0,\ \theta \in [0, 2\pi),\ \mathbf{b} \in \mathbf{R}^2$. Then

$$\phi_{a,\theta,\mathbf{b}}(\mathbf{x}) = a^{-1}\phi(a^{-1}\rho_\theta(\mathbf{x} - \mathbf{b})),$$

where ρ_θ is the rotation around the origin by angle θ in the anti-clockwise direction, and

$$\rho_\theta(\mathbf{x}) = (x_1\cos\theta - x_2\sin\theta,\ x_1\sin\theta + x_2\cos\theta),$$

where $f(\mathbf{x}) = f(x_1, x_2)$ represents the 2D image.

The wavelet transform of $f(\mathbf{x})$ based on the Euclidean group is [8]

$$W_{\phi_M} f(a,\rho,\mathbf{b}) = \frac{1}{a}\int_{\mathbf{R}^2} e^{i<\rho^{-1}\mathbf{k},\frac{\mathbf{x}-\mathbf{b}}{a}>} e^{\frac{-|\mathbf{x}-\mathbf{b}|^2}{a^2}} f(\mathbf{x}) d\mathbf{x}. \quad (1)$$

There are three parameters to be determined. They are $\mathbf{k}$, ρ, and a.

2.2 Discretization

Denote $\mathbf{k}=(0,k_0)$, $a=a_0, \theta=\theta_0, \mathbf{b}=(b_1,b_2)$, the discretization of (1) is

$$W_{\phi_M} f(a_0,\rho_{\theta_0},\mathbf{b}) = \frac{1}{a_0} \sum_{x_1=b_1-\frac{N_{x_1}}{2}}^{b_1+\frac{N_{x_1}}{2}} \sum_{x_2=b_2-\frac{N_{x_2}}{2}}^{b_2+\frac{N_{x_2}}{2}} f(x_1,x_2)$$
$$e^{i\frac{k_0}{a_0}[(x_1-b_1)\sin\theta_0+(x_2-b_2)\cos\theta_0]} e^{-\frac{1}{2a_0^2}[(x_1-b_1)^2+(x_2-b_2)^2]}, \quad (2)$$

where N_{x_1}, N_{x_2} are the sample numbers along the x_1 and the x_2 axis, respectively. The discretization steps for both x_1 and x_2 are 1. x_1 runs over all the integers in the interval $[b_1-\frac{N_{x_1}}{2}, b_1+\frac{N_{x_1}}{2}]$, and x_2 runs over all the integers in the interval $[b_2-\frac{N_{x_2}}{2}, b_2+\frac{N_{x_2}}{2}]$. $f(x_1,x_2)$ is the image.

Euler's formula $e^{i\theta}=\cos\theta+i\sin\theta$ is used, where the real and imaginary parts of (2) denoted by R_W and I_W, respectively.

Denote $er(x)=\frac{1}{\sqrt{2/\pi}}\int_{-\infty}^{x} e^{-\frac{t^2}{2}} dt$. From the mathematical table, we can find

$$er(\infty)=1, \quad er(2)=0.9772.$$

Therefore

$$\frac{1}{\sqrt{2/\pi}}\int_{-2}^{2} e^{-\frac{t^2}{2}} dt = er(2)-(1-er(2)) = 0.9544.$$

This suggests that the "effective support" of the function $\frac{1}{\sqrt{a/\pi}}\frac{1}{a_0}e^{-\frac{t^2}{2a_0^2}}$ is $[-2a_0, 2a_0]$. Indeed

$$\int_{-2a_0}^{2a_0} \frac{1}{\sqrt{2/\pi}}\frac{1}{a_0}e^{-\frac{t^2}{2a_0^2}} dt = \frac{1}{\sqrt{\pi/2}}\int_{-2}^{2} e^{-\frac{t^2}{2}} dt = 0.9544.$$

Therefore, we choose $\frac{N_{x_1}}{2}=\frac{N_{x_2}}{2}=2a_0$. The direction based edge detector is,

$$R_W = \frac{1}{a_0}\sum_{x_1=b_1-2a_0}^{b_1+2a_0}\sum_{x_2=b_2-2a_0}^{b_2+2a_0} f(x_1,x_2)$$
$$\cos\frac{k_0}{a_0}[(x_1-b_1)\sin\theta_0+(x_2-b_2)\cos\theta_0]\, e^{-\frac{1}{2a_0^2}[(x_1-b_1)^2+(x_2-b_2)^2]} \quad (3)$$

$$I_W = \frac{1}{a_0}\sum_{x_1=b_1-2a_0}^{b_1+2a_0}\sum_{x_2=b_2-2a_0}^{b_2+2a_0} f(x_1,x_2)$$
$$\sin\frac{k_0}{a_0}[(x_1-b_1)\sin\theta_0+(x_2-b_2)\cos\theta_0]\, e^{-\frac{1}{2a_0^2}[(x_1-b_1)^2+(x_2-b_2)^2]}. \quad (4)$$

Then $|W_{\phi_M} f(x_1,x_2)| = \sqrt{R_W^2+I_W^2}$. (5)

Theoretically, signals are represented by the components of large wavelet coefficients. Noise is evenly distributed across wavelet coefficients and is generally small [9]. Thresholding technique is used to select the wavelet coefficients. If pixel value is greater than the specified threshold, the pixel is considered to be part of an edge.

2.3 Determination of the Parameters

Paper [8] proved that the direction based edge detector can effectively detect the edge pieces which, after applying certain rotation, becomes nearly perpendicular with the pre-selected direction $\mathbf{k_0}$. There are three parameters to be determined.

i. Choose a small parameter *a*. It was proved in [8] that there is certain relation between the width of the edge and *a*. When the width of the edge is zero, only when $a \to 0$ can the edges be efficiently extracted. So the determination of *a* would have to be based on the experience of dealing with the same kind, or close kinds, of signals. Usually, if the signals are from the same source, for example, shot by using the same kind of cameras at similar objects, etc. then the corresponding admissible values for *a* should be similar. In fact, it is a general problem of almost all other sophisticated edge detectors, including the Canny edge detector [5]. Experiment in paper [5] also proved that the optimal parameter settings of an edge detector are strongly dependent on the image, and the edge quality of an edge detector varies greatly with a fixed parameter choice. In the experiments in below it shows that $a \le \frac{1}{8}$ works well.

ii. Choose a vector $\mathbf{k}_0$ of a large modulus and fix it all the time. In below we choose $\mathbf{k}_0 = (0, k_0)$ with $k_0 = 6.9$.

iii. Choose θ_0 to be of 90° difference from the direction pre-selected by the user. This angle θ_0 is used to detect edge pieces nearly parallel to the selected direction. If the user would like to detect the whole edges in an image, θ_0 is suggested to be $0^\circ, 45^\circ, 90^\circ$ and 135°, respectively. The four sets of edge segments corresponding to the four choices of the rotation angles can complement each other and unbroken edges can be composed. The flowchart of the proposed direction based edge detector is shown in Fig.1.

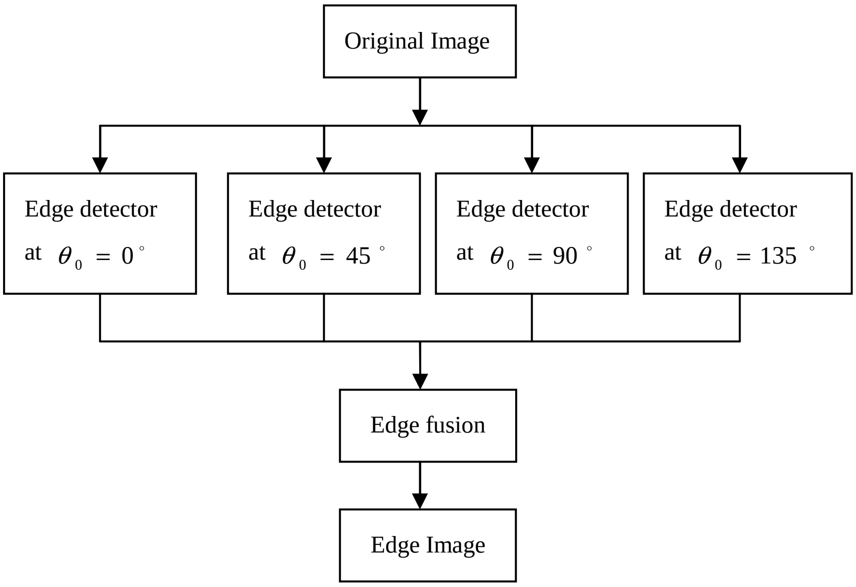


Fig.1 Diagram for direction based edge detector

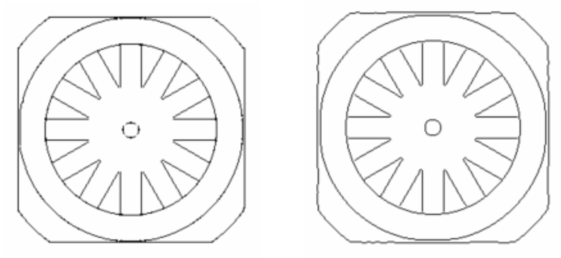
2.4 Edge Detector Algorithm Efficiency Analysis

We use O-notation to help characterize the time of running algorithm.

Based on the discretization formula (2), N_{x_1} and N_{x_2} are the sample numbers along the x_1 and the x_2 axis, respectively. The computational complexity for each pixel is $O(N_{x_1} N_{x_2} \ln(N_{x_1} N_{x_2}))$, where N_{x_1} and N_{x_2} are fixed numbers. In the experiment below, $N_{x_1} = N_{x_2} = 4a_0 = 4 \times 4 = 16$. Then $N_{x_1} N_{x_2} \ln(N_{x_1} N_{x_2}) \approx 1434$. To an image with size $n \times n$, the algorithm is $O(1434 n^2)$. Based on the O-notation, our algorithm is $O(n^2)$. So the computational complexity is moderate.

3 Experiment and Results

There are three kinds of images used in the experiment. They are regular pattern image, portrait, and remote sensing image. They cover typical examples users usually come across. Fig.2, Fig.5 and Fig.7 illustrate the three different kinds of original images. Fig.3 presents edges of four directions extracted from typical pattern image by directions based edge detector, which shows that the detector can efficiently extract edges of different direction. Fig.4, Fig.6 and Fig.8 show the whole edges extracted by the direction based edge detector and the Canny detector.

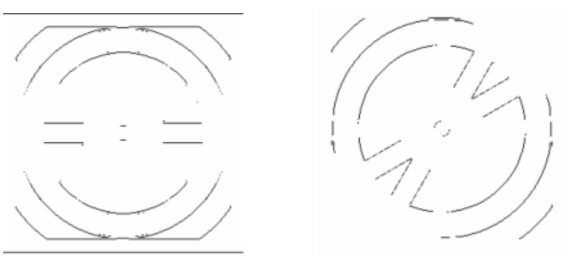


(a) Edges extracted by the proposed detector   (b) Edges extracted by Canny detector

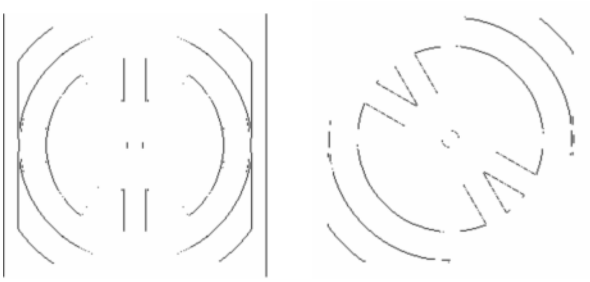
Fig.4 Extracted edges in a regular pattern image

Fig.2 Original Wheel Image

Fig.5 Original Lenna Image

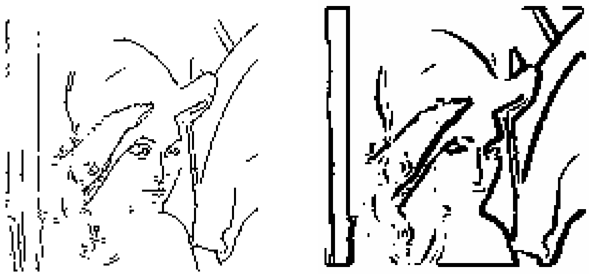


(a) Edges of 0°   (b) Edges of 45°



(c) Edges of 90°   (d) Edges of 135°

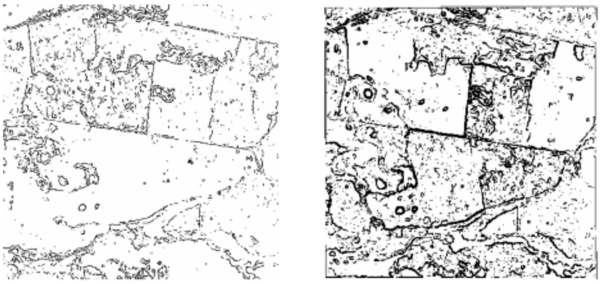
Fig.3 Edges in Four Directions



(a) Edges extracted by the proposed detector   (b) Edges extracted by Canny detector

Fig.6 Extracted edges in a portrait image

Fig.7 Original remote sensing image



(a) Edges extracted by the proposed detector

(b) Edges extracted by Canny detector

Fig.8 Extracted edges in a remote sensing image

4 Conclusion

A novel direction based edge detector is proposed in this paper. It can emphasize the singularities of images in preselected directions. Though the continuous wavelet transform is used as its mathematical foundation, the computational complexity is proved to be moderate. The Canny edge detector is used in the experiments to evaluate the performance of the proposed edge detector. The purpose of the comparison is not to identify the optimal edge detector in relation to the proposed due to the parameters setting problem mentioned in this paper before. The experiment results, however, show the proposed edge detector is promising in application.

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