Asymmetry and Long Memory Volatility: some empirical evidence using GARCH

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Abstract: - This paper investigates the asymmetry and long memory volatility behaviour of the Malaysian Stock Exchange daily data over a period of 1991 to 2005. The long-spanning data set enable us to examine piecewise before, during and after the economic crisis encountered in the Malaysian stock market. The daily index returns are adjusted for infrequent trading effect and we employed the variance time plot and R/S approaches to test the fractal scaling behavior of the volatility. The estimated Hurst parameter allows us to rank the market efficiency across the periods. The leverage effect, clustering volatility and long memory behaviour of the volatility are fitted by the asymmetry GARCH models. Across the periods, the results show the mixture of symmetry and asymmetry GARCH modelling.

Key-Words: - market efficiency, fractal, volatility, non-linearity, GARCH, BDS test, sign and size bias test.

1 Introduction
The study of volatility is one of the prevailing features in financial markets. Since the introduction of ARCH model by Engel[1] and GARCH by Bollerslev[2], there has been a numerous extension of ARCH models developed in specifying the conditional mean and conditional variance models. The recent survey of GARCH models are available in Degiannakis[3], Engle[4,5], and Poon and Granger[6]. The presence of conditional heteroskedasticity variance has an important implication and definition of market efficiency. Campbell et al.[7] categorized this process as a random walk 3(RW3) model with uncorrelated increments, but dependent with its squared increments. The least restrictive RW3 provides the most interesting property that attracts market participants, where for information on the variance of historical prices is able to predict the future volatility of the equity markets.

This findings have extended the traditional definition of market efficiency such as fractal market hypothesis by Peter[8], heterogeneous market hypothesis by Mullier et al[9] and mixture of distribution hypothesis by Andersen et al[10]. In Mandelbrot[11], he suggests that the weak form market efficiency is rejected if the stock returns present long-range dependence behavior. The traditional definition of market efficiency assumes that the market is composed of homogeneous participants who response according to the rational expectation strategy regardless of the amount of available information. This assumption seems to be not reasonable in the real financial market. No all market participants are provided with equivalent information. As a conclusion, the above studies suggest that the presence of long-range dependence behavior in the equity markets and this fractal scaling behavior is important in measuring the volatility, market efficiency and market risk.

In this paper, we include the infrequently trading adjustment, asymmetry and long-range dependence behaviors of the stock market in the model specifications. In addition, the long memory estimation is used to rank the market efficiency across the predefined periods. We consider the asymmetry and long memory GARCH models such as asymmetry component GARCH(CGARCH) by Ding and Granger[12], Engle and Lee[13] and Ding et al[14], symmetry and asymmetry fractionally integrated GARCH(FIGARCH and FIAPARCH) by Baillie et al[15] and Tse[16]. To analyse the performance of the asymmetry and long memory GARCH model, we estimate and compare them to a
benchmark symmetry and short persistence GARCH model for a long spanning daily returns series with four separate periods.

The non-linearity dependence is analyzed by using the BDS test to ensure the presence of iid condition. The results show mixture of GARCH models across the predefined periods. As a result, the interesting case study of an emerging market using KLSE may imply significant contributions to the theoretical modelling and predictability of financial time series

2 Methodology
2.1 Long Memory Behavior Analysis
The long-range dependence behavior is measured by the Hurst parameter(H) proposed by Hurst[17]. The long-range dependence behavior is measured by using variance time plot and R/S analysis. The details of other common estimators are discussed in Mandelbrot[11] and Beran[18]. The absolute returns have been selected as the volatility proxy. The positive value of volatility component converges to a constant level which behaves like a GARCH model. The long-run volatility component converges to a constant level with the value of \( \omega(1-\gamma_{t}) \) which follows an AR(1) process if \( 0<\gamma_{t}<1 \). Under this circumstance of \( 0<\alpha+\beta<\gamma_{t}<1 \), the permanent components has a much slower mean-reverting rate than the short run component. The asymmetric CGARCH(1,1) can be estimated by including the asymmetric parameter \( \phi \) in the transitory equation. For instance, the specific case for \( \delta=2 \), the transitory equation is estimated in the form:

\[
\sigma_{t,2}^{2} = \gamma_{10}\sigma_{t-1,2}^{2} + \gamma_{20}(a_{t-1}^{2} - \sigma_{t-1,2}^{2}) + \phi_{1}(a_{t-1}^{2} - \sigma_{t-1,2}^{2})l_{t-1} \quad (4)
\]

where \( l \) is the dummy variable indicating negative innovation. The positive value of \( \phi \) shows that the presence of transitory leverage effects in the conditional variance.

2.2 Long Memory GARCH
The conditional mean equations of KLSE stock returns are an AR(1) model of \( \{r_{t}\} \) as below:

\[
r_{t} = \mu + \phi r_{t-1} + a_{t}, \quad (1)
\]

where the \( a_{t} \) is serially uncorrelated, but dependent to its lagged values or the conditional variance components as follow: \( a_{t} = \sigma\varepsilon_{t} \quad (2) \)

2.2.1 Component GARCH
Ding and Granger[12] and Engle and Lee[13] introduced the CGARCH that can capture the high persistence in volatilities. Specifically, the CGARCH is decomposed into two components with one component captures the short-run innovation impact and the other captures the long-run impact of a transitory as follow:

\[
\sigma_{t}^{2} = \sigma_{t,1}^{2} + \sigma_{t,2}^{2}
\]

\[
\sigma_{t,1}^{2} = \omega + \gamma_{10}\sigma_{t-1,1}^{2} + \gamma_{20}(a_{t-1}^{2} - \sigma_{t-1,1}^{2}) \quad (3)
\]

\[
\sigma_{t,2}^{2} = \gamma_{10}\sigma_{t-1,2}^{2} + \gamma_{20}(a_{t-1}^{2} - \sigma_{t-1,2}^{2})
\]

The \( (a_{t-1}^{2} - \sigma_{t-1}^{2}) \) term is influences both the permanent and transitory volatility components. The short-run transitory effect is mean-reverts to zero at a rate of \( (\alpha+\beta) \) under the condition of \( 0<\alpha+\beta<1 \) which behaves like a GARCH model. The long-run volatility component converges to a constant level with the value of \( \omega\left(1-\gamma_{t}\right) \) which follows an AR(1) process if \( 0<\gamma_{t}<1 \). Under this circumstance of \( 0<\alpha+\beta<\gamma_{t}<1 \), the permanent components has a much slower mean-reverting rate than the short run component. The asymmetric CGARCH(1,1) can be estimated by including the asymmetric parameter \( \phi \) in the transitory equation. For instance, the specific case for \( \delta=2 \), the transitory equation is estimated in the form:

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\]

where \( l \) is the dummy variable indicating negative innovation. The positive value of \( \phi \) shows that the presence of transitory leverage effects in the conditional variance.

2.2.1 Fractionally Integrated GARCH
We start with the GARCH(p,q) model introduced by Bollerslev[2] which can be written in the form of backshift/lag notation:

\[
\sigma_{t}^{2} = \frac{\alpha_{0}}{1-\beta(B)} + \left\{ 1 - \frac{\phi(B)(1-B)}{1-\beta(B)} \right\} a_{t}^{2}, \quad (5)
\]

where \( \phi(B) = \frac{1-\alpha(B)-\beta(B)}{1-\beta(B)} \), is of order \([\max\{p,q\}-1] \).

In Baillie et.al[15], the model’s coefficients of \( \beta(B) \) and \( \alpha(B) \) capture the shot-run of volatility while the fractional difference parameter \( d \) models the long run characteristics of volatility. This model is named as fractional integrated GARCH or FIGARCH(p,d,q). The conditional variance of FIGARCH(p,d,q), see Baillie et.al[15] is written in the form:

\[
\sigma_{t}^{2} = \frac{\alpha_{0}}{1-\beta(B)} + \left\{ 1 - \frac{\phi(B)(1-B)^{d}}{1-\beta(B)} \right\} a_{t}^{2}, \quad (6)
\]

with \( 0 \leq d \leq 1 \). If \( d=0 \), the model will become \( \beta(B)\sigma_{t}^{2} = \alpha(B)a_{t}^{2} \), which is a GARCH model. If \( d=1 \), the model \( \beta(B)(1-B)^{2}\sigma_{t}^{2} = \alpha(B)a_{t}^{2} \), will follow a IGARCH model. And when \( d \) is 0<\(d<0.5\), the term \( (1-B)^{d} \) has an infinite binomial distribution for non-integer powers.

Finally the FIAPARCH(p,d,q), seeTse[16] is given by:

\[
\sigma_{t}^{\delta} = \frac{\alpha_{0}}{1-\beta(B)} + \left\{ 1 - \frac{\phi(B)(1-B)^{\delta}}{1-\beta(B)} \right\} (a_{t} - \phi a_{t-1})^{\delta}, \quad (7)
\]

2.3 Diagnostic Test
The adequacies of the models are tested by using Ljung-Box statistics for both standardized and
squared standardized residuals. In addition, the Engle LM ARCH test is implemented to ensure the absence of ARCH effect. The BDS portmanteau test[21] for time based dependence in the standardized residuals is used to check whether the series are iid. However, Brooks et al. [22,23] claimed that for a asymmetry conditionally heteroskedastic models, the BDS test is unable to detect a common mis-specification. Due to this, the reliability of BDS test is further examined by using Engle and Ng[24] to determine the asymmetry volatility models response to news.

3 Empirical Result

The stock market price is taken from the daily closing price of Kuala Lumpur stock exchange(KLSE) through Bank Negara Malaysia[25]. This price index is weighted by market capitalisation with the base year 1977 of 100 listed companies. The selection guidelines of Kuala Lumpur composite index(KLCI) component can be found in the official website of Bursa Malaysia[26]. The continuously compounded daily return at time $t$ is defined as:

$$ return_t = \log(index_t) - \log(index_{t-1}) $$

The sample period starts from 1$^{\text{st}}$ January 1991 to 14$^{\text{th}}$ April 2005 with 3516 observations. The long-spanning data set consists of 14 years enable us to run various tests with reliable statistical results. This is suggested by Taylor[27] that large sample size of stock prices series may improve the error variance and increase the power of random walk tests. The KLSE trades five days a week, start from Monday through Friday. In order to observe and analyze the behavior of the stock market, we split the overall sample data into four distinct periods for prices index as follows:

I: pre-crisis(1-1-1991-31-12-1996)
II: during crisis(1-1-1997-31-8-1998)
III: USD pegged to RM(1-9-1998-31-12-2000)
IV: post-crisis(1-1-2000-14-4-2005)

<table>
<thead>
<tr>
<th>Table 1. Descriptive statistics of Stock returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>period</td>
</tr>
<tr>
<td>--------</td>
</tr>
<tr>
<td>overall</td>
</tr>
<tr>
<td>I</td>
</tr>
<tr>
<td>II</td>
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<tr>
<td>III</td>
</tr>
<tr>
<td>IV</td>
</tr>
</tbody>
</table>

* denotes 1% level of significance

A glance at the statistical behaviours of the five different time periods is illustrated in Table 1. The descriptive statistics measure the moments, skewness, kurtosis and normality of the returns series. The returns series show highest standard deviation and volatile in the crisis period(from 1997 to 1998). The huge magnitude of Jarque-Bera statistics in all periods, enable us to reject the return series is normally distributed by referring to the $p$-values. The returns series exhibit the fat tailed phenomenon and the kurtosis is less than 3 for all the periods.

After the correction of thin trading effect, the adjustment appears to have eliminated the apparent serial correlation of the linear model across all the periods as shows in Table 2. The coefficient, $a_1$, is insignificantly different from zero at the 1% level for all the periods. Across all the periods, only the residuals during the crisis period indicate a white noise process. On the other periods, the diagnostic tests show mixture of autocorrelation, unconditional and conditional heteroscedasticity effect. This leads to the conclusion of random walk process during the crisis period and acceptance of less stringent random conditions in others periods.

The fractal behaviour is examines using different time-scale for example hourly, daily, weekly or monthly. The sample autocorrelation function(SACF) is plotted as the preliminary analysis. The Hurst’s parameters for the different sub-sample periods are determined by using variance-time plot and R/S analysis. Several studies of financial time series reported that the absolute values of returns exhibit long-range dependence behaviours. Due to this, we concentrate our analysis in the absolute returns series only.
The results show that the highest inefficiency is during the crisis period, follows by pre-crisis, post-crisis and USD pegged period. The post-crisis period indicates the lowest inefficient as indicated in Table 3. With the evidence of long-range dependence property, we reject the random walk hypothesis Malaysian stock market.

Table 3. Ranking of Hurst’s parameter

<table>
<thead>
<tr>
<th>Period</th>
<th>Hurst parameter ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-crisis</td>
<td>0.612</td>
</tr>
<tr>
<td>Crisis</td>
<td>0.642</td>
</tr>
<tr>
<td>USD pegged</td>
<td>0.389</td>
</tr>
<tr>
<td>Post-crisis</td>
<td>0.609</td>
</tr>
</tbody>
</table>

* denotes 1% level of significance

Table 4 report the estimated results for GARCH(1,1), CGARCH(1,1) and FIGARCH(1,1,d). The asymmetry volatilities are significantly different from zero in the pre-crisis and USD pegged periods in the asymmetry CGARCH(1,1) and FIGARCH(1,1,d) models. The fat tailed property of the volatility is fitted by assuming the, $\varepsilon_t$, as a GED distribution. The ARCH and GARCH effects are represented by $\alpha$ and $\beta$ for GARCH and FIGARCH. In CGARCH, the persistent and transitory components are determined by the $\gamma_h$ and $\gamma_s$ in the CGARCH model respectively.

In Table 4, the CGARCH and FIGARCH provide a slightly statistical improvement over a symmetric GARCH based on the log likelihood and Akaike information criteria (AIC) evaluations. Furthermore, the leverage effect of volatility is captured by $\phi$ and $\phi_s$ which statistically significant in pre-crisis and USD pegged periods. This concludes that downward movements (shock) in the stock market are followed by a greater volatilities than upward movements of the same magnitude.

The CGARCH(1,1) models show that the persistent components are all significant with $\gamma_h$ close to one. The persistency is strongest in crisis period, follows by pre-crisis, post-crisis and weakest in USD pegged period. This result is consistent with the values of $d$ estimated in FIGARCH. The long-range dependence coefficient is 0.49 and rose to the highest value at crisis period ($d=0.76$) and then dropped to 0.09 in the USD pegged period and 0.41 in the post-crisis period.

Table 4: Estimation results

<table>
<thead>
<tr>
<th>Test</th>
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</tr>
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<tbody>
<tr>
<td>$Q_{(10)}$ on $\hat{u}_t$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CGARCH</td>
<td>14.66</td>
<td>6.75</td>
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<td>14.03</td>
</tr>
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<td></td>
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<tr>
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</tr>
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</table>

Ljung Box Serial Correlation Test (Q-statistics) on $\hat{u}_t$ and $\hat{u}_t^2$: Null hypothesis – No serial correlation, LM ARCH test: Null hypothesis - No ARCH effect, * indicate significance at 10%.

In Table 5, the diagnostic tests for the specifications in GARCH models indicate no significant serial correlations and ARCH effect in the mean and variance equations at the 5% level respectively.

During the currency crisis period, the market shows the greatest strength of long-range dependence. The weak performance of RM has caused the investors fled from the stock market. As a result, most of the market participants tend to follow the trends and leave the stock market. This is consistent with the study of Shiller[27] argued that most market participants are not ‘smart investor’ but rather followers to trends and fashions. The strength of the long-range dependence decreases after the USD pegged or recovery period. This may be caused by the heterogeneous participants’ reactions respected to their optimistic or pessimistic views of the future prospect to Malaysian stock market. The estimated models are consistent with the trend of our global Hurst estimations in all the periods.

Table 5: Serial correlation and ARCH effect Diagnostic

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</tbody>
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Ljung Box Serial Correlation Test (Q-statistics) on $\hat{u}_t$ and $\hat{u}_t^2$: Null hypothesis – No serial correlation, LM ARCH test: Null hypothesis - No ARCH effect, * indicate significance at 10%.
The overall BDS tests are illustrated in Table 6. In the BDS test for detecting non-linear dependences in the standardized residuals, we selected the values of the distance, \(d\), as 1.0 times the standard deviation; the embedding dimensions \(m\) are up to 5 for observations less than or equal to 500 and up to 10 for sub-period exceed 500. The acceptance of null hypothesis indicates the standardized residuals series are random with no non-linear dependences of any kind. We start with the benchmark GARCH(1,1) models result where the BDS test statistics for pre-crisis and USD pegged periods are statistically significant in several dimensions. Thus, this concludes that the symmetry GARCH has been found to be inadequate in the model non-linearity specification in the pre-crisis and USD pegged periods. On the other hand, the CGARCH and FIGARCH are not statistically significant at the level of 5% across all the period. This implies that the chaos behavior of volatility ‘seems’ has been captured by the long-range dependence components embedded in the CGARCH and FIGARCH. However, the BDS test is suffer for low detection of asymmetry volatility behavior. Therefore, the sign-bias and negative test are implemented to ensure the possible neglected asymmetries in the standardized residuals in the BDS test.

In Table 7, the symmetry GARCH(1,1) model shows significant negative size-bias test statistics in overall, pre-crisis and post-crisis periods. This result indicates that the GARCH model is not able to explain the large negative innovations impact in volatility. The sign-bias test is only significant at level 10% in USD pegged period shows that the different impacts that positive and negative innovations have on volatility which are not predicted by the GARCH model. On the other hand, the test for size and sign bias show significant negative size-bias in all the models for the overall period. The large negative innovations impact is not sufficiently modelled even by the asymmetry and long memory FIGARCH model. From the estimation results of Table 7, the asymmetry volatility exists in the pre-crisis and USD pegged periods only. Due to alternating presence of asymmetry volatility across the periods, implementing the FIGARCH model to the overall data may cause the model mis-specification in accounting the asymmetry volatility. On the other hand, after separating the overall data in phenomenon events such as crisis etc., the results show that the size and leverage effect are not statistically significant at level 1% in all sub-periods for CCGARCH and FIGARCH.

As a result, in the fitted standard GARCH model, both or alternately, the BDS and sign/size tests strongly reject the iid hypothesis since the fitted GARCH model is mis-specified in that it does not include the asymmetries manifest and long-memory behaviours in the data for all the periods. In contrast, the CGARCH and FIAPARCH show no evidence of unexplained non-linearity, sign or size bias in the negative side with the exception in the overall period.

### 4 Conclusion

This paper studies the asymmetry and long memory volatility in the Malaysian stock market by using the component GARCH model and fractionally integrated GARCH model. Both the long memory GARCH models provide good description of the long memory behavior in the Malaysian stock market volatility compare to the standard GARCH model. In addition, the presence of long memory volatility enables us to rank the degree of market inefficiency which also leads to the rejection of efficiency market hypothesis in Malaysian stock market. The diagnostic tests indicate better specification in CGARCH and FIGARCH models with no significant of iid except in the overall period. As a conclusion, the long memory GARCH provides a better framework for volatility modeling.

### Table 6: BDS test

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Overall</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH</td>
<td>0.0042</td>
<td>0.0047</td>
<td>0.0041</td>
<td>0.0046</td>
<td>0.0023</td>
</tr>
<tr>
<td>CGARCH</td>
<td>0.0025</td>
<td>0.0023</td>
<td>0.0028</td>
<td>0.0024</td>
<td>0.0036</td>
</tr>
<tr>
<td>FIGARCH</td>
<td>0.0067</td>
<td>0.0065</td>
<td>0.0062</td>
<td>0.0064</td>
<td>0.0035</td>
</tr>
</tbody>
</table>

### Table 7: Sign and size bias test

<table>
<thead>
<tr>
<th>Overall</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>GARCH</td>
<td>-0.7643</td>
<td>-0.5796</td>
<td>-0.3724</td>
<td>-1.9041</td>
</tr>
<tr>
<td>Negative size-bias test</td>
<td>-4.3831</td>
<td>-2.5664</td>
<td>-0.7218</td>
<td>-0.6558</td>
</tr>
<tr>
<td>Joint test</td>
<td>7.0606</td>
<td>2.9880</td>
<td>0.5850</td>
<td>1.0074</td>
</tr>
<tr>
<td>CGARCH</td>
<td>-0.7551</td>
<td>-0.2102</td>
<td>-0.4056</td>
<td>-1.9909</td>
</tr>
<tr>
<td>Negative size-bias test</td>
<td>-4.4052</td>
<td>-1.6813</td>
<td>-2.0072</td>
<td>-2.0573</td>
</tr>
<tr>
<td>Joint test</td>
<td>7.6058</td>
<td>3.0099</td>
<td>0.3484</td>
<td>0.9673</td>
</tr>
<tr>
<td>FIGARCH</td>
<td>-0.4887</td>
<td>-0.4505</td>
<td>-0.4552</td>
<td>-0.0806</td>
</tr>
<tr>
<td>Negative size-bias test</td>
<td>-2.6677</td>
<td>-1.4738</td>
<td>-0.8210</td>
<td>0.1995</td>
</tr>
</tbody>
</table>

The values represent the t-statistics. \(a\), \(b\) and \(c\) denote 10%, 5% and 1% level of significance.
However, for long spanning data sets, the volatility modeling should take into account the market structures and important events (boom or recession) which may trigger the time varying asymmetry and long memory volatility.

References: