Trading Behavior under Public Disclosure Regulations

MINH T. VO
Department of Economics and Management
University of Minnesota-Morris
600 E. 4th Street, Morris, Minnesota 56267
USA

Abstract: - This paper shows that under public disclosure regulations, in order to keep their information advantage, insiders may manipulate the market by trading against their information and by adding noise to their trades. However, outsiders can learn more information from insiders' disclosed orders. This makes the competition for profit among them more intense. The market becomes more efficient. The paper also shows that disclosure regulations transfer gains from insiders to outsiders.

Key-Words: - Trading behavior; Market microstructure; Insider trading; Asymmetric information.

1 Introduction
Corporate insiders routinely trade in the stock of the company with which they are affiliated. In order to diminish this unfair advantage, the U.S. Congress enacts a law requiring insiders associated with a firm to report any equity transactions they make in the stock of that firm to the Securities and Exchange Commission. The reports are filed after the trade is completed, at which time they become publicly available. This paper examines the effects of the mandatory disclosure regulations on the strategic trading of informed traders in financial markets. More specifically, it investigates how effective the mandatory disclosure is against the abuse of inside information, how it helps outsiders infer inside information, and how this affects the trading behavior of insiders. Toward that end, the paper compares the (perfect Bayesian) equilibrium where disclosure is mandatory to the equilibrium where insiders do not have to disclose their trades. It shows that the goal of Congress is, indeed, achieved: compared to the benchmark case - where insiders are not required to disclose their trades - more information about the stock is revealed to the public. As a result, the market becomes more efficient, risk associated with the stock is lower and marginal trading cost is lower too. In addition, it shows that mandatory disclosure could create incentives for an insider to manipulate the market by trading against his information and adding noise to his order. In contrast, in the benchmark case, the insider’s actions do not involve any contrarian trading or any random element (and he would never trade in a manner which is inconsistent with his private information).

The paper also demonstrates that disclosure regulations transfer gain from insiders to outsiders. The remainder of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes the equilibrium in the disclosure case and the non-disclosure case. Section 4 uses a numerical example to discuss the implications of the model. Section 5 concludes the paper.

2 The Model
There are two assets in the economy: a risky stock and a risk-free bond. The interest rate of the bond is normalized to zero so that we do not need to discount the future cash flows. Market participants include an informed insider trader, an informed outsider trader (for example, institutional investors or investors with good research capability), a market maker, and a number of liquidity traders. These traders buy or sell the stock in two periods. The informed traders are assumed to be risk neutral. The liquidation value of the stock (i.e., its value in period 3) is a random variable \( \tilde{\nu} \). The paper assumes that \( \tilde{\nu} \) is the sum of two other random variables \( \tilde{a} \) and \( \tilde{b} \), which are independently normally distributed with mean zero and variance \( \sigma^2 \). Variables \( \tilde{a} \) and \( \tilde{b} \) could represent, for example, a new discovery, a new contract, legal allegations, etc. Given the distribution of \( \tilde{a} \) and \( \tilde{b} \), \( \tilde{\nu} \) is normally distributed with mean zero and variance \( 2\sigma^2 \). In this paper, a tilde is used to distinguish a random variable from its realization.
Before the first trading takes place, both informed traders receive a signal \( s_0 \) which is the realization of \( \tilde{b}(s_0 = b) \). The insider, trader 1, receives, in addition, the value \( v \) of the realization of \( \tilde{v} \). There is no new information in the second period. In period 3, the liquidation value of the stock is announced and stock holders are paid accordingly. This information structure is common knowledge. Liquidity traders buy or sell shares for reasons exogenous to the model. The quantity traded by liquidity traders in period \( t \), denoted by \( \tilde{u}_t \), is normally distributed with mean zero and variance \( \sigma_u^2 \) and is independent of all other random variables.

Denote the quantities traded in period \( t \) by the insider - trader 1, the outsider - trader 2, and liquidity traders by \( x_t, y_t, \) and \( u_t \) respectively. The aggregate order flow in period \( t \):

\[
    w_t = x_t + y_t + u_t.
\]

The market maker observes the aggregate order flow but does not know which orders come from which traders. He sets the price equal to the expected value of the stock, conditioned on the history of aggregate orders he received up to that time, and he trades the quantity necessary to clear the market at this price. Specifically,

\[
    p_t = E(\tilde{v}|w_t) \quad \text{and} \quad p_{E}(\tilde{v}) = E(\tilde{v}|w_1, w_2).
\]

In order to derive the resulting demands, \( x_t, y_t, \) and the prices \( p_t \), I represent the above economy as an extensive form game with imperfect information, and employ the notion of Perfect Bayesian Equilibrium (PBE). This equilibrium notion is studied because it captures the fact that informed traders are rational and forward-looking. That is, each of them takes into account that his demand will be used by other traders to update their beliefs concerning the value of the stock. More specifically, I will explore the linear equilibrium in this game because in addition to its appeal and tractability, given the normality assumption of all random variables, prices are linear functions of the history of aggregate order flows. Thus, traders can infer the aggregate demands \( w_t \) from price \( p_t \).

The information structure of the insider, trader 1, is not affected by the requirement to disclose his trade \( x_t \). (Of course, the equilibrium value of \( x_t \) will depend on whether or not he has to disclose it.) By converting the price \( p_t \), trader 1 can learn \( w_t \). Since he also knows trader 2’s information, \( s_0 \), he can infers trader 2’s order. Thus, trader 1 knows, at the end of period 1, the values of \( x_t, y_t, \) and \( u_t \).

Trader 2’s information structure does depend on whether or not the disclosure is enforced. In the case of no disclosure, trader 2 learns from the price \( p_t \) the aggregate order flow \( w_t \) and therefore \( x_t + u_t \) (since he knows his own order) and uses them to infer more information about the stock. His updated evaluation of \( \tilde{v} \) after the first trading round is:

\[
    s_t = E(\tilde{v}|s_0, x_t, u_t).
\]

We now turn to the case of disclosure, where trader 1’s order is publicly disclosed after the trade is completed. To distinguish between the two cases, this paper uses the superscript \( d \) to denote variables in the disclosure case.

The outsider, trader 2 learns the aggregate order flow \( w_t^d \) by converting \( p_t^d \), and \( x_t^d \) from the disclosure by trader 1. Using this information, he updates his belief about the value of the stock as follows:

\[
    s_t^d = E(\tilde{v}|s_0, x_t^d).
\]

He then uses (1) to form his trading strategy in period 2.

**3 Characterization of Equilibrium**

**Definition** A PBE of the trading game is given by a strategy profile \( [(x_t, x_t^d), (y_t, y_t^d)] \) and a price system \( \{p_t, p_t^d\} \) such that the following conditions hold:

1. **Profit maximization:**
   \[
   x_t \in \arg \max_{x_t} E\left[x_t (\tilde{v} - p_t)^2 \right], \\
   y_t \in \arg \max_{y_t} E\left[y_t (\tilde{v} - p_t)^2 \right], \\
   y_t \in \arg \max_{y_t} E\left[y_t (\tilde{v} - p_t)^2 \right],
   \]

2. **Market efficiency**
   \[
   p_t = E(\tilde{v}|I_t^m), \\
   p_t^d = E(\tilde{v}|I_t^d),
   \]
   where \( I_t^m, I_t^d \) are the information sets of trader 1, trader 2, and the market maker respectively in period \( t \). The conditional expectations are derived
using Bayes’ rule to ensure that the beliefs are consistent with the equilibrium strategy.

This paper uses variance to measure the amount of information. Specifically, define:

$$\Sigma_i = \text{var}(\bar{v} | w_1, ..., w_t) = \text{var}(\bar{v} - p^*_t),$$

$$\Sigma_i$$ is the variance of the asset value given the market maker’s information. It measures the total amount of information that has not been incorporated into price after $$t$$ trading round(s). A high value of $$\Sigma_i$$ indicates that informed traders retain a large amount of information. Accordingly, in the disclosure case, we have:

$$\Sigma_{id} = \text{var}(\bar{v} | w_1, ..., w_t, x^d_1, ..., x^d_t) = \text{var}(\bar{v} - p^*_t),$$

where $$p^*_t$$ is the updated price in period $$t$$ after the market maker sees the disclosure of the insider. How the market maker updates the price after the disclosure will be discussed later in this Section.

The amount of inside information after $$t$$ trading round(s) in the non-disclosure case and disclosure case is measured by $$\Lambda_i$$ and $$\Lambda^d_i$$ respectively, where

$$\Lambda_i = \text{var}(\bar{v} | s_0, x^i_1 + u_i, ..., x^i_t + u_i) = \text{var}(\bar{v} - s_i),$$

$$\Lambda^d_i = \text{var}(\bar{v} | s_0, x^d_1, ..., x^d_t) = \text{var}(\bar{v} - s^d_i).$$

$$\Lambda_i, \Lambda^d_i$$ are variance of the stock value given trader 2’s information. Thus, they measure the amount of information known only to trader 1.

The amount of common information shared between traders 1 and 2 is measured by the difference between the total information and the inside information. Specifically,

$$\Pi_i = \Sigma_i - \Lambda_i = \text{var}(s_i - p_i),$$

$$\Pi^d_i = \Sigma^d_i - \Lambda^d_i = \text{var}(s^d_i - p^*_i).$$

3.1 The Equilibrium in Non-Disclosure Case

For comparison purposes, this paper first derives the equilibrium in the non-disclosure case. It is based on the model of Foster and Viswanathan [1].

**Proposition 1** A PBE in which all trading strategies and pricing rule are of linear form is given by:

$$x_1 = \beta_1 (v - s_0) + \gamma_1 s_0,$$  (2)

$$x_2 = \beta_2 (v - s_1) + \gamma_2 (s_1 - p_1),$$  (3)

$$y_1 = \theta_1 s_0,$$  (4)

$$y_2 = \theta_2 (s_1 - p_1),$$  (5)

$$p_1 = \lambda_1 w_1,$$  

$$p_2 = p_1 + \lambda_2 w_2,$$

where $$(\beta_1, \beta_2, \gamma_1, \gamma_2, \theta_1, \theta_2, \lambda_1, \lambda_2)$$ is a solution of the following system of equations

$$\beta_1 = \frac{2\lambda_2 - \phi + 2\lambda_2 \mu (\phi - \lambda_1)}{2\lambda_2 \rho},$$

$$\beta_2 = \frac{1}{2\lambda_2},$$

$$\gamma_1 = \frac{(1 - \lambda_1 \theta_1) [1 + 2\xi (\phi - \lambda_1) - \mu \phi]}{2\rho},$$

$$\theta_2 = \gamma_2 = \frac{1}{3\lambda_2},$$

$$\lambda_1 = \frac{1 - 2\psi / \lambda_1}{\lambda_1 [3 + 2\psi (\phi - 2\lambda_1)]},$$

$$\lambda_2 = \frac{9\lambda_1 + 8\Pi}{36\sigma^2_u},$$

$$\xi = \psi = \frac{\mu}{3} = \frac{1}{9\lambda_2},$$

$$\phi = \frac{\beta_1 \sigma^2}{\beta^2_1 \sigma^2 + \sigma^2_u},$$

provided the following second order conditions:

$$\lambda_1 (1 - \psi \lambda_1) > 0,$$

$$\rho = \lambda_1 - \frac{\phi^2}{4\lambda_2} - \xi (\phi - \lambda_1)^2 + \mu \phi (\phi - \lambda_1) > 0$$

are satisfied.

(The proof is available upon request.)

Equations (2) and (3) show the trading strategy of trader 1. His orders consist of two parts. The first one is based on the difference between the true value known only to him and the signal he shares with trader 2. The intensity at which she trades on this part is $$\beta_1$$. The second one is based on the difference between the signal he shares with trader 2 and the previous price (assuming the price is zero before the trading game starts). The intensity at which she trades on this part is $$\beta_2$$. The proof is available upon request.

In the following analysis, I will show that in this context, trader 1 could trade more intensely on the shared information at first
($\gamma_1 > \theta_1$); however, this is reversed in the second trading round. This strategy makes it harder for trader 2 to learn the information he does not have.

**Proposition 2** Let $\pi_i^1$ be the expected profit of trader $i$ in period $t$, then

$$\pi_1^1 = \beta_1 (1 - \lambda_1 \beta_1) \sigma^2 + \gamma_1 [1 - \lambda_1 (\gamma_1 + \theta_1)] \sigma^2,$$

$$\pi_2^1 = \beta_2 (1 - \lambda_2 \beta_2) \Lambda_1 + \gamma_2 [1 - \lambda_2 (\gamma_2 + \theta_2)] \Pi_1,$$

$$\pi_1^2 = \theta_1 [1 - \lambda_1 (\gamma_1 + \theta_1)] \sigma^2,$$

$$\pi_2^2 = \theta_2 [1 - \lambda_2 (\gamma_2 + \theta_2)] \Pi_1.$$

(The proof is available upon request.)

The expected profits of trader 1, $\pi_1^1$ and $\pi_1^2$, consist of two parts. The first part represents the profit resulting from the inside information while the second part is the profit resulting from the common information shared between 2 informed traders. These profits are proportional to the trading intensities and the corresponding information amount that has not been released yet. By the same token, the expected profits of trader 2, $\pi_2^1$ and $\pi_2^2$, are proportional to the amount of common information he shares with trader 1, and his trading intensity. Both traders face a trade-off. If they trade more intensely on their information at first, their profits in period 1 will be higher but more information is released to the public ($\Lambda_1$ and $\Sigma_1$ will be lower). With less retained information, their profits will be lower in period 2. In equilibrium, they trade such that their total profits are maximized.

The expressions in Propositions 1 and 2 provide a benchmark against which to compare the equilibrium for the case where disclosure is mandatory.

### 3.2 The Equilibrium in Disclosure Case

Using the notion of dissimulation introduced by Huddart et al. [2], I show in this section that there exists an equilibrium in which trader 1’s order in period 1 consists of an information-based component and a random noise component $z$, which is normally distributed with mean zero and variance $\Sigma_z$. The random component may be a buy or a sell, independent of trader 1's information. Because of it, trader 1 sometimes buys (sells) when his information indicates that the asset is overvalued (undervalued). He may also buy or sell more aggressively than would be the case under no disclosure. This is to diminish the ability of other market participants to draw inference from the public records. However, this is not without costs because at times trader 1 has to trade in a manner inconsistent with his private information. We will see that the random noise reduces his profit in period 1. In contrast, in the non-disclosure case, the insider’s actions do not involve any contrarian trading nor random element and he would never trade in a manner which is in consistent with his private information.

To derive the equilibrium, I first postulate that the demands of trader 1 and trader 2, and the market maker’s pricing rule in period 1 take the form:

$$x_1^d = \beta_1 (v - s_0) + \gamma_1 s_0 + z,$$

where $z \sim N(0, \Sigma_z)$,

$$y_2^d = \theta_2 s_0,$$

$$p_1^d = E(\tilde{v} | w_2^d) = \lambda_2 w_2^d.$$

Based on the public disclosure of trader 1, the market maker updates his belief formed on the basis of the first period aggregate order flow. Specifically, let $p_1^*$ be the updated price in period 1 after the market maker sees the disclosed order, then

$$p_1^* = E(\tilde{v} | x_1^d, y_2^d + u_1) = \beta x_1^d + \eta (y_2^d + u_1).$$

Based on $p_1^*$ and the aggregate order flow $w_2^d$, the market maker sets the price in period 2. Thus,

$$p_2^d = E(\tilde{v} | p_1^*, w_2^d) = p_1^* + \lambda_2 w_2^d.$$

The disclosure of trader 1 also allows trader 2 to update his information. Specifically,

$$s_1^d = E(\tilde{v} | s_0, x_1^d) = s_0 + \phi x_1^d. \quad (6)$$

Since period 2 is the last period market participants can trade before the value of the stock is known to the public, disclosure after this round does not affect trader 1’s profit. Therefore, trader 1 need not use a random noise to hide his information. Thus, his period 2 demand has the following form:

$$x_2^d = \beta_2 (v - s_1^d) + \gamma_2 (s_1^d - p_1^*).$$

Trader 2, after updating his signal from $s_0$ to $s_1^d$ using (1), uses $x_1^d$ and the updated price $p_1^*$ to form his second period demand. I hypothesize that:

$$y_2^d = \theta_2 (s_1^d - p_1^*).$$

To derive the equilibrium, I use backward induction to obtain the informed traders’ trading strategies and expected profits as a function of the price. Then, I use the market efficiency conditions to solve for the pricing rules of the market maker. Proposition 3 characterizes the equilibrium under disclosure.

**Proposition 3** A PBE in a setting with mandatory disclosure of insider trades is given by:
where is a solution of the following equation system:

\[
\begin{align*}
\beta_i \gamma_i &\theta_i \lambda_i \phi_i \varsigma_i = \frac{1}{\beta_i} \left( \gamma_i - \theta_i \right), \\
\theta_i & = \frac{1}{2 \phi + 2 \sigma^2}, \\
\phi & = 6 \lambda - 2 \theta_i, \\
\gamma_i & = \frac{1}{2} \left( \beta_i + \gamma_i \right) \sigma^2 \left( \beta_i + \gamma_i \right) \sigma^2 + \varsigma_i, \\
\eta & = \frac{3 \lambda}{2 \theta_i} - \frac{1}{2 \theta_i}, \\
\varsigma_i & = \beta_i \left( 1 - \beta_i \phi \right) \sigma^2.
\end{align*}
\]

(The proof is available upon request.)

4 Discussions

This Section uses a numerical example to discuss the implications of the model. In particular, it will discuss how disclosure regulations change the trading behavior of the insider, how information is revealed to the public over time and how the informed traders’ profits change under disclosure. Two parameters that fully describe the economic environment are the variance of the random variables and , and the variance of the liquidity trades . These parameters are chosen to be 1 and 2 respectively.

If trader 1 is not required to disclose his order, then the unique linear equilibrium is given by:

\[
\begin{align*}
x_1(s_0) & = 0.62 \left( v - s_0 \right) + 0.63 s_0, \\
x_2(s_1, p_1) & = 1.35 \left( v - s_1 \right) + 0.9 \left( s_1 - p_1 \right), \\
y_1(s_0) & = 0.58 s_0, \\
y_2(s_1, p_1) & = 0.9 \left( s_1 - p_1 \right), \\
p_2 & = p_1 + 0.37 w_2.
\end{align*}
\]

We observe that trader 1’s demands and are functions of both his inside information (the first terms in equations (7) and (8)) and the common information (the second terms in equations (7) and (8)). In the first period, he even trades more intensely on the common information (0.63 vs. 0.62); however, this is reversed in the second period (0.9 vs. 1.35). This helps him save more of his private information for the next round. We also see that both traders trade less intensely on the common information in period 1 than in period 2. This strategy reduces the amount of common information released to the market in period 1. In period 2, both traders increase their profit by trading more intensely on all information they possess. Because of this, the expected profit of trader 1 in period 1 is
lower than that in period 2 (0.70 vs. 0.91). It is the same for trader 2 (0.24 vs. 0.34).

The unique linear equilibrium under disclosure is given by:

\[ x_1^d(v, s_0) = -0.02(v - s_0) + 0.67s_0 + z, \]

where \( z \sim N(0, 0.025) \),

\[ x_2^d(v, s_1^d, p_1^d) = 1.47(v - s_1^d) + 0.98(s_1^d - p_1^*), \]

\[ y_1^d = 1.05s_0, \]

\[ y_2^d(s_1^d, p_1) = 0.98(s_1^d - p_1^*), \]

\[ p_1^d = 0.34w_1^d, \]

\[ p_1^* = 1.33y_1^d + 0.04(y_1^d + u_1), \]

\[ p_2^d = p_1^* + 0.34w_2. \]

We can observe that when disclosure is mandatory, trader 1 does not want his order in the first period \( x_1^d \) to fully reveal his perfect information. He obtains this goal by trading against his inside information (the first term in (12) is negative), and adding a random noise \( z \) to his order. In contrast, if disclosure were not required, trader 1 would never trade in a manner which is inconsistent with his information. In the second period, he trades very intensely on the information that is not revealed yet to maximize his profit.

For trader 2, we observe that his trading intensity on common information is almost double in the disclosure case (0.58 vs. 1.05). Trader 1, in response to that, also raises his trading intensity on the common information from 0.63 to 0.67. A possible explanation is that under disclosure, the public can learn some information from the disclosed orders, so both try to exploit it right from the beginning. After the first trading round, trader 2 observes the disclosure of trader 1 and uses it to update his signal from \( s_0 \) to \( s_1^d \). Since he has information advantage over the market maker, trader 2 infers more information from the disclosure than the market maker does.

Using Propositions 2 and 4, one can calculate expected profits of informed traders in both cases. (The detailed calculations are available upon request.) The total profits of both informed traders reduce by 10% under disclosure. Since this is a zero sum game between informed traders and liquidity traders, liquidity traders are better off in general. The total profits of trader 1 decrease by 24%, about half of which goes to trader 2 and the other half goes to liquidity traders. This is reasonable given the fact that trader 2 incurred some costs to acquire information which the paper does not model here.

The contrarian trading and the random noise reduces the period 1 profit of trader 1 by 64%; however, they help trader 1 keep his information from being revealed completely to other market participants. The retained information increases trader 1’s profit in period 2 by 8%.

Trader 2, who, under disclosure, trades very heavily on the common information sees that his profit increase significantly (79%). However, in doing so trader 2 makes almost all common information incorporated into the price. The consequence is that his profit in period 2 is 10% lower under disclosure. Despite that, the total expected profit of trader 2 increases by 28%.

We also see that the mandatory disclosure regulations reduce marginal trading cost. In the non-disclosure case, the marginal trading costs in periods 1 and 2 are 0.48 and 0.37 respectively while in the disclosure case, they are 0.34 in both periods. Thus, the market can benefit from lower trading costs.

For information revelation, we can show that mandatory disclosure regulations make more information released to the public than would be the case without disclosure \( (\Sigma_t > \Sigma_t^d) \). Thus, more information is incorporated into the price. This makes the market more efficient.

5 Conclusion

This paper shows that under disclosure an informed insider could add a random noise and trade against his information to maintain his information superiority. However, disclosure helps outsiders obtain more information to compete with insiders. Overall, the market is more efficient. It also shows that disclosure transfers gains from insiders to outsiders.

References: