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Application of Rate Equations to model growth in biological systems

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Abstract

We have presented here a mathematical model of disease propagation .To make the model we need to make some assumptions about the disease we are going to model. First, we assume that the disease is self-limiting or curable, so that everyone who gets it recovers in a matter of weeks. Secondly, we assume that the disease confers permanent immunity, so that once someone recovers he or she cannot get the disease again. These are reasonable assumptions which are actually true of many real diseases, for example measles***.***

The progress of a disease in a population consists of 2 processes, people getting sick, and people recovering. To consider these processes mathematically, that is in terms of numbers, one begins by dividing the population into groups. Thus, for example, the mathematical description of a person getting well would be the decrease of the group of sick people by one, and the increase by one of the group of recovered people. So the first step in making a model of the disease is to define the groups. If one considers such a disease in some population, the population can be divided into three groups, at any given time.

Key Words :- Rate equations , Population , Rate of transmission , Recovery rate, Immunity.

MATHEMATICAL MODEL

The first step in making a model of the disease is to define the groups . If one considers such a disease in some population, the population can be divided into three groups, at any given time.

Susceptible

The most obvious division is into the sick and the not sick. But the people who are not sick can be further divided into those who have already had the disease, and those who have not. Those who have not yet had the disease are those who are susceptible to it.

Recovered

The well people who have already had the illness and are immune are the recovered part of the population.

Infected

Those people who currently have the disease and are capable of transmitting it to others are infected.

Let us use S, I, and R to denote the number of people in each of the susceptible, infected, and recovered groups.

As time passes people will move from one of these groups to the other. If someone gets sick she moves from the S group to the I group. If someone recovers she moves from the I group to the R group. Since each one of the groups will be changing we will have a rate of change for each group. These rate of change will be denoted by the ' symbol. So the 3 rates are S', I' and R'. There will be some relationship between the 3 rates of change since we are assuming that the total number of people in the population is constant.

The Recovery Rate

The recovery rate is the simplest rate to consider because we are assuming that recovery from an illness is only a matter of time. If we consider some disease like measles, it takes on the average about 14 days to recover. So if we look at the entire infected population today, we can expect to find some who have been infected less than one day, some who have been infected between one and two days, and so on, up to fourteen days. Those in the last group will recover today. In the absence of any definite information about the fourteen groups, let's assume they are the same size. Then 1/14-th of the infected population will recover today: Thus the rate equation for R' says that R' will be equal to 1/14 of the infected population, I.

$$R' = (1/14) \times I$$

In order to make the model more general, we can define b as the recovery coefficient, which in our specific case we chose to be (1/14). So we get the general formulation

$$R' = b \times I$$

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Rate of Transmission

The rate of transmission, which is the rate of growth of the infected population, **I**, is not so simple. Since we are considering a contageous disease, in order for a person to become sick she must come into contact with someone who is already sick. If I am a uninfected person, one of the susceptible group, my chances of coming into contact with a sick person will depend on how many sick people there are. If I am a sick person, my chance of coming into contact with a susceptible person will be greater if there are many susceptible people, and smaller if there are few susceptible people.

Here's a way to model the transmission rate. First, let's consider a single susceptible person on a single day and let's say that on that day there are 5000 sick people in the population. Since not every person in the population comes into contact with every other person each day, we should assume that the our healthy subject comes into contact with only a small fraction of the sick population. Suppose there are 5000 infected people, so $I = 5000$. We might expect only a couple of them--let's say 2--will be in the same classroom with our ``average'' susceptible. So the fraction of contacts is

$$p = 2/I = 2/5000 = .0004.$$

The 2 contacts themselves can be expressed as

$$2 =(2/I) \times I \leq p \times I$$

contacts per day per susceptible.

To find out how many daily contacts the whole susceptible population will have, we can just multiply the average number of contacts per susceptible person by the number of susceptible: this is

$$p \times I \times S = pS\,I$$

Not all contacts lead to new infections; only a certain fraction q do. The more contagious the disease, the larger q is. Since the number of daily contacts is pSI, we can expect $p \times q\,SI$ new infections per day (i.e., to convert contacts to infections, multiply by q .

This becomes $a\,SI$ if we define **a**, for convenience, to be the product $q\,p$.

Thus the conclusion is that the rate of new infections is

$$a \times S \times I$$

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