

# Possibilistic Cournot Equilibrium for Electricity Markets

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*Abstract:* - Cournot non-cooperative game theory is one of the theoretical approaches more used to model market behavior in the electricity industry. However this approach is highly influenced by the residual demand curves of the market agents, which are usually not precisely known. Imperfect information has normally been modeled with Probability Theory, accepting to treat it as randomness. However, Possibility Theory might sometimes be more helpful than Probability Theory in modeling uncertainty, imprecision and vagueness. A Possibilistic Cournot equilibrium formulation is proposed, and two dual and complementary approaches are applied to simplify from fuzzy to deterministic, to compute a robust solution, when the residual demand uncertainty is modeled with a possibility distribution.

*Key-Words:* - Electricity market, Cournot game theory, Possibility Theory, Fuzzy programming, Chance constraints

## 1 Introduction

Most electricity oligopoly markets are being regulated on the base of competition among the companies or agents, to establish auto-regulated price fixing mechanisms. One important challenge is the proper modeling of the market behavior to forecast agent offer strategies. One of the main modeling approach is the so-called *market equilibrium theory* [1], and in particular, *Cournot equilibrium* approach, possibly the theoretical scheme most commonly used [2], [3]. However, there are few published works where the Cournot approach takes into account the electricity market uncertainty (see [4] for different sources of uncertainty). Most of them do not take into account its potentially high sensibility with respect to the residual demand curves of each agent [5], and assume that a probabilistic estimation for these curves is always available [6][7][8]. For instance, [9] only considers that operating cost of rival producers are probability distributions.

However (see 3.1) several drawbacks prevent from using the probabilistic approach, being sometimes more natural to choose a more flexible uncertainty model such as Possibility Theory, although less informative. Evidence Theory helps to fill the gap between possibility and probability, by providing a frequentist interpretation of *possibility distribution* [14]. It reduces to Possibility Theory, when focal elements are nested intervals, and to Probability Theory, when they are singleton [15]. Finally, it should be said that possibility distributions model not only uncertainty but also

imprecision and vagueness [16], and can be well suited for linguistic information. That is why in this paper possibility distributions have been chosen to model the residual demand curves uncertainty.

When modeling real electricity markets, it becomes convenient to obtain sensible energy agent productions when they face up unfavorable, but possible residual demand scenarios. These types of solutions, popularly called robust solutions, have been widely studied in operation research, leading to several similar interpretations of *robustness* [17][18]. In this paper new possibilistic Cournot equilibrium is proposed, and the approaches introduced in [20] for possibilistic objective optimization problems and applied in [21] to Cournot equilibrium are reviewed.

Next section describes the bases of Cournot equilibrium. Section 3 compares probability and possibility for uncertainty modeling of the residual demand curves. Section 4 proves that Cournot equilibrium under possibilistic uncertainty can be formulated as a possibilistic optimization problem. Section 5 applies two approaches to compute a robust Cournot equilibrium. Finally conclusions are given.

## 2 Classical Cournot Equilibrium

Consider an electricity market in which  $E$  producers offer  $P_e$  quantities of energy ( $e=1,\dots,E$ ) to a large number of consumers, whose demand  $D$  is a linear function of price market  $\lambda$ , with inelastic demand  $d$  and slope  $\rho$ , i.e.,  $D(\lambda)=d-\lambda\cdot\rho$ . Assume that productions  $P_e$  do not respond to changes in the market price ( $P_e(\lambda)=P_e$ ), and that the generation costs of each producer are represented by functions  $C_e(P_e)$ . Cournot

game theory provides a general approach to the problem of finding the agents behavior in the market, when producers make decisions independently and simultaneously and don't cooperate with each others. The solution, in the sense of Nash [22], provides a production  $P_e^*$  for each producer  $e$ , that maximizes its individual profit when the productions of the others agents are supposed to be fixed:

$$B_e(P_1^*, \dots, P_e^*, \dots, P_E^*) = \max_{P_e} B_e(P_1^*, \dots, P_e, \dots, P_E^*) \quad \forall e = 1, \dots, E \quad (1)$$

Individual profit  $B_e(P_e)$  is calculated by subtracting agents costs from incomes, and can be formulated in terms of  $P_e$  and  $\lambda$ , or in terms of  $P_e$  and  $D$ :

$$B_e(P_e) = \lambda \cdot P_e - C_e(P_e) \Leftrightarrow B_e(P_e) = \mu \cdot (d - D) \cdot P_e - C_e(P_e) \quad \forall e = 1, \dots, E \quad (2)$$

where  $\mu$  is the inverse of the slope of the demand curve ( $\mu=1/\rho$ ) and the demand  $D$  is also interpreted as function of the productions  $\{P_e\}$  through the power balance equation:

$$d - \lambda \cdot \rho = \sum_{e=1}^E P_e \Leftrightarrow D = \sum_{e=1}^E P_e \quad (3)$$

Cournot equilibrium can be obtained by differentiating and zeroing  $B_e(P_e)$ , subject to the balance equation:

$$\frac{\partial B_e(P_e)}{\partial P_e} = 0 \quad \forall e = 1, \dots, E \quad (4)$$

s.t. balance equation

Or equivalently:

$$\left\{ \begin{array}{l} \lambda = \frac{\partial C_e}{\partial P_e} + \mu \cdot P_e \quad \forall e = 1, \dots, E \\ \lambda = \mu \cdot \left( d - \sum_{e=1}^E P_e \right) \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{l} -\frac{\partial C_e}{\partial P_e} + \mu \cdot (d - P_e - D) = 0 \quad \forall e = 1, \dots, E \\ D = \sum_{e=1}^E P_e \end{array} \right\} \quad (5)$$

Clearly, in the equilibrium marginal costs ( $\partial C_e / \partial P_e$ ) are equal to marginal incomes ( $\lambda - \mu \cdot P_e$ ).

### 3 Uncertain residual demand curves

The residual demand curve (RDC)  $D_e(\lambda)$  of an agent  $e$  quantifies the amount of production  $D_e$  that the agent is able to sell at market price  $\lambda$  [23]. The RDC is calculated by subtracting the aggregated production of the rest of agents (denoted by  $P_{E-\{e\}}$ ) from the demand  $D$ , and is a function of the market price  $\lambda$ :

$$D_e(\lambda) = D(\lambda) - P_{E-\{e\}}(\lambda) \quad \forall e = 1, \dots, E \quad (6)$$

It is common to express the market price  $\lambda$  as a

function of the agent sales,  $\lambda(D_e)$ , in which case the RDC of each agent  $e$  is the inverse of  $D_e(\lambda)$ . In particular, when an oligopoly Cournot model is considered, then  $P_e(\lambda)$  is independent of the market price  $\lambda$  and therefore the same applies to  $P_{E-\{e\}}(\lambda)$ . If in addition the demand curve  $D$  is supposed to be linear, then  $D_e(\lambda)$  is also linear with negative slope  $\rho$ :

$$D_e(\lambda) = (d - \lambda \cdot \rho) - P_{E-\{e\}} \quad \forall e = 1, \dots, E \quad (7)$$

In this case the RDC (inverse function of  $P_e(\lambda)$ ) is also linear but with negative slope  $\mu$  (inverse of  $\rho$ ) and equal for all the agents:

$$\lambda(D_e) = \mu \cdot (d - D_e - P_{E-\{e\}}) \quad \forall e = 1, \dots, E \quad (8)$$

Since Cournot models assume that rivals do not respond to price changes [24], the results may be extremely sensitive to the form of the RDC. That is why in this paper RDC slope has been considered uncertain (from now on denoted by  $\tilde{\mu}$ ) in order to obtain a robust and crisp Cournot equilibrium, more insensitive to  $\mu$  variability.

#### 3.1 Probabilistic RDC uncertainty modeling

Although the probabilistic approach is the most used to represent uncertainty, several drawbacks suggest the convenience to use possibility distribution for uncertainty modeling. When fitting probability density functions (pdf), more than one type of pdf may satisfy the test conditions used to accept the fitting, reducing the validity of this approach. See [25] for a good example of how the high sensitivity of the results with respect to changes in the parameters of the accepted probability distributions suggests the use of possibility distribution instead. Moreover, sufficient historical information to fit a RDC slope pdf is often not available, due to, for example, market rule modifications (more frequent than expected, reducing the validity of historical data), data confidentiality (three months for bidding curves in Spain), etc. Additionally, computationally efficient probabilistic model requires simplifying hypotheses, which can be far from reality (such as normality of the pdf of the RDC slope  $\tilde{\mu}$ ) reducing the rigor of the probabilistic approach. Finally, probability is not well suited for representing subjective linguistic inputs provided by the experts about the future behavior of the RDC.

#### 3.2 Possibilistic RDC uncertainty modeling

LR possibility distributions have been chosen to model the RDC slope. Let  $\pi \equiv \tilde{\mu} = (\mu^L, \mu^R, \alpha \mu^L, \alpha \mu^R)_{LR}$  be a LR possibility distribution [26]. Then:

$$\pi(\mu) = \begin{cases} L((\mu^L - \mu) / \alpha \mu^L), & \mu \leq \mu^L \\ 1, & \mu^L \leq \mu \leq \mu^R \\ R((\mu - \mu^R) / \alpha \mu^R), & \mu^R \leq \mu \end{cases} \quad (9)$$

where  $L: \mathfrak{R} \rightarrow [0,1]$  and  $R: \mathfrak{R} \rightarrow [0,1]$  are defined in  $\mathfrak{R} = (-\infty, +\infty)$ , continuous and differentiable, and strictly decreasing in  $(0,1)$  and such that  $L(s)=R(s)=1$  for  $s \leq 0$  and  $L(s)=R(s)=0$  for  $s \geq 1$ . The support  $\{u/\pi(\mu) > 0\}$  is assumed to be bounded. When  $L(s)=R(s)=1-s$  for  $s \in [0,1]$ , trapezoidal possibility distributions are obtained.

The main advantage of  $LR$  distributions is their closure under some operations performed via the extension principle. For example the addition and the multiplication by a scalar are  $LR$  distributions:

$$(m^L, m^R, \alpha^L, \alpha^R)_{LR} + (n^L, n^R, \beta^L, \beta^R)_{LR} = \quad (10)$$

$$(m^L + n^L, m^R + n^R, \alpha^L + \beta^L, \alpha^R + \beta^R)_{LR}$$

$$x \cdot (m^L, m^R, \alpha^L, \alpha^R)_{LR} =$$

$$\begin{cases} (x \cdot m^L, x \cdot m^R, x \cdot \alpha^L, x \cdot \alpha^R)_{LR} & x \geq 0 \\ (x \cdot m^R, x \cdot m^L, -x \cdot \alpha^R, -x \cdot \alpha^L)_{RL} & x < 0 \end{cases}$$

This fact contributes to make algebraic operations with possibility distributions very efficient. Besides, possibilistic computation is much easier than probabilistic computation where integrals are typically involved.

#### 4 Uncertain Cournot equilibrium

Let's suppose that the RDC slope is modeled with a possibility distribution  $\tilde{\mu}$ , so that the profit of each agent  $e$  is also given by the following possibility distribution ( $D$  is a function of the productions  $\{P_e\}$  through the power balance equation, denoted by  $D(P_e)$ ):

$$\tilde{B}_e(P_e) = \tilde{\mu} \cdot P_e \cdot (d - D(P_e)) - C_e(P_e) \quad (11)$$

$$\forall e = 1, \dots, E$$

If  $\tilde{\mu}$  is a  $LR$  possibility distribution, using (10) each firm profit is also a  $LR$  possibility distribution:

$$\tilde{B}_e(P_e) = (B_e^L(P_e), B_e^R(P_e), \alpha B_e^L(P_e), \alpha B_e^R(P_e))_{LR} \quad (12)$$

$$\forall e = 1, \dots, E$$

where  $\forall e = 1, \dots, E$  and  $H \in \{L, R\}$  it is:

$$B_e^H(P_e) = \mu^H \cdot P_e \cdot (d - D(P_e)) - C_e(P_e) \quad (13)$$

$$\alpha B_e^H(P_e) = \alpha \mu^H \cdot P_e \cdot (d - D(P_e))$$

Then, the Cournot equilibrium can be obtained by fuzzy differentiation and fuzzy zeroing:

$$\frac{\partial \tilde{B}_e(P_e)}{\partial P_e} \cong 0 \quad \forall e = 1, \dots, E \quad (14)$$

Where  $\cong$  is a fuzzy equality, since strict equality with crisp  $P_e$  would lead to non-existing equilibriums. Differentiation of possibility-valued mappings ([27]) allows computing the derivative of the profit of each agent  $e$  as:

$$\frac{\partial \tilde{B}_e(P_e)}{\partial P_e} = \begin{cases} \left( \frac{\partial B_e^L(P_e)}{\partial P_e}, \frac{\partial B_e^R(P_e)}{\partial P_e}, \frac{\partial \alpha B_e^L(P_e)}{\partial P_e}, \frac{\partial \alpha B_e^R(P_e)}{\partial P_e} \right)_{LR} & \text{if } d - D - P_e \geq 0 \\ \left( \frac{\partial B_e^R(P_e)}{\partial P_e}, \frac{\partial B_e^L(P_e)}{\partial P_e}, -\frac{\partial \alpha B_e^R(P_e)}{\partial P_e}, -\frac{\partial \alpha B_e^L(P_e)}{\partial P_e} \right)_{RL} & \text{otherwise} \end{cases} \quad (15)$$

where  $\forall e = 1, \dots, E$  and  $H \in \{L, R\}$  it is:

$$\frac{\partial B_e^H(P_e)}{\partial P_e} = -\frac{\partial C_e}{\partial P_e} + \mu^H \cdot (d - D(P_e) - P_e) \quad (16)$$

$$\frac{\partial \alpha B_e^H(P_e)}{\partial P_e} = \alpha \mu^H \cdot (d - D(P_e) - P_e)$$

According to the extension principle, (15) can be condensed in a simpler function of  $\tilde{\mu}$ , which substituted in (14) results in the possibilistic Cournot equilibrium equations proposed in this paper:

$$-\frac{\partial C_e}{\partial P_e} + \tilde{\mu} \cdot (d - D - P_e) \cong 0 \quad \forall e = 1, \dots, E \quad (17)$$

$$D = \sum_{e=1}^E P_e$$

Generalizing similar results in [19] for the crisp case, solving these equations is equivalent to optimizing the following fuzzy programming model (the balance equation and the non-negative conditions of  $P_e$  are denoted by region  $F$ ):

$$\tilde{Min}_{(\bar{P}, D) \in F} \left\{ \tilde{C}(\bar{P}, D, \tilde{\mu}) = \sum_{e=1}^E C_e(P_e) + \frac{\tilde{\mu}}{2} \cdot S(\bar{P}, D) \right\} \quad (18)$$

where  $\tilde{Min}$  is some kind of fuzzy minimization to be defined, and where:

$$\bar{P} = \{P_e\} = (P_1, \dots, P_E) \quad \text{and} \quad (19)$$

$$S(\bar{P}, D) = D^2 + \sum_{e=1}^E P_e^2 - 2 \cdot D \cdot d$$

The equivalence in the fuzzy case can be obtained as in [19], but in this case, optimizing (18) is equivalent to optimizing the following fuzzy Lagrange function:

$$\tilde{L}(\bar{P}, D, \tilde{\lambda}, \tilde{\mu}) = \tilde{C}(\bar{P}, D, \tilde{\mu}) + \tilde{\lambda} \cdot \left( D - \sum_{e=1}^E P_e \right) \quad (20)$$

By fuzzy differentiation and fuzzy zeroing, the fuzzy equation (17), the  $LR$  possibility distribution of market price  $\tilde{\lambda}$ , and the balance equation are again obtained:

$$\left[ \begin{array}{l} \frac{\partial \tilde{L}}{\partial P_e} \cong 0 \Rightarrow -\frac{\partial C_e}{\partial P_e} + \tilde{\mu} \cdot (d - P_e - D) = 0 \\ \forall e = 1, \dots, E \\ \frac{\partial \tilde{L}}{\partial D} \cong 0 \Rightarrow \tilde{\lambda} = \tilde{\mu} \cdot (d - D) \\ \frac{\partial \tilde{L}}{\partial \tilde{\lambda}} \cong 0 \Rightarrow D = \sum_{e=1}^E P_e \end{array} \right] \quad (21)$$

outlining the main steps to prove the equivalence. Note that fuzzy equality must be the same in (14) and (21), which in turn determines the fuzzy minimization criterion of (18). Using (10) it is possible to compute

the LR distribution of the fuzzy cost of (18):

$$\tilde{c}(\bar{P}, D, \tilde{\mu}) = \begin{cases} (C^L(\bar{P}, D), C^R(\bar{P}, D), \alpha C^L(\bar{P}, D), \alpha C^L(\bar{P}, D))_{LR} & \text{if } S(\bar{P}, D) \geq 0 \\ (C^R(\bar{P}, D), C^L(\bar{P}, D), -\alpha C^R(\bar{P}, D), -\alpha C^L(\bar{P}, D))_{RL} & \text{otherwise} \end{cases} \quad (22)$$

where for  $H \in \{L, R\}$  it is:

$$C^H(\bar{P}, D) = \sum_{e=1}^E C_e(P_e) + \frac{\mu^H}{2} \cdot S(\bar{P}, D) \quad (23)$$

$$\alpha C^H(\bar{P}, D) = \frac{\alpha \mu^H}{2} \cdot S(\bar{P}, D)$$

Since  $d \geq D$  and due to the power balance equation (3), it is:

$$S(\bar{P}, D) \leq -D^2 + \sum_{e=1}^E P_e^2 = -2 \cdot \sum_{e < e'} P_e \cdot P_{e'} \leq 0 \quad (24)$$

Therefore, model (18) is equivalent to minimizing the following fuzzy cost, which measures the system efficiency in the fuzzy approach:

$$\tilde{Min}_{(\bar{P}, D) \in F} (C^R(\bar{P}, D), C^L(\bar{P}, D), -\alpha C^R(\bar{P}, D), -\alpha C^L(\bar{P}, D))_{RL} \quad (25)$$

As in [19] additional constraints can be included into the model, such as the technical constraints of each agent generation groups. A more temporal detail can also be considered, such as a multi-period case for stationalities and different load levels.

Summarizing, it has been proved that it is possible to generalize the deterministic crisp Cournot equations showed in (5) to the fuzzy case using fuzzy problem (25). Besides, since  $\{P_e\}$  are supposed crisp, a crisp optimization criterion must be defined to solve problem (25).

This paper applies two different resolution approaches introduced in [21] to compute robust Cournot equilibriums, but other alternatives can be found in [28]. Most of them are based on different criteria for ranking fuzzy numbers [29] applied to fuzzy objectives, fuzzy constraints, or both.

Given any solution of (25), the agent productions obtained can then be used to compute the possibility distribution of each agent profit or the possibility distribution of the market price:

$$\tilde{B}_e(P_e) = \tilde{\mu} \cdot P_e \cdot (d - D) - C_e(P_e) \quad \forall e = 1, \dots, E \quad (26)$$

$$\tilde{\lambda} = \tilde{\mu} \cdot (d - D)$$

Both distributions can be used to quantify the risk assumed by each agent and the volatility of the price, and will be computed in the case study for results comparison.

## 5 Robust deterministic formulation

A robust equilibrium can be interpreted as the equilibrium resulting from agents with risk aversion, looking for stable results when faced to unexpected but possible inputs; in this case due to the RDC slope variability.

When the equilibrium is solved for a given RDC slope  $\mu$ , but the real demand is more elastic (lesser slope  $\mu^r$ ), according to the equilibrium constraints (5), if the agents keep the same productions  $P_e$  (Fig. 1), then a higher market price is obtained ( $\lambda$  instead of  $\lambda^r$ ), less generation is needed ( $P_e$  instead of the optimum  $P_e^r$ ), and lesser global agent profits can be observed (although some individual profits could decrease with respect to the optimum) as well as a loss of the demand utility due to a larger amount of not satisfied demand ( $D$  instead of  $D^r$ ).

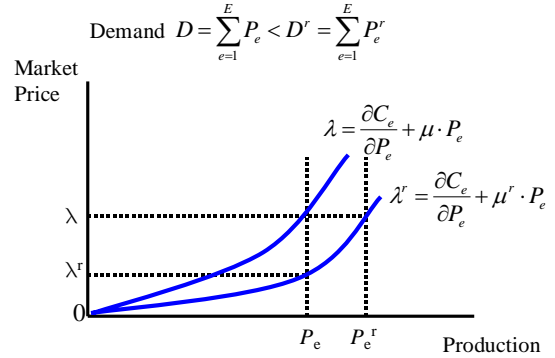


Fig. 1: Equilibrium conditions with  $\mu^r < \mu$

Risk averse agents should protect themselves against this case, trying to reduce the risk of having non-unexpected low profits.

### 5.1 The primal approach

If each agent looks for a set of productions that maximizes a pessimistic (minimum) profit  $B_{e,\min}(P_e, \alpha^0_{\max})$ , such that the possibility of having profits lesser than  $B_{e,\min}(P_e, \alpha^0_{\max})$  is equal to a same value  $\alpha^0_{\max} \in [0, 1]$  for any agent, then it is:

$$Max_{P_e} \left\{ B_{e,\min}(P_e, \alpha^0_{\max}) = Min_{B_e(P_e, \alpha^0_{\max})} B_e(P_e, \alpha^0_{\max}) \right\} \quad (27)$$

$$\forall e = 1, \dots, E$$

$$s.t. \quad \Pi \{ \tilde{B}(P_e) \leq B_e(P_e, \alpha^0_{\max}) \} = \alpha^0_{\max}$$

Every agent is then ready to assume the same amount of possibilistic risk  $\alpha^0_{\max}$ , which is coherent with classical game theory where players behave the same way. For each production  $P_e$  the pessimistic profit  $B_{e,\min}(P_e, \alpha^0_{\max})$  is given by the minimum profit  $B_e(P_e, \alpha^0_{\max})$  with possibility  $\alpha^0_{\max}$  (Fig. 2):

$$B_{e,\min}(P_e, \alpha^0_{\max}) = Min_{B_e(P_e, \alpha^0_{\max})} B_e(P_e, \alpha^0_{\max}) \quad (28)$$

$$s.t. \quad \Pi \{ \tilde{B}(P_e) \leq B_e(P_e, \alpha^0_{\max}) \} = \alpha^0_{\max}$$

$$\Downarrow \forall e = 1, \dots, E$$

$$B_{e,\min}(P_e, \alpha^0_{\max}) = B_e^L(P_e) - L^{-1}(\alpha^0_{\max}) \cdot \alpha B_e^L(P_e)$$

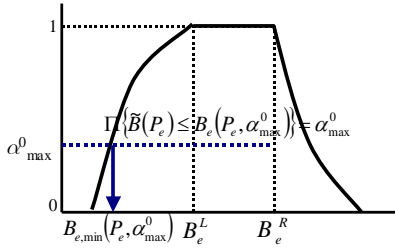


Fig. 2: Minimum profit with possibility  $\alpha_{max}^0$

According to the extension principle,  $B_{e,min}(P_e, \alpha_{max}^0)$  is then reached for  $\mu = \mu^L \cdot L^{-1}(\alpha_{max}^0) \cdot \alpha \mu^L$ . This means that the possibilistic problem (27) leads to a simpler programming model with no integral calculus, and, since  $\mu = \mu^L \cdot L^{-1}(\alpha_{max}^0) \cdot \alpha \mu^L$ , the original fuzzy model can be simplified into a crisp one that can be solved as in [19], where the maximum cost in (25) with possibility  $\alpha_{max}^0$ , denoted by  $C_{max}()$ , is minimized according to the following problem:

$$\begin{aligned} & \text{Max}_{P_e} \left\{ \begin{array}{l} B_{e,min}(P_e, \alpha_{max}^0) = \\ B_e^L(P_e) - L^{-1}(\alpha_{max}^0) \cdot \alpha B_e^L(\bar{P}, D) \end{array} \right\} \quad \forall e = 1, \dots, E \quad (29) \\ & \quad \quad \quad \updownarrow \\ & \text{Cl}_{max}^0 = \text{Min}_{(\bar{P}, D) \in F} \left\{ \begin{array}{l} C_{max}(\bar{P}, D, \alpha_{max}^0) = \\ C^L(\bar{P}, D) - L^{-1}(\alpha_{max}^0) \cdot \alpha C^L(\bar{P}, D) \end{array} \right\} \end{aligned}$$

## 5.2 The dual approach

Instead of maximizing a pessimistic profit, a dual approach can be defined if agents look for a set of productions minimizing the possibility of having profits lesser than a minimum target  $B_{e,min}^0$ , fixing the productions of the other agents (Fig. 3):

$$\begin{aligned} & \text{Min}_{P_e} \left\{ \alpha_{e,max}(P_e, B_{e,min}^0) = \Pi \left\{ \tilde{B}_e(P_e) \leq B_{e,min}^0 \right\} \right\} \quad (30) \\ & \quad \quad \quad \forall e = 1, \dots, E \end{aligned}$$

This is equivalent to maximize the necessity of having profits greater than  $B_{e,min}^0$ , that is:

$$\begin{aligned} & \text{Max}_{P_e} \left\{ \beta_{e,min}(P_e, B_{e,min}^0) = N \left\{ \tilde{B}_e(P_e) \geq B_{e,min}^0 \right\} \right\} \quad (31) \\ & \quad \quad \quad \forall e = 1, \dots, E \end{aligned}$$

Each agent tries to necessarily reach a minimum profit, protecting against the risk of having unacceptable low profits, coherent with classical game theory where players are self-interested profit maximizers, taking decisions for good-defined objectives. For simplicity same possibility levels  $\alpha(P_e, B_{e,min}^0)$  can be assumed for all agents. The equivalence of this problem with the next non-linear one is straight forward:

$$\text{Min}_{P_e} \left\{ \alpha(P_e, B_{e,min}^0) = L \left( \frac{B_e^L(P_e) - B_{e,min}^0}{\alpha B_e^L(P_e)} \right) \right\} \quad (32)$$

$$\forall e = 1, \dots, E$$

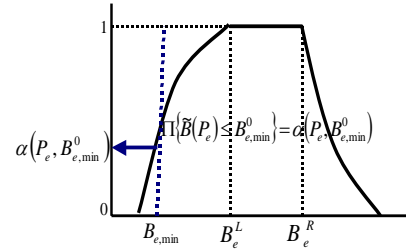


Fig. 3: Possibility of a fixed profit  $B_{e,min}^0$

In the optimum, a bad choice of profits  $B_{e,min}^0$  could provide arguments for  $L()$  outside the interval  $(0,1)$  (smaller than zero if  $B_{e,min}^0$  is large, and larger than one if  $B_{e,min}^0$  is small). Since  $L()$  is not monotonous outside  $(0,1)$ , this situation corresponds to a degenerated case not considered. In this case it is possible to replace  $L()$  by a new function  $L_a()$  monotonous on  $\mathfrak{R}$ , with a non-pieceswise formulation. When trapezoidal fuzzy numbers are considered it is useful to consider  $L_a(s) = 1 - s$ , for all  $s \in \mathfrak{R}$ . However, without lack of generality it can be assumed that arguments for  $L()$  are restricted to the interval  $(0,1)$ . It is easy to prove that equation (32) is equivalent to the following non-linear programming model:

$$\alpha_{max}^0 = \text{Min}_{(\bar{P}, D) \in F} \left\{ \begin{array}{l} \alpha_{max}(\bar{P}, D, C_{max}^0) = \Pi \left\{ \tilde{C}(\bar{P}, D) \geq C_{max}^0 \right\} \\ = L \left( \frac{C^L(\bar{P}, D) - C_{max}^0}{\alpha C^L(\bar{P}, D)} \right) \end{array} \right\} \quad (33)$$

where, if  $P_{e,min}^0$  and  $\mu_{min}^0$  are the productions and the RDC slope that give profits  $B_{e,min}^0$ , then  $C_{max}^0$  is:

$$\begin{aligned} & C_{max}^0 = \sum_{e=1}^E C_e(P_{e,min}^0) + \frac{\mu_{min}^0}{2} \cdot S \left( \{P_{e,min}^0\}, \sum_{e=1}^E P_{e,min}^0 \right) \quad (34) \end{aligned}$$

A combined approach can also be defined [21] as a compromise solution between primal and dual approaches, maximizing simultaneously the profit  $B_{e,min}(P_e, \alpha_{max}^0)$  and the necessity  $\beta(P_e, B_{e,min}^0) = 1 - \alpha(P_e, B_{e,min}^0)$ .

## 6 Conclusions

The results presented in this paper, together with the real numerical application (to the Spanish hydrothermal electric power system, see [21]) have shown that Possibility Theory can effectively be used to model the uncertainty of the residual demand curve (RDC) for an electricity market, where each agent maximizes its benefit based on the Cournot equilibrium conjecture. It has been proved that Cournot equilibrium can be obtained by solving a fuzzy programming model, which generalizes the approach proposed in [19]. Since it is not possible to

find an optimal equilibrium for each possible RDC, criteria proposed in [20] and [21] can be used to obtain a robust Cournot equilibrium.

The authors are currently preparing a paper on a conjectural-variation equilibrium with uncertain (possibilistic) residual demand slopes, leading to a more general and powerful market modeling under uncertainty.

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