Algorithm for increase the transportation network capacity with minimum cost

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Abstract: - This paper presents the problem of increasing the capacity of a transportation network with minimal costs. Supposing we have a transportation network that simulates a real situation and we need to increase the capacity of network with a specified quantity from the substance transported. The most algorithms that solve transportation network consider the arcs transportation capacity for determining the maximum of the capacity of the network. But real network are formed by real elements (roads, pipes etc.). If we need to increase the capacity of the network we have two solutions: increase the capacity of one or many existing arcs or installing a new arc between two nodes. This problem involves a total cost of the operations that depend on the selected solution. The scope of this work is to determine the minimum cost for increasing the capacity of a transportation network with a determined value.

Key-Words: - **transport network, arc, node, flux, capacity**

1 Introduction

A transportation network is an oriented graph that simulate real network that facilitate the transport of something (substance, cars, peoples etc.) between two or many points.

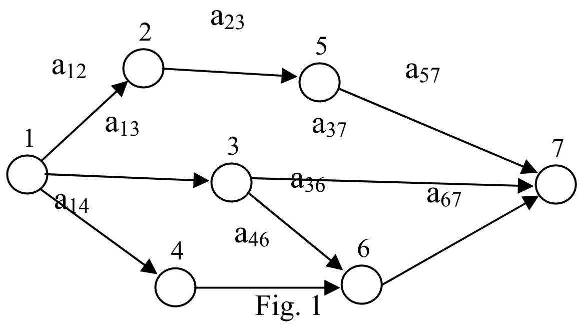

Fig. 1

The network is formed by nodes and oriented arcs.
By every arc flows a quantity of substance or a number of cars in every time unit.
Every arc has a limited capacity of transportation expressed in flux unit.
The node 1 is the inner node for the network and the node 7 is the outer node.
We have some arcs noted in the figure 1 with a_{ij} where i and j represent the node i and j.
The capacity of every arc is noted with c_{ij} and represents the maximum flux by the arc.
The entire network has a maximum capacity that can be determined with one of these methods: dinamic programming, Ford-Fulkerson method and preflux method.
If we need to increase the capacity of the network we can generate a solution analyzing the network configuration.
But real network use arcs that have specified length and the operations for increase the capacity of the arc determine a specified cost. In case of real network the solution for increasing the capacity of the network need to consider these characteristics of the arcs.

2 Problem Formulation

Suppose we have a transportation network that simulates a real network like a pipes network for fluid transportation, a roads network from a city or from a country, an electrical network for electrical signals transfer etc. (fig. 1).
We need to increase the entire network capacity with a specified value of flux but with a minimum cost for the operations.
Because every arc had a correspondent in the real world we need to consider the necessary cost for increasing the every arc capacity.
But an arc had a specified length and a specified cost for increase his capacity.
In case of real correspondent of the arcs we analyze roads or pipes or other structures. To extend the capacity of a road is necessary to extend the width of the road. To extend the capacity of a pipe is

necessary to replace the pipe with other, with a large diameter.
These operations involve many costs that can not be ignored if we need to increase the transportation network capacity.
For this reason we need to consider these costs when we want to solve the problem of increasing the network capacity.
The total cost of operations depends by the next factors:

- the arcs combination selected for increasing capacity;
- the length of every arc selected;
- the cost of operations for every arc selected.

Because the physical characteristics of the arcs could be differed for every arc we consider the cost per length unit and per flux unit, and we note this cost with CU_{ij}.
The entire cost for increasing the capacity of arc a_{ij} depend the difference between the final capacity of the arc and the initial capacity.
If we note this difference with Δ_{ij} we obtain the equation:

$$CA_{ij} = L_{ij} \times \Delta_{ij} \times CU_{ij} \quad (1)$$

where L_{ij} is the length of arc a_{ij},
CA_{ij} – the entire cost for the a_{ij} arc.
For entire network the cost of operation is:

$$C = CA_{ij}+CA_{kp}+CA_{ln}+ \dots +CA_{qr} \quad (2)$$

We can observe, from equation (2), that the arcs that induce the increase of the entire network capacity can not be determined using specified rules.
For a complex transportation network we can not use a deterministic algorithm for selecting the arcs that minimize the cost of increasing the transportation network capacity. The only possibility is to use heuristic algorithms.

3 Problem Solution

The characteristics of this problem permit to include them in the Greedy type problems.
The greedy algorithms are simple and are used to obtain the solution for optimisation problems like the minimum length rute for a graph, the minimum time of attendance etc.
The solution for a greedy optimisation problem consists by a combination of a lot number of elements.
These problems consist of:

- a lot number of candidats for solution;
- a function that verify if a set of candidats could be a problem solution not necessary optimale;
- a function that verify if a solution set could be completed with another candidat and the resulted set still be a solution for the problem;
- a selection function that permit to select the optimum candidate from the unused candidates;
- a function purpose that offer the value of one solution and represent the function that can need to be optimized.

A greedy algorithm constructs the solution step by step. At the beginning the candidat set is blankness. At every step the algorithm try to append to this set the optimum candidate from unused candidats using section function.
If, after the append of the last candidate, the set is not acceptable the last candidate is eliminated and will not be considered never. If, after the append, the set of candidats is acceptable the process continue with the next step.
Every time when are extended the candidats set we verify if this set consist a solution for the problem.
If greedy algorithm work corect, the first solution accepted will be an optimal solution for the problem.
The optimal solution is not necessary unique. Could be possible that objective function have the same optimale solution for more solution sets.
Formal description of a general greedy algorithm is presented to the next:

function *greedy*(*C*)
{*C* is the multitude of candidats }
S ← Ø {*S* is the solution set}
while not *solution*(*S*) **and** *C* ≠ Ø **do**
{ *x* ← element from *C* who minimizes the function *select*(*x*)
C ← *C* \ {*x*}
if *acceptable*(*S* ∪ {*x*}) **then** *S* ← *S* ∪ {*x*}
}
if solution(*S*) **then return** *S*
else return “not exists solution”

A greedy algorithm doesn’t obtain every time the optimal solution or a solution. It is only a general method that will be necessary in every case to establish if the solution is optimum or not.
In the case of a transportation network we need to establish the functions for verifying if an arc could be into solution set and if the selected set is an acceptable solution.
An arc could be in one of this situation:

- the capacity could be increased with a Δ_{ij} value, determined by the capacity of the neighbours of the arc;
- the capacity could be increased with a value less than Δ_{ij} for technical reasons;
- the capacity of the arc could not be increased for technical reasons.

The Δ_{ij} value represents the minim difference between the arc capacity and the capacity of the neighbour arcs. For example, considering the network from fig. 1, the Δ_{23} value is the minim value of the next differences:

$$d_1 = c_{12} - c_{23} \quad (3)$$

$$d_2 = c_{57} - c_{23} \quad (4)$$

It is logical because if the arc a_{23} is saturated and will increase this capacity we can not exceed the capacity of the arcs a_{12} or a_{57}. We can increase the capacity of the arc a_{23} until the arc a_{12} or the arc a_{57} are saturated.

From Ford-Fulkerson method we know that the network maxim capacity is obtained when a number of arcs are saturated (i.e. the flux is equal with the arc capacity).

It's obviously that the first arcs that can determine an increasing of the transportation network capacity are the saturated arcs.

For this reason we consider these arcs for solving the problem.

We start to solve the problem calculating the Δ_{ij} for every saturated arcs and the cost for increasing every saturated arcs with the calculated value Δ_{ij}.

Will obtain a list of costs CA_{ij} for every saturated arc, list that will be sorted ascending.

Beginning from this list we start the selections of the arcs that can be part of the final solution.

The condition for finishing the selection process is to obtain a total increased transportation network capacity great or equal with the necessary value.

The algorithm is presented to the next:

1. establish the multitude of arc candidats (the list of saturated arcs, obtained with Ford-Fulkerson algorithm)
2. calculate the value Δ_{ij} for every arc from the list;
3. calculate the value CA_{ij} for every arc and create the list with these values;
4. sort ascending the list of CA_{ij} costs;
5. *S* ← Ø { *S* is the solution set of arcs}
6. **while not** *solution*(*S*) **and** CA ≠ Ø
   { *S* ← first element from CA
   CA ← CA – first_element_of_the_list
   S ← *S* ∪ {a_{ij}}
   }
7. **if** solution(S) **then write** *S*
   else write "don't have solution"

The function *solution(S)* evaluates the increased capacity of the network and compares them with the value of the necessary network capacity. If the capacity generated by increased arcs from *S* is great or equal with the new capacity need for the network the process will stopped. Other will continue with the next arc from the list. Maximal value of network capacity could be determined with Ford-Fulkerson method.

The list of candidat arcs will be extracted from results of Ford-Fulkerson algorithm and corrected by eliminating the arcs that could not be increased.

Beginning with this candidate arcs we are sure that we can obtain a minimum cost but not necessary the most minimum possible.

4 Conclusion

In the previous paragraph we present an algorithm for determining a minimal cost for increasing the transportation network capacity of a mentioned network with to a mentioned value.

The presented algorithm has the advantage of simplicity but can not assure the obtaining of the optimal result in every case.

Could be situations when the optimal solution need modifications of the network topology or need to extend and the capacity of other arcs, not only the saturated arcs.

Other disadvantage of the method is represented by the necessity to use repeatedly the Ford-Fulkerson algorithm. This algorithm needs a large number of operations in case of complex network and that can affect the eficiency of the proposed algorithm.

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