Using The Orthogonal Wavelet Transform to Identify Fatigue Features in Variable Amplitude Fatigue Loadings

S. ABDULLAH, A. ZAHARIM
Engineering Faculty, Universiti Kebangsaan MalaysiaDepartment
43600 UKM Bangi, Selangor
MALAYSIA

Abstract: - This paper describes a technique to identify the important fatigue features (or also known as fatigue damaging events or bumps) in fatigue road load data that cause the majority of the total damage. Using this technique, called Wavelet Bump Extraction (WBE), these features were identified in the frequency bands by means of the orthogonal wavelet transform. For this case, the 12th order of Daubechies wavelet functions was used. In this paper, bumps identification has been evaluated using a variable amplitude fatigue loading with WBE. This loading was measured on a road vehicle lower suspension arm. The related findings suggested that this wavelet type was suitable for analysing variable amplitude fatigue loadings.

Key-Words: - Wavelet, Fatigue, WBE, Fatigue loadings, Automotive, Daubechies, Variable amplitude.

1 Introduction

Fatigue damage analysis is one of the key stages in the design of vehicle structural components. One of the vital input variables in the fatigue assessment of consumer products is the load history. For ground vehicles, which have an extremely wide range of uses, a representative road load time history can be hard to quantify. Since it is generally the large amplitude cycles presence in the load histories that cause the majority of damage, they should be retained for durability testing [1]. Several approaches for retaining high amplitudes cycles have been introduced in various domains: time, peak and valley, frequency, cycles, damage and histogram. The most commonly applied procedures in the research literature have been based in the time and the frequency domains [2]. The latest approach applied for the fatigue data editing is using the wavelet transform, which is a part of the subject of this paper.

A new wavelet-based fatigue data editing method introduced by the author for summarising the road load fatigue data [3] by identifying fatigue features (or also known as fatigue damaging events or bumps) and extracting them from the history whilst preserving the load cycles sequence is important. This method was specifically designed to identify and extract the fatigue features using the orthogonal wavelet transform by means of the 12^{th} order of the Daubechies family. This algorithm is called Wavelet Bump Extraction (WBE) and its objective is to maintain the fatigue damage of the mission signal (the shortened output signal) as close as possible to that of the original signal.

Accordingly, the objective of this paper is to show the suitability of the orthogonal wavelet transform to identify fatigue features in the experimental fatigue loadings. Since these are nonstationary data sets, hence, the analysis is more accurate with the use of the wavelet transform.

2 The Orthogonal Wavelet Transform in the WBE Algorithm

Wavelet Bump Extraction (WBE) is a wavelet-based fatigue data editing technique which is used to produce a shortened mission signal of similar behaviour. A flowchart describing the WBE processing is presented in Fig. 1. Three main stages can be observed in the flowchart: the application of the wavelet decomposition process, the identification of fatigue features and the production of the mission signal.

Many experimental signals exhibit time-varying, or nonstationary characteristics, which provide a challenge in signal analysis. Traditional approach for the frequency domain analysis of the time series was performed using the Fourier transform. However, it is not suitable for a nonstationary signal because it cannot provide any information of the spectrum changes with respect to time. Referring to this limitation, therefore, the wavelet transform has been found to be a suitable transform for nonstationary signals [4]. The wavelet transform is the functions in the time-scale domain and it is a significant tool for presenting local features of a signal. The wavelet transform gives a separation of components of a signal that overlap in both time and frequency and it gives a more accurate local description of the signal characteristics. Using the wavelet domain analysis, the time and frequency of an oscillating signal can be detected.

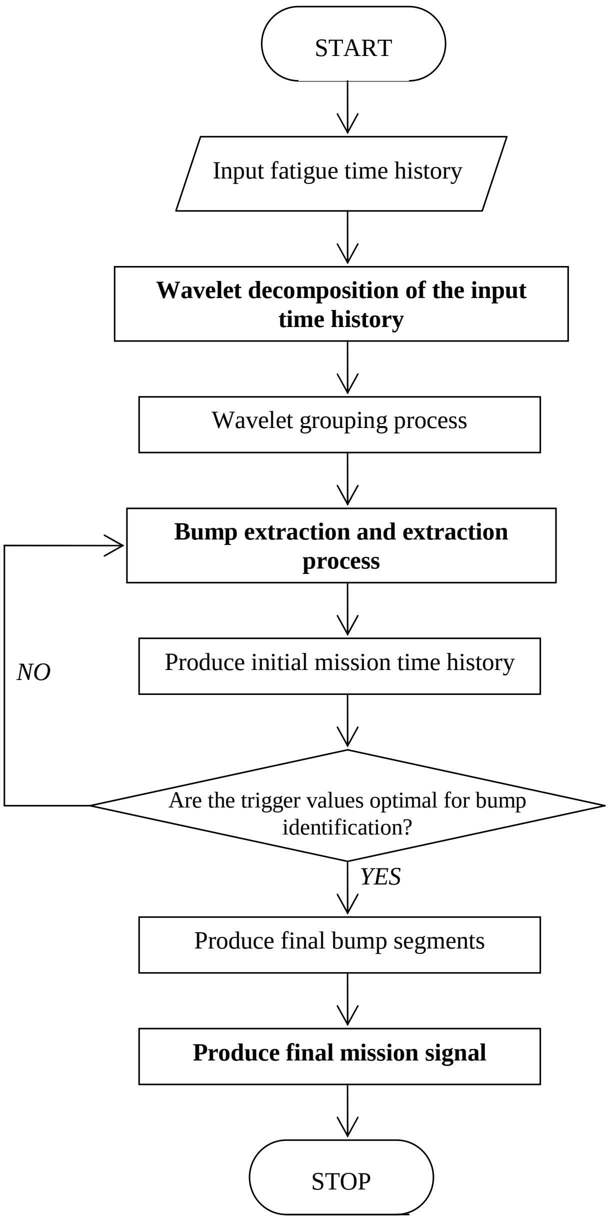

Fig. 1. Flowchart of the WBE algorithm.

Wavelets are analytical functions $\psi(t)$ which are used to decompose a signal $x(t)$ into scaled wavelet coefficients $W_{\psi}(a,b')$. The continuous wavelet transform is a time-scale method that can be expressed as

$$W_{\psi}(a,b') = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t)\psi^{*}\left(\frac{t-b'}{a}\right) \mathrm{d}t \tag{1}$$

where $\psi_{a,b}(t)$ are the scaled wavelets and ψ^{*} is the complex conjugate of ψ. The basis wavelet $\psi(t)$ can be chosen from a number of functions which satisfy a set of admissibility conditions. The admissibility conditions is mathematically defined as

$$\int_{-\infty}^{\infty} \frac{|\Psi(\omega)|^{2}}{|\omega|} d\omega = C_{\psi} < \infty \tag{2}$$

where $\Psi(\omega)$ is the Fourier transform of the mother wavelet. This condition is used for the inversion process of the wavelet transform.

A natural extension of continuous analysis is the discretisation of time *b’* and scale *a* according to $a = a_0^{\ m}$, $b' = na_0^{\ n} b_0$ where *m* and *n* are integers, $b_0 \neq 0$ is the translation step. This implies the construction of a time-scale grid, and thus a discrete wavelet transform can be defined by

$$W_{\psi}(m,n) = \int_{-\infty}^{\infty} x(t) a_0^{-m/2} \psi^{*}\left(\frac{t - nb_0 a_0^{\ m}}{a_0^{\ m}}\right) \mathrm{d}t \tag{3}$$

When the wavelets $\psi_{m,n}(t)$ form a set of orthonormal functions there is no redundancy in the analysis, i.e. the procedure can be precisely inverted. The discrete wavelet transform based on such functions is called the Orthogonal Wavelet Transform (OWT).

There are many wavelet families and one of the most famous is the Daubechies [5] that has the orthogonal basis functions. It is the discrete wavelet transform that allows the decomposition and reconstruction of the input signal in order to observe the signal characteristics within the specific frequency band. Various applications of the orthogonal wavelet transform can be found in fatigue studies, i.e. the compression of nonstationary signals measured from a light railway train component [6] and the automobile data sets [7].

In the first stage of WBE, the power spectral density (PSD) of the input signal is calculated in order to determine its vibrational energy distribution in the frequency domain. This PSD approach is applied in the wavelet decomposition process of the input signal. 12th order of Daubechies’ wavelets were chosen as the basis functions to form an orthogonal set due to the efficiency in providing a large number of vanishing statistical moments. The wavelet levels produced in the wavelet decomposition consist of the reconstructed signals for a given value of scale a_0^{-m} and each level describes the time behaviour of the signal within a specific frequency band.

The number of discrete sampling points in the time history determines how many wavelet levels can be decomposed. When the number of sampling points N is equal to 2^n ($N = 2^n$), the number of levels obtainable from the wavelet decomposition is $n + 1$. A wavelet grouping stage in WBE permits the user to group wavelet levels into single regions of significant energy. Each wavelet group is defined by the user to cover frequency regions of specific interest, such as high energy peaks caused by a subsystem resonances. Fig. 2 shows the concept of the grouping technique based on the PSD plot. This subdividing of the original signal permits analysis to be performed for each frequency region independently, avoiding situations where small bumps in one region are concealed by the greater energy of other regions of the frequency spectrum. Detail of the WBE operation can be found in the related literature [3].

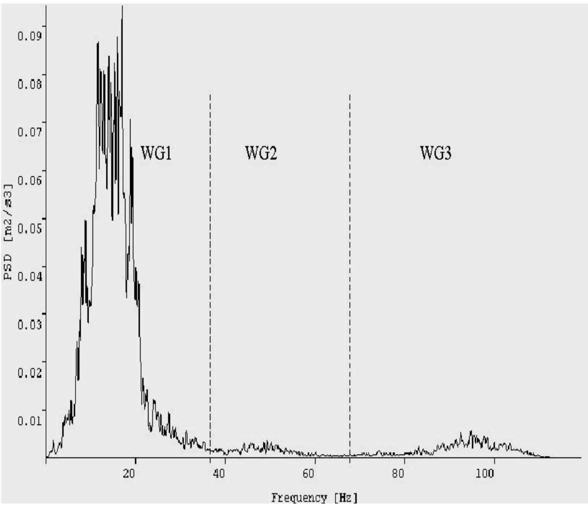

Fig. 2. A PSD plot showing the dividing of a frequency spectrum into three wavelet group (WG)

3 The WBE Application and Results

The accuracy of the fatigue features identification process was evaluated by the application to a variable amplitude fatigue loading which was measured on vehicle suspension arms. This signal (Fig. 3) was measured on a van driven over a pavé test track and it was sampled at 500 Hz with a record length 46 seconds. Using the WBE algorithm, this loading was decomposed into 12 wavelet levels. These levels were later assembled into four wavelet groups.

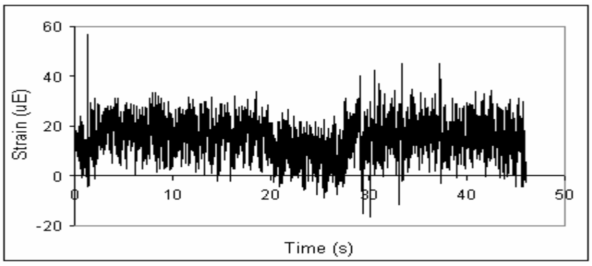

Fig. 3. A fatigue time histories used as the input to the WBE algorithm

The wavelet coefficients generated from the wavelet composition were then used to construct its time history (Fig. 4a). In addition, the locations of bumps present in each wavelet group were also shown in Fig. 4c. The individual bumps in each wavelet group are identified within ±10% of global signal statistics between the original and mission signals (Details of this operation and explanation can be found in [3]). The extracted fatigue features were joined based on their original sequence in order to produce the WBE shortened or mission signal.

To verify the effectiveness of the 12th order of Daubechies in the WBE analysis, the fatigue damage of the original and shortened loadings were compared. The damage values were calculated by applying the Palmgren-Miner's cumulative linear damage rule with two strain-life models with mean stress correction effects. The selected models chosen for this comparison study were Smith-Watson-Topper (SWT) and Morrow, and were respectively defined as:

$$\sigma_{\max}\varepsilon_a = \frac{\left(\sigma'_f\right)^2}{E}\left(2N_f\right)^{2b} + \sigma'_f\varepsilon'_f\left(2N_f\right)^{b+c} \qquad (4)$$

$$\varepsilon_a = \frac{\sigma'_f}{E}\left(1 - \frac{\sigma_m}{\sigma'_f}\right)\left(2N_f\right)^b + \varepsilon'_f\left(2N_f\right)^c \qquad (5)$$

where E is the material modulus of elasticity, ε_a is a true strain amplitude, σ_m is the mean stress, σ_{max} is the maximum stress for the particular cycle, $2N_f$ is the number of reversals to failure, σ'_f is a fatigue strength coefficient, b is a fatigue strength exponent, ε'_f is a fatigue ductility coefficient and c is a fatigue ductility exponent.

Fig. 5 shows the level of fatigue damage for the bump segments, the original signal and the mission signal. For the comparison of fatigue damage between the original signal and its mission signal, at least 98% of fatigue damage was retained when the shortened loading (comprised all extracted fatigue features) is 40% of the original loading time length. The finding showed that most of the original damage was retained in the mission signal, and it indicates the suitability of the 12th order of the Daubechies wavelet transform to be used for analyzing fatigue loadings, by means for the purpose of fatigue data editing applications.

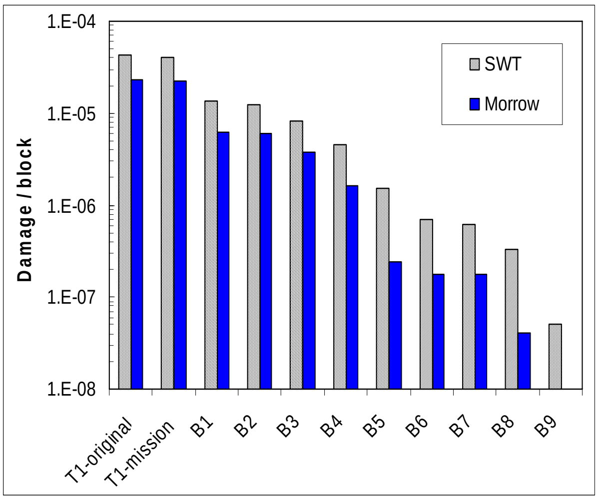

Fig. 5. Damage distribution for the original signal, the mission signal and bump segments (B1-B9 are the bump numbers, as shown in Fig. 4c)

4 Conclusion

Wavelet Bump Extraction (WBE) is an algorithm that was developed to identify fatigue features or bumps, so as to extract them from the original time history. Using the WBE procedure the total damage produced by the combination of the extracted fatigue features was close to that of the original data set. From the obtained results, it is suggested that are a potential and better approach of using the orthogonal wavelet transform (the 12th order of the Daubechies wavelet) to identify and extract fatigue features in variable amplitude fatigue loadings.

References:

[1] A. Conle, TH. Topper, Overstrain effects during variable amplitude service history testing, *International Journal of Fatigue*, Vol. 2, No. 5, 1980, pp. 130-136.

[2] W. El-Ratal, M. Bennebach, X. Lin, R. Plaskitt, Fatigue life modelling and accelerated test for components under variable amplitude loads, *Symposium on Fatigue Testing and Analysis Under Variable Amplitude Loading Conditions*, France, 29-31st May 2002.

[3] S. Abdullah, JR. Yates, JA. Giacomin, Wavelet Bump Extraction (WBE) algorithm for the analysis of fatigue damage, *Proc of the 5th Int Conf on Low Cycle Fatigue (LCF5)*, Berlin, Germany, 9-11th September 2003, pp 445-450.

[4] DE. Newland, *An Introduction to Random Vibrations Spectral and Wavelet Analysis*, 3rd Edition, Longman Scientific and Technical, 1993.

[5] I. Daubechies, *Ten Lectures on Wavelets*, SIAM, 1992.

[6] C-S. Oh, Application of wavelet transform in fatigue history editing, *International Journal of Fatigue*, Vol. 23, 2001, pp. 241-250.

[7] S. Abdullah, JA. Giacomin, JR. Yates, A mission synthesis algorithm for fatigue damage analysis, *Proc of the Instn of Mech Engrs, Part D, Journal of Automobile Engineering*, Vol. 218, No. D3, 2004, pp. 243-258.

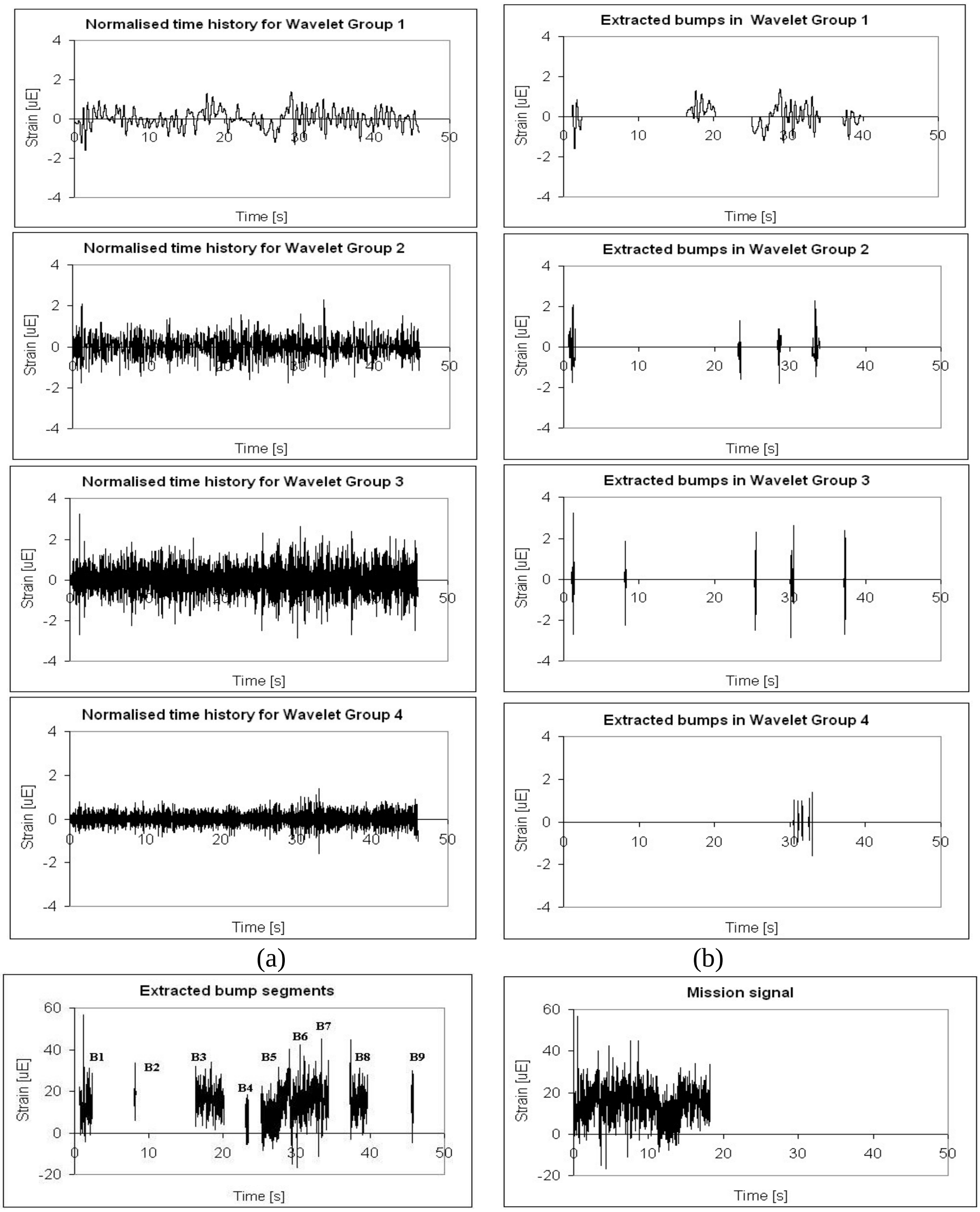

Fig. 4. Normalised time history for all wavelet groups (b) Location of bumps in all wavelet groups, (c) The extracted bump segments (in original scale) at their original location of the input fatigue signal, (d) The mission signal