Optimal design of a linear antenna array using particle swarm optimization

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Abstract: An optimal design of a linear antenna array is presented. The antenna array is optimized using a particle swarm optimization based method in order to produce a radiation pattern that has the minimum possible side lobe level and the maximum possible gain at the desired direction. The method has been applied to collinear wire-dipole arrays and seems to be suitable for improving the radiation patterns in many practical applications.

Key-Words: Antenna arrays, Antenna array synthesis, Antenna beam-forming, Particle swarm optimization

1. Introduction

Antenna arrays have been widely studied due to their importance in communications industry. Many techniques have been proposed on the design of antenna arrays in order to produce radiation patterns that satisfy several requirements [1-3]. In many practical applications, the radiation pattern is required to satisfy three basic conditions: First, the main lobe of the pattern has to provide the maximum possible power gain. Also, the main lobe must be oriented to the desired direction depending on the specific application. Last but not least, the side lobe level must be as low as possible because side lobes are responsible for the power loss caused by spatial spread of radiated power. The above conditions are satisfied by choosing a suitable antenna array geometry and by defining the appropriate excitation applied on the array elements.

In cases of linear antenna arrays, the geometry concerns the distance between the elements, while the excitation concerns the amplitude and phase of the currents applied on the elements by an appropriate feeding network. According to many methods, the crucial condition of low side lobe level is easily achieved by applying a non-uniform excitation distribution on the array elements. One of the most popular excitation distributions is the Chebyshev distribution, which is calculated by applying the design method of Dolph [4]. However, a non-uniform excitation distribution is not easily implemented in practice, because the feeding network needed for such types of excitation is the very complex and quite inefficient.

In order to avoid such feeding networks, the present work introduces an alternative method to produce radiation patterns with low side lobe level. The basic idea of the method is to assume unequal distances between adjacent elements, considering that the excitation currents applied on the elements have the same amplitude. In this case, the feeding network is very simple and easily implemented in practice. Also, the condition for the maximum possible power gain is taken into account by the proposed method. In addition, if the main lobe direction is not normal to the array axis, the method assumes that the excitation currents have different phases. The method has been applied to collinear antenna arrays composed of wire dipoles. The arrays are analyzed by using the Method of Moments [5,6]. The appropriate interelement distances and the phases of the excitation currents are calculated by applying a Particle Swarm Optimization (PSO) algorithm [7-16] developed by the authors. The objective of the PSO algorithm is the maximization of a particular mathematical function called “fitness function”. The fitness function is suitably determined according to the above three conditions. When these conditions are satisfied, the fitness function finds its global maximum value and the algorithm terminates with success.

2. Formulation

The PSO method is based on the intelligence and movement of swarms. According to the PSO terminology [7-16], every individual in the swarm is called “particle”. The number S of the particles is called “population size”. The experience suggests that a value of S between 10 and 50 is a good choice for many problems. All the particles move in the search space and update their velocity according to the best positions already found by themselves and by their neighbors, trying to find an even better position. Each particle is considered as point in an N-dimensional space. The position of the i-th particle (i=1,…,S) is represented as $\vec{x}_i = \left(x_{i1}, x_{i2}, ..., x_{iN}\right)$

the position coordinates. The coordinates x_{in} may be limited between two boundaries, U_n and L_n ($L_n \le x_{in} \le U_n$). The difference $R_n = U_n - L_n$ is called "dynamic range" of the n-th dimension. The performance of each particle is measured according to a mathematical function called "fitness function", which depends on the position coordinates, i.e., $F = F(\vec{x}_i)$. Actually, as the value of the fitness function is increased, the particle position is improved. The best previous position (pbest position) of the i-th particle is $\vec{p}_i = (p_{i1}, p_{i2}, ..., p_{iN})$. After a time step, the new position of the particle is given by

$$\vec{x}_i(t+1) = \vec{x}_i(t) + \vec{v}_i(t+1) \tag{1}$$

where $\vec{v}_i = (v_{i1}, v_{i2}, ..., v_{iN})$ is the velocity of the particle.

Two models have been developed in particle swarm theory, the "gbest" and the "lbest" model. In the gbest model, every particle is attracted to the best position found by any particle of the swarm. This position is the gbest position $\vec{g} = (g_1, g_2, ..., g_N)$ and corresponds to the maximum value $F_{max} = F(\vec{g})$ of the fitness function found so far by the swarm. In the lbest model, each i-th individual is attracted to the best position found by its K_i neighbors. This position is the lbest position $\vec{\ell}_i = (\ell_{i1}, \ell_{i2}, ..., \ell_{iN})$ and corresponds to the maximum value $F_{max,i} = F(\vec{\ell}_i)$ of the fitness function found so far by the K_i neighbors. The research has shown that using the gbest model the swarm tends to converge more rapidly on optima, but it is more susceptible to convergence on local optima. Thus, the present work adopts the lbest model.

Many techniques have been suggested for the calculation of the particle velocity. An efficient way of calculation is presented by [11]. According to the lbest model, the velocity is given by

$$\vec{v}_i(t+1) = k\{\vec{v}_i(t) + \varphi_1 \cdot rand(t) \cdot [\vec{p}_i(t) - \vec{x}_i(t)] + \varphi_2 \cdot rand(t) \cdot [\vec{\ell}_i(t) - \vec{x}_i(t)]\} \tag{2}$$

In the above equation, rand(t) is a function that generates random numbers drawn from a uniform distribution between 0.0 and 1.0. The parameter k is the "constriction coefficient" and is defined by:

$$k = \frac{2}{\left|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}\right|} \tag{3}$$

where the parameter φ, called "acceleration constant", must be greater than 4 ($\varphi > 4$) and is calculated by:

$$\varphi = \varphi_1 + \varphi_2 \tag{4}$$

The parameter φ_1 determines how much the particle is influenced by the memory of its best location, while the parameter φ_2 determines how much the particle is influenced by its neighbors. A good choice recommended in [10] for both φ_1 and φ_2 is 2.05.

An undesirable effect concerning the velocity is that the particle's trajectory can expand into wider cycles eventually approaching infinity. One method of solving this problem is to define a maximum allowed velocity $\vec{v}_{max}$. The choice of $\vec{v}_{max}$ depends on the problem. For example, the particle will be trapped if a step larger than $\vec{v}_{max}$ is required to escape a local optimum. However, in approaching an optimum it is better to take smaller steps. It must be mentioned that the use of the constriction coefficient was an attempt to eliminate the need for $\vec{v}_{max}$, but most authors agree that it is still better to use $\vec{v}_{max}$ in order to keep the particles in bounds.

However, the above parameters are not always able to confine the particles within the search space. To solve this problem, several boundary conditions have been suggested. According to the "absorbing walls" condition used by the present work, when a particle hits the upper or the lower boundary of the search space in one of the N dimensions, the velocity component in this dimension is zeroed and the particle is pulled back toward the search space, i.e., if $x_{in}>U_n$ then $x_{in}=U_n$ and $v_{in}=0$, and also if $x_{in}<L_n$ then $x_{in}=L_n$ and $v_{in}=0$. In that manner, the energy of the particles that try to escape the search space is considered to be absorbed by the boundary walls.

Using the theory described above, a PSO algorithm was developed by the authors in order to improve the radiation patterns produced by collinear wire-dipole antenna arrays (Fig. 1). The positions of the dipoles as well as the phase of the excitation currents applied on the dipoles are considered as position coordinates x_{in} of the particles. Given the values of x_{in} (n=1,…,N), a corresponding value of the fitness function $F(x_{i1}, x_{i2}, ..., x_{iN})$ is derived for the position of the i-th particle. The algorithm aims at finding the gbest coordinates g_n that correspond to the maximum value F_{max} of the fitness function. The g_n coordinates are actually the dipole positions and the phases of the excitation currents that produce the desired radiation pattern. A swarm size of 30 particles (S=30) is used in the algorithm. The particle velocity is calculated by equation (2), where each particle is affected by 3 neighbors (K_i=3, for i=1,…S). The parameters φ_1 and φ_2 are chosen equal to 2.05, and thus φ=4.10. The constriction coefficient results from equation (3), and its value (k=0.73) as well as the values of φ_1 and φ_2 are used in equation (2). The algorithm makes use of the maximum allowed velocity. Each coordinate of this velocity is set equal to 20% of the dynamic range of the respective dimension, i.e., $v_{max,n}=0.2R_n$ (n=1,…,N). Finally, the boundary condition of the absorbing walls is used in the algorithm in order to confine the particles within the search space.

In short, the PSO algorithm is described by the following steps:

1. Randomly initialize the particle positions inside the search space as well as the particle velocities according to the value of the maximum allowed velocity.
2. Evaluate the fitness function for all the particles.
3. Set the first position of each particle as pbest position ($\vec{p}_i = \vec{x}_i$).
4. Find the gbest position $\vec{g}$ that corresponds to the maximum value F_{max} of the fitness function.
5. Find randomly K_i neighbors for each particle.
6. Find the lbest position $\vec{\ell}_i$ among the K_i neighbors.
7. Update the particle velocities using equation (2) and make the appropriate corrections by taking into account the maximum allowed velocity.
8. Update the particle positions using equation (1) and make the appropriate corrections by taking into account the absorbing walls condition.
9. Evaluate the fitness function for all the particles. If $F(\vec{x}_i) > F(\vec{p}_i)$ then the new position becomes pbest position of the i-th particle ($\vec{p}_i = \vec{x}_i$).
10. If $F(\vec{p}_i) > F(\vec{g})$ then update the gbest position ($\vec{g} = \vec{p}_i$).
11. If $F(\vec{g})$ was increased repeat the process from the 6^{th} step. If $F(\vec{g})$ was not increased repeat the process from the 5^{th} step (meaning that the K_i neighbors must be reinitialized).

The above process is repeated until a predefined maximum number of iterations is reached.

The PSO algorithm was applied on the linear antenna array of Fig. 1. The array consists of M collinear wire dipoles. All the dipoles have the same radius of 0.001λ, where λ is the wavelength, and the same length of 0.478λ, which is the resonant length of a wire dipole with radius of 0.001λ in free space. Due to the vertical orientation of the dipoles, the radiation pattern is omni-directional on the horizontal plane and thus the antenna array can be used by a communications base station located at the center of the service area. The radiation pattern on the vertical plane depends on the geometry of the array as well as on the excitation currents applied in the middle of the length of the dipoles. The geometry of the array is determined by the positions z_m of the dipoles with respect to the position of the 1^{st} dipole which is considered fixed at the origin of the coordinate system (z_1=0). Each excitation current is specified only by its phase a_m, because the amplitude distribution is considered to be uniform (I_m=1). Given the positions z_m of the dipoles and the corresponding excitation phases a_m, the antenna array is modeled as a wire grid and is analyzed by applying the Method of Moments (MoM) [5,6]. The results derived from the MoM is the side lobe level (SLL) and the power gain (PG) at the desired direction, which is determined in the spherical coordinate system by the elevation angle θ. The values of SLL and PG in deciBel (dB) are used by the PSO algorithm in order to estimate the fitness function. The fitness function is evaluated by the expression

$$F = w_1 \cdot SLL + w_2 \cdot PG \quad (5)$$

The coefficients w_1 and w_2 are weight factors and they declare the importance of the corresponding terms that compose the fitness function. Provided that $w_1<0$ and $w_2>0$ (note that SLL<0dB and PG>0dB), the fitness function increases, as the radiation pattern is improved (decrease of SLL and increase of PG). When the fitness function finds its global maximum value, the PSO algorithm terminates successfully.

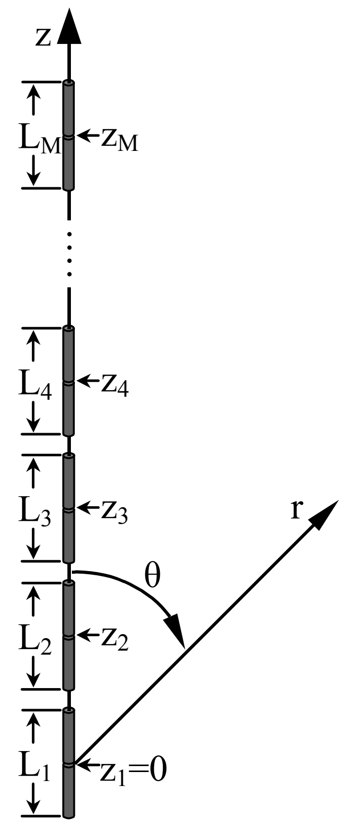

Fig.1: Collinear wire-dipole antenna array.

3. Results

The PSO algorithm was used in several cases in order to present some optimized structures that can be used in practical applications. In each case, the optimization procedure is applied on the collinear antenna array and the results are summarized in a table, while the radiation pattern on the vertical plane is shown in a corresponding diagram. In particular, each table shows the positions of the dipoles and the corresponding excitation phases. It must be noted that the 1st dipole (located at the position z_1=0) is considered as reference dipole. Thus, the excitation phases of the rest dipoles are presented, respectively, with reference to the excitation phase of the 1st dipole (a_1=0). Also, the radiation patterns are normalized with reference to the PG of the antenna array.

In the first three cases, the optimization is focused on the design of broadside antenna arrays, meaning that the main lobe direction is required to be normal to the array axis (θ=90°). From antenna theory, it is well known that all the elements of a broadside array have the same excitation phase. Thus, it is considered that a_m=0 (m=1,…,M) and the optimization procedure has to find only the appropriate element positions that improve the radiation pattern. In these three cases, the antenna arrays consist, respectively, of 5, 10, and 20 wire dipoles. The results of the optimization are given, respectively, in Tables 1, 2, and 3, while the radiation patterns are presented, respectively, in Figs. 2, 3, and 4. It is obvious that an increase in the number of elements results in a respective increase in the value of PG and a decrease in the value of SLL. In other words, the number of elements is capable of improving the radiation pattern.

Table 1: Structure characteristics of an optimized broadside antenna array of 5 collinear wire dipoles.

Number of dipole	Dipole position (λ)	Excitation phase (degrees)
1	0.000	0.0
2	0.626	0.0
3	1.105	0.0
4	1.584	0.0
5	2.211	0.0
Power gain: 7.61 dB	SLL: –19.15 dB	

Table 2: Structure characteristics of an optimized broadside antenna array of 10 collinear wire dipoles.

Number of dipole	Dipole position (λ)	Excitation phase (degrees)
1	0.000	0.0
2	0.816	0.0
3	1.416	0.0
4	1.936	0.0
5	2.415	0.0
6	2.896	0.0
7	3.375	0.0
8	3.894	0.0
9	4.495	0.0
10	5.313	0.0
Power gain: 10.64 dB	SLL: –22.10 dB	

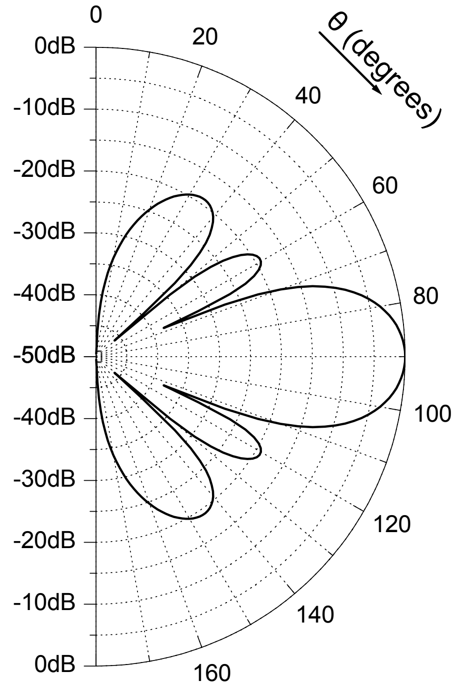

Fig.2: Radiation pattern of an optimized broadside antenna array of 5 collinear wire dipoles.

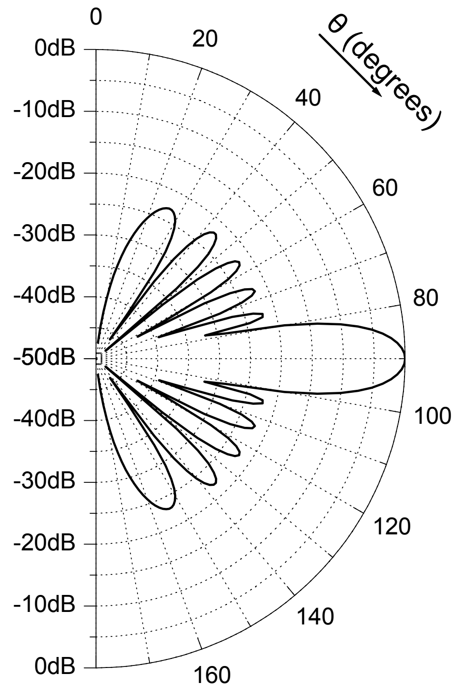

Fig.3: Radiation pattern of an optimized broadside antenna array of 10 collinear wire dipoles.

In the next two cases, the optimization is focused on the design of antenna arrays where the main lobe is not normal to the array axis. The desired main lobe direction is chosen at θ=100°. In these cases, the antenna arrays consist, respectively, of 5 and 10 wire dipoles. The results of the optimization are given, respectively, in Tables 4 and 5, while the radiation patterns are presented, respectively, in Figs. 5 and 6.

Table 3: Structure characteristics of an optimized broadside antenna array of 20 collinear wire dipoles.

Number of dipole	Dipole position (λ)	Excitation phase (degrees)
1	0.000	0.0
2	0.977	0.0
3	1.955	0.0
4	2.596	0.0
5	3.150	0.0
6	3.828	0.0
7	4.307	0.0
8	4.849	0.0
9	5.328	0.0
10	5.846	0.0
11	6.325	0.0
12	6.813	0.0
13	7.292	0.0
14	7.835	0.0
15	8.368	0.0
16	8.893	0.0
17	9.579	0.0
18	10.316	0.0
19	11.294	0.0
20	12.271	0.0
Power gain: 13.82 dB SLL: –24.24 dB		

Table 4: Structure characteristics of an optimized antenna array of 5 collinear wire dipoles with main lobe direction at θ=100°.

Number of dipole	Dipole position (λ)	Excitation phase (degrees)
1	0.000	0.0
2	0.578	38.5
3	1.057	69.7
4	1.536	92.1
5	2.071	136.0
Power gain: 7.39 dB SLL: –17.33 dB		

Table 5: Structure characteristics of an optimized antenna array of 10 collinear wire dipoles with main lobe direction at θ=100°.

Number of dipole	Dipole position (λ)	Excitation phase (degrees)
1	0.000	0.0
2	0.783	57.5
3	1.489	96.7
4	2.018	140.2
5	2.531	166.7
6	3.054	-161.3
7	3.533	-118.9
8	4.086	-87.2
9	4.758	-47.3
10	5.512	8.9
Power gain: 10.80 dB SLL: –19.90 dB		

Even in cases of non-broadside antenna arrays, the optimization procedure has the ability to improve the radiation pattern. Of course, an increase in the number of elements is always a reasonable way to achieve better values for both the power gain and the side lobe level.

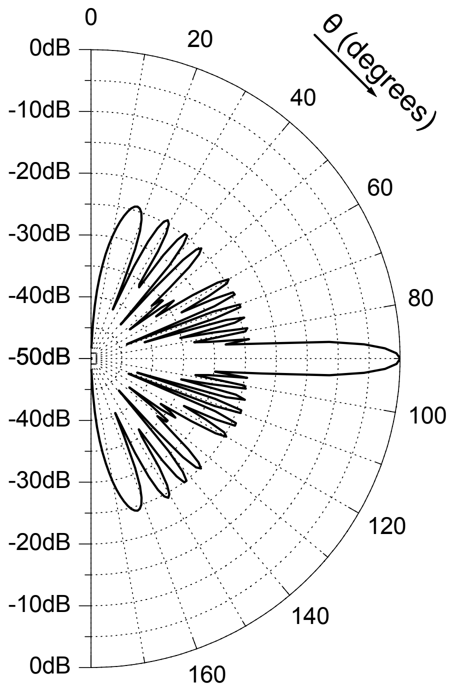

Fig.4: Radiation pattern of an optimized broadside antenna array of 20 collinear wire dipoles.

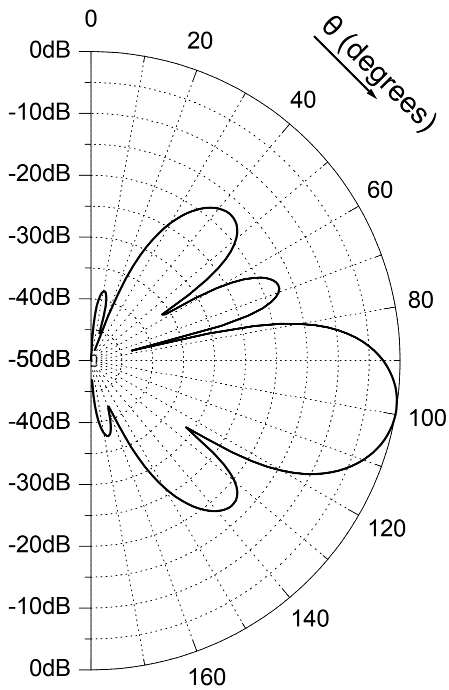

Fig.5: Radiation pattern of an optimized antenna array of 5 collinear wire dipoles with main lobe direction at θ=100°.

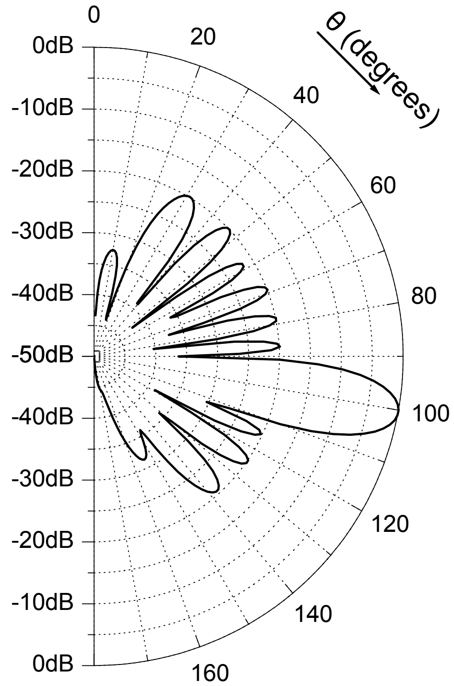

Fig.6: Radiation pattern of an optimized antenna array of 10 collinear wire dipoles with main lobe direction at θ=100°.

4. Conclusions

A PSO based algorithm was used in order to improve the radiation pattern of linear antenna arrays according to specific requirements. Several cases of broadside and non-broadside arrays were tested in order to exhibit the robustness of the proposed method. The optimization procedure was applied under the restriction of uniform current excitation and under the requirements for the maximum possible power gain at a desired direction and for the minimum possible side lobe level. The results show that the proposed method is very promising and capable of optimizing any type of antenna array under any restriction or any requirement demanded in practice.

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