Partial volume correction for positron emission images using the wavelet transform

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Abstract: A method for partial volume correction of positron emission tomography (PET) images is proposed. The method is based on combining information from PET and high resolution anatomical images. Two different algorithms fast wavelet transform and $\grave{a}$ *trous* were implemented in order to decompose the PET and high resolution images. Results obtained using several simulated images show a significant partial volume reduction and image enhancement. The relative errors in the partial volume corrected image are always less than 3,6% with respect to 16% of the original image.

Key–Words: Positron emission tomography, wavelet transform, partial volume correction

1 Introduction

Positron emission tomography (PET) images provide important functional information however the poor spatial resolution with respect to other imaging techniques such as Computed Tomography (CT) and Magnetic Resonance (MR) images lead to partial volume effect (PVE). The net results of PVE is an incorrect measurement of the true radiotracer concentration. As outlined by Aston *et al.* [1] PVE can be divided into two effects: tissue-fraction and point-spread effect. The tissue fraction effect arises from tissue heterogeneity because the region of interest (ROI) used to determine radiotracer concentration contains signals from different tissues (for example gray and white matter). The point-spread effect arises from the finite resolution of PET cameras. The spatial resolution of a clinical PET tomograph is about 4-5 mm, while for example the resolution of CT, MR images is about 1-2 mm. Correction for PVE is of great importance for both semi-quantitative and quantitative measurements. Typically semi-quantitative measurements involve the estimation of the Standard Uptake Value (SUV) or the Tumor to background ratio (TBR). In both cases a ROI is drawn on the lesion and the mean value of the radiotracer concentration is measured. PVE typically reduces the value of the measured tracer concentration. Quantitative and more advanced approaches such as compartmental analysis are applied to measure the exchange of substances between several compartments. PVE can lead to large errors in the estimated kinetic rate constants.

In order to correct for PVE a wavelet-based approach was proposed in [2, 6]. It allows not only PVE correction, but also images enhancement by adding high-resolution information obtained by a multi-resolution analysis of PET and high resolution (coregistered) images. In the following a more general approach is proposed considering two types of wavelet decomposition algorithms such as the $\grave{a}$ *trous* [3] and the fast wavelet transform (FWT) introduced by Mallat [4].

The paper is organized as follow: in section 2 the necessary mathematical formalism is introduced, in section 3 the main results are presented, conclusions then follow.

2 Material and Methods

2.1 Image wavelet transform

A wavelet base of $L^2(\mathbb{R})$ can be constructed simply by taking all the dilated and shifted versions of a mother wavelet:

$$\psi_{j,n} = \frac{1}{\sqrt{2^j}}\psi\left(\frac{t-2^j n}{2^j}\right) \qquad (1)$$

The wavelet transform (WT) of a function *f(t)* is defined as the inner product of the function with the base functions:

$$c_{j,n} = \frac{1}{\sqrt{2^j}} \int f(t) \psi^* \left(\frac{t - 2^j n}{2^j} \right) dt \tag{2}$$

Where * denotes the complex coniugate. In order to obtain the wavelet transform two different algorithms were applied: FWT and the *à trous* wavelet decomposition. The FWT algorithm is well known and will not be described here. Generally speaking the *à trous* wavelet decomposition has several advantages such as simplicity, shift invariance and accurate reconstruction without any loss of information. The performance of the two approaches have been investigated.

The wavelet decomposition using the *à trous* algorithm is based on iterative convolutions with a low-pass filter *h* obtained from a scaling function ϕ.

$$I_{j+1}(k,l) = \sum_{m,n} h(m,n) I_j(k + m2^j, l + n2^j) \tag{3}$$

where $I_j(k,l)$ is the image at iteration *j*. The inverse WT is performed using the following equation:

$$I = I_K + \sum_{j=1}^{K} w_j \tag{4}$$

where $w_j = I_{j-1} - I_j$ are the detail wavelet coefficients, *K* is the number of iterations and *I* is the original image. Wavelet decomposition was performed by using a 5x5 bicubic-spline filter.

2.2 PET images partial volume correction

In this section the proposed PVE correction algorithm for PET images is described. It useful to define two images one low resolution I_l (typically a PET image) and one high I_h resolution image (typically CT or MR images). I_h and I_l are vectors belonging to two Hilbert spaces U and V. As shown by equation 4 the images can be decomposed as follow:

$$I_l = I_{r_1+1,l} + w_{r_1+1,l} \tag{5}$$

$$I_h = I_{r_2+1,h} + w_{r_2+1,h} \tag{6}$$

where $r_1 \geq r_2$ are respectively the resolution levels of the low and high resolution images. Alternatively equation (6) is also equal to:

$$I_h = I_{r_1+1,h} + \sum_{i=1}^{r_1 - r_{2+1}} w_{r_2+i,h} \tag{7}$$

As one can see by looking at equation (7) the difference between I_h and I_l is a function of $\sum_{i=1}^{r_1-r_{2+1}} w_{r_2+i,h}$ and, thus, the resolution content of I_l can be increased by adding a similar term. More precisely:

$$\hat{I}_l = I_{r_1+1,l} + H(\sum_{i=1}^{r_1-r_{2+1}} w_{r_2+i,h}) \tag{8}$$

where $\hat{I}_l$ is the partial volume corrected PET image and H is a functional between the Hilbert spaces U and V.

There are several possible choices of the functional H, a simple solution is to consider $w_{i,l} = \alpha \cdot w_{i,h}$, where α is a proportional constant. An alternative approach is to take the maximum between the detail wavelet coefficients of the two images.

2.3 Image simulations

The PVE correction approach described in section 2 was tested using different images. The first set of measurements were performed using a test pattern with objects of known intensity and shape. Secondly montecarlo simulated brain PET images and MR images were used. Synthetic high-resolution test image (256x256 pixels) includes a first series of 5 squares with decreasing sizes (30x30, 20x20, 15x15, 10x10 and 5x5 pixels) with constant pixel intensity. Another series of 7 squares of the same size (10x10 pixels) and decreasing intensities were also included. In order to simulate partial volume effect typical of a PET scanner, images were smoothed with a Gaussian filter and Gaussian-random noise was also added. In order to study the degree of recovery offered by our approach, regions of interest (ROI) were drawn on each square in the simulated PET image and in the recovered one. The mean value intensities and the corresponding relative errors were calculated.

Montecarlo simulated PET images were obtained using the SORTEO [5] PET simulator, the simulations are based on the MR image shown in figure 6.

3 Results

Figures 1 and 2 show respectively the original (high resolution) and the smoothed test pattern images. Figure 3 shows the recovered image using the wavelet decomposition obtained with the *à trous* algorithm. As one can see by looking at the edges of the squares inside figure 3 the recovered image is comparable with the true one. A more detailed analysis on the recovered image was performed by drawing a profile along the squares with the same intensity but with different dimensions. Figure 4 shows a comparison between the line profiles of the true, smoothed and recovered images. The profile of the recovered image is more similar to the true profile.The mean values of the ROIs in the partial volume corrected images show a significant improvement with respect to the original smoothed image as shown in figure 5. The relative errors in the recovered image are always less than 3,6% with respect to 16% of the smoothed image. Figure 6 shows respectivlely a simulated Montecarlo PET images and the corresponding MR image. In figure 7 is shown the PVE correted PET image, as one can see the spatial resolution is improved. Figure 8 shows a PVE corrected PET image where the wavelet decomposition was performed with the FWT algorithm. The image was obtained by taking the maximun of the wavelet transform coefficients of the PET and MR images.

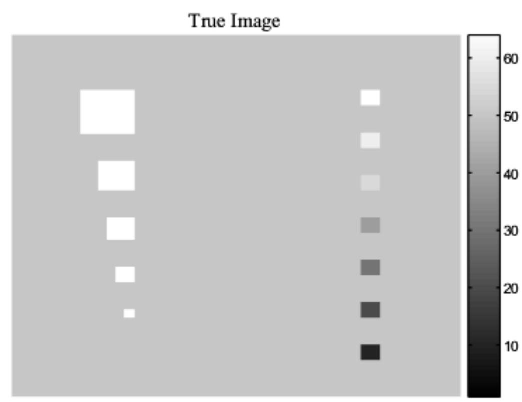

Figure 1: Synthetic high-resolution test image. The image includes a first series of 5 squares with decreasing sizes with constant pixel intensity (white squares on the left). A series of 7 squares of the same size and decreasing intensities was also included.

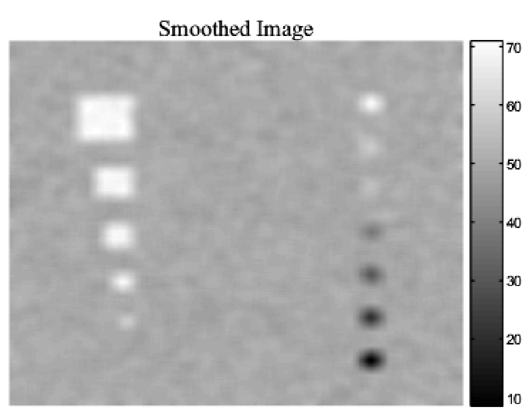

Figure 2: Test image smoothed with a Gaussian filter. Gaussian-random noise was also added in order to simulate a more realistic image.

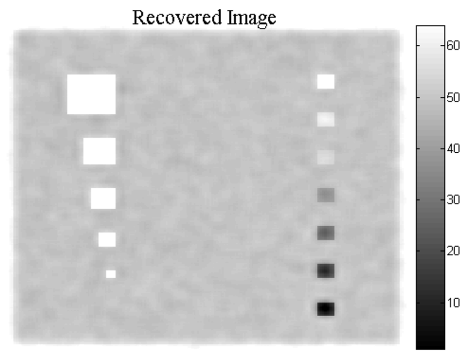

Figure 3: The figure shows the recovered image using the partial volume correction method described in section 2.1. The wavelet decomposition was performed using the $\grave{a}$ *trous* algorithm.

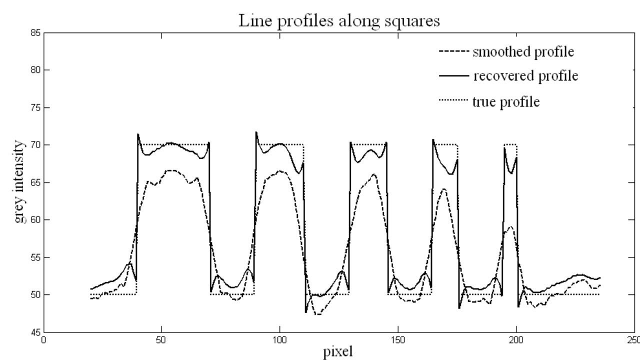

Figure 4: The plots show a comparison between the line profiles of the true, smoothed and recovered images.

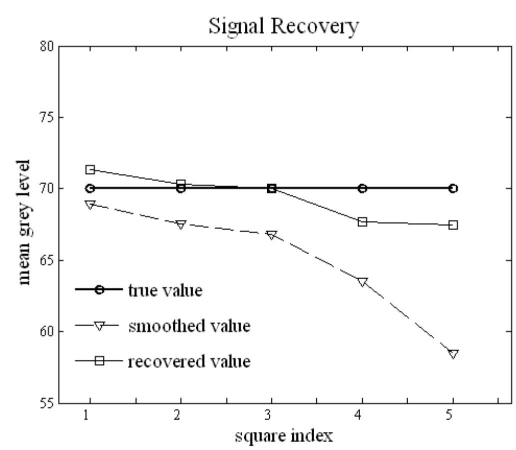

Figure 5: The plots show the mean values of the ROIs measured in the partial volume corrected and the original images. The mean value of the PVE corrected image is always very closed to the true value, the bias being less than 3.6%.

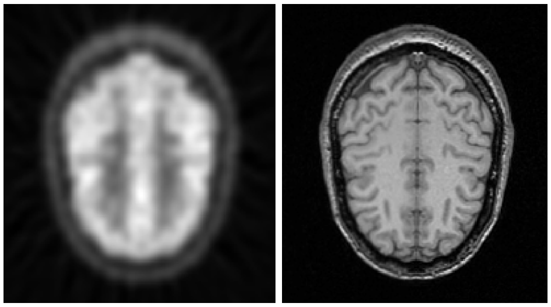

Figure 6: The image on the left shows a PET slice of the Zubal brain phantom obtained using Montecarlo simulations on the right the corresponding MR image is shown.

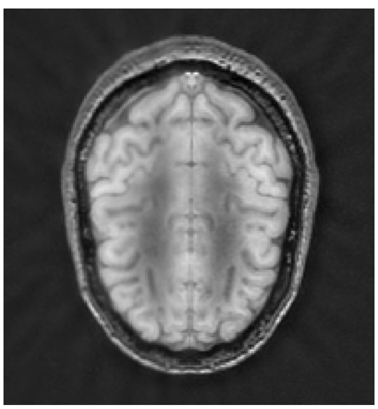

Figure 7: Partial volume corrected PET image using the *à trous* algorithm.

Figure 8: Partial volume corrected PET image using the FWT algorithm. The image was obtained by taking the maximun of the wavelet transform coefficients of the PET and MR images.

4 Conclusions

We showed that by using a wavelets decomposition of simulated PET and MR images it is possible to perform accurate PVE correction.
Results obtained using a test image with objects of different size and intensity show that the mean gray level errors in PVE corrected image are always less than 3,6% with respect to 16% of the original image. The PVE correction method described in this paper was applied to simulated PET brain images combined with high resolution MR. The resulting partial volume corrected PET images show a significant contrast enhancement with respect to the original image.
We believe that the proposed PVE correction method will have a significant impact on the improvement of clinical PET images.

Acknowledgements: This research was supported by the Fondazione del Monte of Bologna, Italy.

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