Performances Test Statistics for Single Outlier Detection in Bilinear(1,1,1,1) Models

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Abstract: An outlier detection procedure for BL(1,1,1,1) model is developed based on the maxima of the test statistics measuring the effects of IO, AO, TC and LC. A simulation study is carried out in order to investigate the sampling properties of the maxima of the outlier test statistics . It is associated with the sample size, the type of outlier and the coefficients chosen for BL(1,1,1,1). The results show that, in general, the performance of the detection procedure is good. The outlier detection procedure performs well in detecting AO for large value of $\hat{\omega}_{AO}$. As for IO, the performance of outlier detection procedure is better for model with larger coefficient values. The outlier detection procedure is capable of detecting TC and LC, though the performance is affected if ω is large.

Keywords: models, test statistics, outlier detection procedure, nonlinear least squares.

1. Introduction

Chen [1997] and Zaharim [1996] studied the detection of outliers in bilinear processes. The former used the Gibbs sampling method for general bilinear model to study the additive outlier type. On the other hand, the latter used least squares method for simple bilinear model to detect four type of outliers, the additive outlier (AO), innovational outlier (IO), temporary change (TC) and level change (LC). Test statistics for measuring the effects of IO, AO, TC and LC outliers in BL(1,1,1,1) models has been developed by Mohamed (2004) . They were used in designing test criteria for the four types of outlier as well as a procedure for detecting an outlier type at

a time point *t*. This article reports a simulation study to investigate the performance of the test criteria and outlier detection procedure. The sampling behavior of the test statistics is studied to determine appropriate critical values for the detection procedure. The detection procedure is found to perform well in detecting and identifying the most probable type of outliers.

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2. Methodology

Statistics for detecting AO, IO, TC and LC in BL(1,1,1,1) processes is given by

$$\hat{\eta}_{TP} = \max_{t=1,2,\ldots,n} \left\{ \left| \hat{\tau}_{TP,t} \right| \right\},$$

TP being either AO, IO, TC or LC, while

$$\hat{\tau}_{TP,t} = \frac{\left(\hat{\omega}_{TP,t} - \bar{\tilde{\omega}}_{TP,BS,t} \right)}{\tilde{\sigma}_{TP,BS,t}}$$

and

$$\hat{\omega}_{TP} = \frac{\sum_{k=0}^{n-d} \left[\{-1\}^k e_{d+k} A_k \right]}{\sum_{k=0}^{n-d} A_k^2}.$$

The expression for A_k is as in Table 1.

Table 1: Expression for the A_k term for different outlier type.

Outlier Type	Expression for A_k for each Type of Outlier (TP)
AO	$A_k = \begin{cases} 1 & k=0 \\ (a+be_d)+(bY_d+c) & k=1 \\ (bY_{d+(k-1)}+c)A_{k-1} & k \geq 2 \end{cases}$
IO	$A_k = \begin{cases} 1 & k=0 \\ (bY_d+c) & k=1 \\ (bY_{d+(k-1)}+c)A_{k-1} & k \geq 2 \end{cases}$
TC	$A_k = \begin{cases} 1 & k=0 \\ \delta^k - \delta^{k-1}(a+be_{d+(k-1)}) - (bY_{d+(k-1)}+c)A_{k-1} & k \geq 1 \end{cases}$
LC	$A_k = \begin{cases} 1 & k=0 \\ 1-(a+be_{d+(k-1)})-(bY_{d+(k-1)}+c)A_{k-1} & k \geq 1 \end{cases}$

Models shown in Table 2 which represent a broad choice of BL(1,1,1,1) processes were used to generate the test data. The random errors e_t's, were chosen to follow the standard normal distribution. For each model, three sizes of sample i.e., n = 50, 100 and 200 were used. A total of 500 data sets were simulated for each model and each sample size,. The values of the test statistics $\hat{\eta}_{TP}$ for AO, IO, TC and LC were then calculated.

Table 2: The BL(1,1,1,1) Models used in the simulation of data sets

Model	Model Specification
1	$Y_t = -0.3Y_{t-1} - 0.3e_{t-1} - 0.3Y_{t-1}e_{t-1} + e_t$
2	$Y_t = 0.2Y_{t\ 1} + 0.2e_{t\ 1} + 0.2Y_{t\ 1}e_{t\ 1} + e_t$
3	$Y_t = 0.1Y_{t\ 1} + 0.1e_{t\ 1} - 0.2Y_{t\ 1}e_{t\ 1} + e_t$
4	$Y_t = 0.1Y_{t\ 1} + 0.1e_{t\ 1} + 0.2Y_{t\ 1}e_{t\ 1} + e_t$
5	$Y_t = 0.1Y_{t\ 1} - 0.1e_{t\ 1} + 0.2Y_{t\ 1}e_{t\ 1} + e_t$
6	$Y_t = 0.5Y_{t\ 1} + 0.1e_{t\ 1} - 0.2Y_{t\ 1}e_{t\ 1} + e_t$

3. Results

3.1 Determining the 1%, 5% and 10% Critical Values for the test statistic $\hat{\eta}_{TP}$

Estimates of the 1st, 5th, and 10th upper percentiles of the statistic were calculated from the outlier-free simulated series. Comparison of values at the 1st percentiles with sample size n do not reveal any clear pattern. However at the 5th and 10th percentiles, many estimates of $\hat{\eta}_{TP}$ increase with sample size n. The increase was very slight; for instance, for Model 1, when *n* changes from 60 to 200, the 10th percentiles of $\hat{\eta}_{IO}$ changes from 1.93 to 2.07. It was also observed that the estimated 1st, 5th, and 10th percentiles of $\hat{\eta}_{IO}$ are slightly higher than those of $\hat{\eta}_{AO}$. The estimated 5th percentiles for IO ranged from 2.09 to 3.48 whereas the estimated 5th percentiles for AO range from 2.07 to 3.40.

In the case of TC, the estimated percentiles, have clearer trend of an increasing function in *n* compared to the AO and IO cases. The estimated percentiles for model 6 are smaller than those of the other models and are very close together. Model 6, with large coefficient value in the AR term (a = ~0.5) when compared to other models, has estimated 5th percentile which ranged from 1.54 to 3.55.

The estimated percentiles for LC for models 1 and 2 showed a trend of an increasing function in sample size, *n*. However, the trend is not clear for models 3 ~ 5. As observed in the TC case, model 6 had lower values of estimated percentiles compared to the other models. The estimated 5th percentiles for LC range from 1.36 to 4.21.

In general, when *n* changes from 60 to 200, the estimated percentiles of $\hat{\eta}_{AO}$ and $\hat{\eta}_{IO}$ increase slightly. As for TC, the increase of estimates is moderate with the exception of model 6. The estimated percentiles of $\hat{\eta}_{LC}$ increase slightly for model 3 to 5 with small values of coefficients *a*, *c* and *b* but increase moderately for models 1 and 2 with moderate coefficient values. However, for model 6, the estimated percentiles are relatively smaller than the other models. In summary, the 5th percentiles estimates of $\hat{\eta}_{IO}$, $\hat{\eta}_{AO}$, $\hat{\eta}_{TC}$ and $\hat{\eta}_{LC}$ fall in the range of 1.3 to 4.2, which is the range of the estimates for LC.

Based on the results, critical values of 2.5 to 4.0 seem to be appropriate for a series with length of 60 to 200. In practice, more than one critical value is recommended for the analysis.

3.2 Performance of the test statistic $\hat{\eta}_{TP}$

A simulation was carried out for the purpose of looking at the performance of the test statistic for detecting AO, IO, TC and LC individually. The test were applied to cases chosen by varying the combination of the following factors:

(a) Outliers types: IO, AO, TC and LC
(b) BL(1,1,1,1): 1 to 6 (see Table 2)
(c) A single outlier at $t = 40$ in samples of size 100.
(d) Values of outlier effect: $\omega = 6, 8, 10$.
(e) Levels of critical values chosen : 2.0, 2.5, 3.0, 3.5, 4.

Data were generated using Models 1 to 6 to contain one of the outlier types. For each model, 500 series of length 100 were generated using the *rnorm* procedure in S-Plus. In the case involving TC, the value of decaying factor, , was chosen to be 0.9. From the simulated data values of the relative frequency or proportion of correctly detecting the outlier at t = 40 were calculated.

For the AO case three main results are observed. Firstly, with critical values chosen to be 2.0 or 2.5, the test criterion performs well. The proportion of correctly detecting AO is close to unity for every model. Secondly, the performance of the test criterion improves in larger value of ω_{AO}. When the magnitude of ω_{AO} is 10, the proportions are close to unity. Thirdly, it is observed that the performances of Models 3 to5 are better than the performances of models 1, 2 and 6. It is known that the earlier models have smaller coefficient values compared to the later models. Hence, there will be lesser number of large spikes in simulated series for models 2 to 5. This means the AO will stand out better and detected easier. However, when $\omega_{AO} = 10$, this scenario has no bearing on the performance of the test criterion.

In the IO case the performance of almost every model improves when ω_{IO} are large. The performances of models 3 to 5 are better than models 1, 2 and 6. The same reason discussed for the AO case applies here. However, the proportions of correctly detecting IO is lower compared to the AO case.

For TC, it is observed that the proportion of correctly detecting TC is high if critical values of 2.0 or 2.5 are used. However, the proportions do not follow any general pattern of the increase or decrease in ω. In several cases, the test criterion performs better when ω = 6 than when ω = 8 or 10. This can be seen in models 1 and 6. These could be due to the fact that when larger ω is introduced at t = 40, more observations are changed and BL(1,1,1,1) model may no longer be the appropriate model. Consequently, it affects the outcome of the test criterion. It is also observed that $\hat{\eta}_{TC}$ tend to take smaller values than 4; for instance, for model a particular model, 81% of $\hat{\eta}_{TC}$ that detect TC correctly are greater then 3.0, 59% of $\hat{\eta}_{TC}$ are greater than 3.5 and only 37% of $\hat{\eta}_{TC}$ are greater than 4.

The same scenario is seen for LC. The proportions of correctly detecting LC decrease with decrease in ω. The estimation of BL(1,1,1,1) model is affected by LC especially when larger ω is used. This subsequently affects the performance of the outlier detection procedure. However, generally, the proportions give satisfactory results.

3.3 Performance of the Outlier Detection Procedure

The performance of the outlier detection procedure to identify the type of outlier, referred as outlier detection procedure herewith, was also investigated through simulation. The same combination of factors considered in section 2 are used. For example, when an AO is included at time t = 40, outlier detection procedure is applied on the series to see whether an AO, and not other type of outlier, is correctly detected. The relative frequency or proportion of correct detection were calculated. Correct detection is defined as a correct identification of both type and location of an outlier.

For the AO case, in most models, it is observed that the larger value of , the larger percentages of correct detection is observed. For = 10, the percentages is above 60%. In fact, the percentages are monotone in critical values, *C*. This implies that the test statistic tends to be larger than 4. It is observed that models 3 to 5 give better results if compared to models 1, 2 and 6.

For the IO case, proportions for models 3 to 5 are less than 50%. Further investigation on the outputs shows that many IO are wrongly detected as AO. This is due to the fact that the formulations for measuring the AO and IO effects differ with respect to A_1. The formula for A_1 of AO has an extra term $(a + be_d)$ where *a* and *b* are the coefficients of the AR and bilinear term respectively whereas e_d is the original residual at time *d*. If *a* and *b* take small values, then A_1 for both cases are close. There is a tendency that $\hat{\omega}_{AO}$ will supercede $\hat{\omega}_{IO}$ and IO is misclassified as AO. However, for models 1 and 2 with moderate coefficients values, the results are better. For model 6, which has large coefficient in the AR term and moderate coefficients of the bilinear term, the proportions of correct detection are high.

As for TC, the results show that it is correctly detected above 80% when critical values of 2.0 and 2.5 are used. However, the performances decrease slightly when ω is larger.

In the LC case the percentages of correct detection decrease as the value of increase. Studying the outcome of the simulation shows that LC loses ground against IO. The existence of LC shifts the residuals to a new level and with the larger □, the variation of the residuals also become larger. This inflated the value of A_k of IO for $k = 1, 2, ..., n \quad d$ whereas the value of A_k of LC gets smaller. As a result, $\hat{\omega}_{IO}$ has a higher chance of being greater than $\hat{\omega}_{LC}$. This effect is observed in the TC case, but not prominent as in the LC case.

4. Conclusion

In general, the performance of the detection procedure is good. The outlier detection procedure performs well in detecting AO for large value of $\hat{\omega}_{AO}$. As for the IO, the performance of outlier detection procedure is better for model with larger coefficients. The outlier detection procedure is capable of detecting TC and LC, though the performance is affected if ω is large.

In all cases, the performances of the test criteria of AO, IO, TC and LC individually are generally better than the outlier detection procedure. This should be true as the testing criteria are developed for one particular type of outliers. On the other hand, the outlier detection procedures aim to detect the type of outlier amongst AO, IO, TC and LC by looking at the maximum values of the test criteria. In some cases, an AO is wrongly detected as IO or a TC is misclassified as IO. Thus, the performance of the outlier detection procedure cannot be better than the test criteria.

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