# Dynamic Analysis of Electromechanical Converters by means of the Wavelet Transform

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*Abstract:* - The analysis performed in this paper is related to a four-quadrant ac converter used to drive the motors of a railway locomotive in modern traction systems. The paper numerically analyses the starting transient and the instantaneous harmonic content of the input current. The current obtained by the analytical relationships is then studied by means of the wavelet analysis, putting into evidence the starting transient component and the superimposed noise. It is clearly stated in the work how this methodology can extend to the transient analysis the evaluation of the harmonic content evolution during the transient phase and how any change in the power supply frequency influence the system dynamic. These results cannot be found as well by applying the more traditional Fourier transform.

*Key-Words:* - wavelet transform, converters, railway locomotives, harmonic analysis

## 1 Introduction

The fundamental benefit of the wavelet analysis, compared with the Fourier technique, can be found in the possibility of performing specific studies, in the time-frequency domain, of signal spectrum evolution. The Fourier analysis provides information related to the frequency domain, but looses any connection on how each spectral component evolves in the time. So, the Fourier analysis is not able to characterise the transient components of the signal or, more generally, evolutionary spectra. The analysis presented in this paper is related to the electromechanical system realised by connecting a four-quadrant converter to an induction machine. In this example the bi-axial model, including the direct self-control, is studied and the time-frequency behaviour of the drive is putted into evidence thanks to the application of the wavelet mathematical tool.

## 2 Problem Formulation

The analysis performed in this work is based on the use of Morlet wavelets. In order to introduce properly the subject we provide first a short introduction to Haar wavelets, that more then any others provide good tools for electrical system analysis [1, 2]. In the meantime we are also introducing Morlet analysis in parallel to Haar framework. In the simplest form, the functions are defined on a unit domain as follows [3, 4]:

$$H_{0,0}(t)=1 \tag{1-a}$$

$$\underset{(\text{with } 1 \le k \le 2^{s-1})}{H_{s,k}(t)} = \begin{cases} +\sqrt{2^{s-1}} & \text{if } \frac{k-1}{2^s} < x \le \frac{k}{2^s} \\ -\sqrt{2^{s-1}} & \text{if } \frac{k}{2^s} < x \le \frac{k+1}{2^s} \\ 0 & \text{for any other interval} \end{cases} \tag{1-b}$$

Haar functions are labeled by a scale parameter (or order) *s* and a position parameter *k*. Each function has a well defined localization, and functions of higher order are obtained by shifting and scaling functions of lower order.
The Morlet wavelets that belong to the class of orthogonal wavelets are defined as follows:

$$\psi\left(\frac{t-b}{a}\right) = \frac{1}{\sqrt{a}} \cdot e^{j\frac{2\pi(t-b)}{a}} \cdot e^{-\frac{1}{2}\left(\frac{t-b}{a}\right)^2} \tag{2}$$

The family of wavelets to be used for the transform is given by:

$$\psi_{a,b}(t) = \psi\left(\frac{t-b}{a}\right),\ a > 0,\ b \in \Re \tag{3}$$

The time constraints are applied by imposing that:

$$|\psi(t)| \le c(1+|\omega|)^{-1-\varepsilon} \quad \forall \varepsilon > 0 \tag{4}$$

and in the frequency domain it is necessary to guarantee that the Fourier transform of the generic wavelet Ψ(t) be a fast decreasing function:

$$|\Psi(\omega)| \le c \cdot (1+|\omega|)^{-1-\varepsilon} \quad \forall \varepsilon > 0 \tag{5}$$

with:

$$\Psi(\omega) = \int_{-\infty}^{+\infty} \psi(t) \cdot e^{-j\cdot\omega\cdot t} dt \tag{6}$$

The continuous wavelet transform is so defined for a generic signal f(t) as:

$$Tf(a,b) = \langle f, \psi_{a,b} \rangle = F(a,b) = \int f(t) \cdot \overline{\psi}\left(\frac{t-b}{a}\right) \cdot dt \tag{7}$$

In the discrete-time case, the Haar wavelet system can be obtained by uniform sampling of the time-continuous Haar functions. Let us consider a set of $2^N$ samples: we can define a Haar wavelet set composed of $2^N$ independent functions defined on $x[n]$. We will indicate such wavelets as $W_i$. Wavelets of order $s$, with $s>0$, have indexes $i$ between $2^{s-1}+1$ and $2^s$. It can be shown that such set is a complete and orthonormal basis. Hence, any discrete signal composed of $2^N$ samples can be decomposed into the weighted sum of Haar wavelets in a unique way:

$$x[n] = S_i \cdot X_i \cdot W_i[n] \tag{8}$$

Because the $W_i$ are an orthonormal base, the coefficients $X_i$ can be obtained by an expression similar to the DFT. This expression is the Haar transform:

$$X_i = S_i \cdot x[n] \cdot W_i[n] \tag{9}$$

Such a transformation, and the inverse transformation as well, can be represented as a matrix product. The result is the array $X$ of the $2^N$ coefficients $X_i$

$$X = Wx \tag{10}$$

where $W$ is the $2^N$-by-$2^N$ matrix containing the Haar wavelets $W_i$ as its rows.
The $W$ matrix has $2^{N-1}$ rows, corresponding to the wavelets of highest order, with only $2$ non-zero elements, $2^{N-2}$ rows with 4 non-zero elements, and so on, with only $2$ rows that are completely full.
Hence, a Haar transform can be computed in a very efficient way: it has been shown [4] that the number of required additions and subtractions is proportional to $N$, in contrast to the fast Walsh or Fourier transform, where it is proportional to $N \cdot log_2 N$.
The Morlet version of discrete transform can be arrange by assuming:

$$F(a_i, b_k) = \sum_l f(t_l) \cdot \overline{\psi}\left(\frac{t_l \cdot f_s - b_k}{a_i}\right) \tag{11}$$

and it will be used in the next paragraphs to manage the current analysis and its time-frequency representation.
A last aspect can be examined by pointing out that the relationship between Haar and Walsh transforms is based on the resemblance of the respective base functions: both are discontinuous functions, with discontinuities at $n/2^k$, where $k$ is the order of the wavelet, or of the Walsh function [5]. The main difference between Haar and Walsh functions is that Walsh functions only take values $+1$ and $-1$, each of them has support extended over the whole unit interval, while Haar functions are localized, i.e., they take non-zero values only on one interval, which gets smaller and smaller with increasing wavelet order. In spite of that, it is rather easy to express each Walsh function as a linear combination of wavelets. It is interesting to remark that each Walsh function decomposes into the sum of all the wavelets of a single order, with their proper weights.
It is well known that, as both Walsh and Haar transforms are orthogonal transformations, a matrix transformation can be written that links the two. In other words, a matrix $T$ can be obtained that converts the Haar coefficients into the Walsh coefficients. Such a matrix still has a reasonably low number of non-zero elements: thus, an efficient way can be found for computing both Haar and Walsh transforms at the same time. Actually, fast Haar-Walsh transformation algorithms exist which provide both sets of coefficients.

## 3 Electric Drive

The electrical drive used in the following analysis is shown in Fig.1 with the joint interaction of a power converter connected to an induction machine. More in detail it can be noted:

- a traction transformer that adapts the line voltage (15 kV) to the level needed for powering the four-quadrant converter;
- the series of the Four-Quadrant Converter (4QS) and a Voltage Source Inverter (VSI) [6,7] that allow the complete amplitude-frequency regulation for the induction machine.

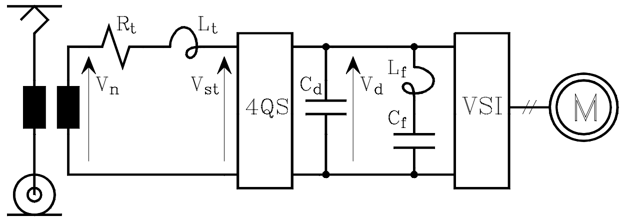


Fig.1. The electrical drive system of one traction unit of the E412 locomotive.

The model adopted for the whole electrical drive system is based on the Depenbrock equations [8] for the power converter and on a 7th order-Park equation model for the induction polystar machine [9]. The whole system is referred to a couple of axes synchronous with the network main frequency.

In the following sections, thanks to the mathematical model defined for the whole system and based on the wavelets theory, a very good and efficient way to develop in depth the transient conditions analysis of the electrical drive is provided. The purpose is to highlight, thanks to the specific performance of wavelets, the harmonic instantaneous content of the transient current.

The wavelet analysis can be performed after evaluating the line current i(t) through the traditional approach given by the driver dynamical theory [10]. Among the different working condition, one in particular (the so called "no-load condition") is analysed. In this way, by assuming zero the mechanical power provided to the wheels, the line power-absorption is limited to the harmonic components due to the modulation provided by the three four-quadrant drivers connected in parallel.

In this situation the converters generate – by using the pulse width modulation technique – the voltage $v_s(t)$ having the amplitude and the phase of the fundamental equal to the no-load voltage on the secondary side of transformer (see Fig.2).

The voltage $V_s$ is linked to dc-side voltage $V_d$ and it is obtained with:

$$V_s = kV_d \tag{12}$$

In this way there is no power adsorption at the fundamental frequency of the line current (fig.2d), thus having nil mean power is nil.

Accordingly to

$$v_1(t) - v_s(t) = \left( R_t + L_t \frac{d}{dt} \right) \cdot i_1(t) \tag{13}$$

there are the harmonic components due to the PWM conversion provided by the 4QS [11,12].

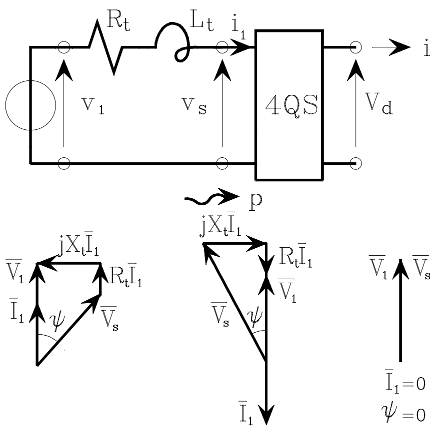


Fig. 2. Four-Quadrant Converter (4QS) model (a). Phasorial diagram corresponding to fundamental harmonic f1=16 2/3 Hz: (b) driving condition; (c) braking condition; (d) cosφ=1 condition.

# 4 Case studies

## 4.1 Current-Analysis Examples

The obtained line current i(t) presents a Fourier spectrum as in Fig. 3. It reveals three distinct groups of peaks that give specific information on the harmonic content. The main harmonic components of these spectra are multiple of the carrier frequency that characterises the PWM technique on every line. In this case it is possible to check how the main current frequency is centred on 1066.7 Hz and the following harmonic packets find their location in multiple integers of this frequency.

From Fourier spectrum, nevertheless, we do not have any significant detail on how the harmonic content evolves in time. In order to find a better representation the wavelets analysis on this signal has been performed.

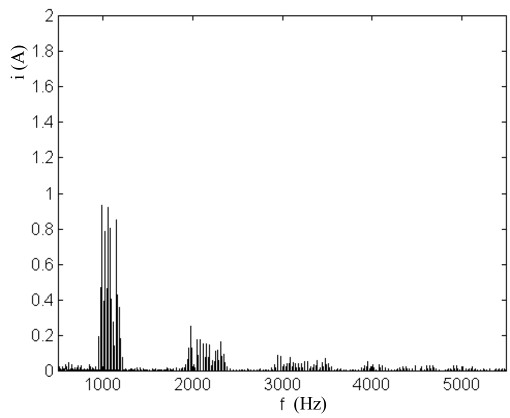


Fig.3. Fourier analysis of the line current.

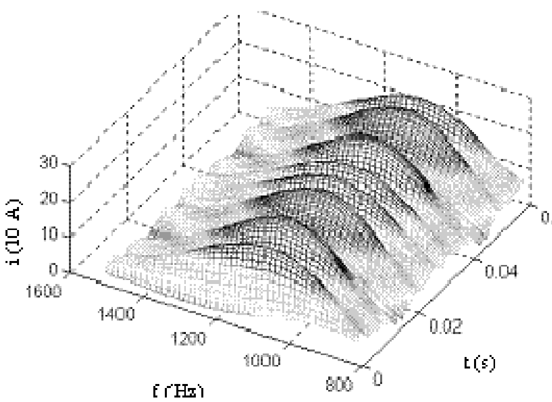


Fig.4. Wavelets analysis of the line current - First spectrum-peaks.

By exploding the first peak frequency it is possible to analyse the time-frequency behaviour presented in Fig.4.

This picture represents the frequency content evolution for the current and gives some additional information. The wavelet analysis put more emphasis on the presence of a frequency component at 1065 Hz showing a non-stationary characterization well-defined in the time axis. The same signal examined by means of Fourier Transform reveals only a less significant distribution of the harmonic content in the frequency domain.

The results obtained in this paper based on the wavelets analysis of the three 4QS parallel connected drivers are in perfect accordance with those related to a single 4QS converter studied with a classical theoretical method and presented by another author [11]. In this case the analytical evaluation of the maximum value for line-current is strictly related to the preliminary knowledge of the converter-driving technique. The wavelet approach, on the contrary, is based on the analysis of the harmonic characterizing the main frequency of the first group in the signal spectrum and of its amplitude behaviour in the time. This type of information is specific of the wavelet analysis. In this way any specific knowledge related to the converter working strategy can be left away. This approach is mandatory when more converters are working interconnected as happens in our analysis. The wavelet approach, by allowing an in-depth evaluation of the line-current time and frequency behaviour, assumes an important role for the EMC study in complex interconnected converters. It provides, in fact, each instant the complete knowledge of amplitudes and frequency constituting the line current adsorbed by the driver.

## 4.2 Starting transient analysis for the induction machine.

The transient analysis performed by means of the Euler's integration technique on the Park variables leads to the determination of the flowing current in any of the stator phases of the induction machine.

The resulting behaviour is justified by the PWM control implemented by the power converter in order to obtain the magnitude-frequency voltage regulation of Fig.5.

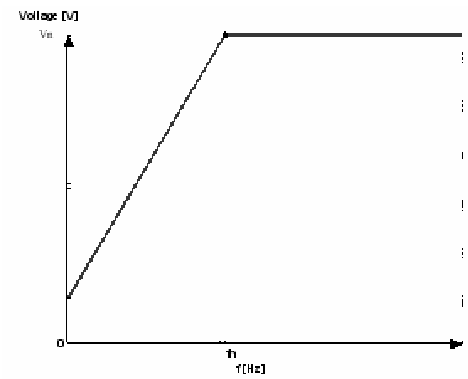


Fig.5. Voltage-frequency PWM control.

In order to analyse the evolution of the harmonic content on the time-frequency plane, through which put into evidence the optimal working condition of the converter, the wavelet analysis of the stator current is arranged. The time-variant properties of the spectrum are so defined and studied and the specific control technique more suitable for a given machine can be found. The wavelet representation of the signals shows how, at low frequency, the spectral content of the stator current be time-variant during the acceleration transient phase leading to the steady state (Fig.6).

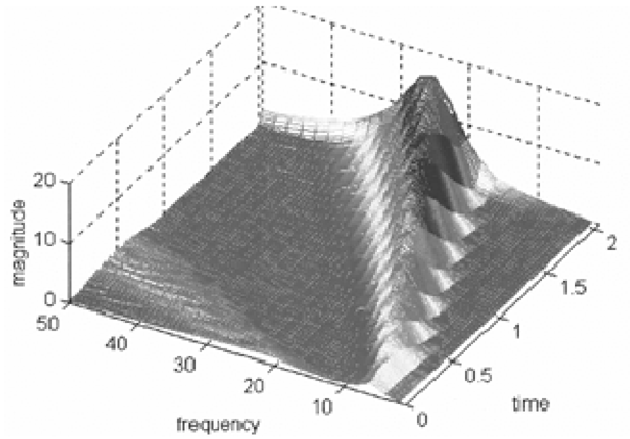


Fig.6. Wavelets analysis of starting transient stator current.

It is important to underline how this way to conduct the analysis does not enhance the level of information contained in the time domain representation of the voltage and current signals, but allows a better presentation of their evolutionary spectra during transient periods. This type of analysis acquire a great importance when electrical machine are powered through power electronic drives, because their specific need to have working conditions arranged in the most suitable and optimal way.

## 4.3 A frequency step

One of the most interesting application of this powerful analytical tool is the possibility offered to study effects due to the frequency variations in the power supply system following faults on the line. For example, when a change of frequency equal to 12% of the standard frequency is imposed in 0.3 s (Fig.7), the corresponding current spectral content in the time-frequency domain is shown in Fig.8.

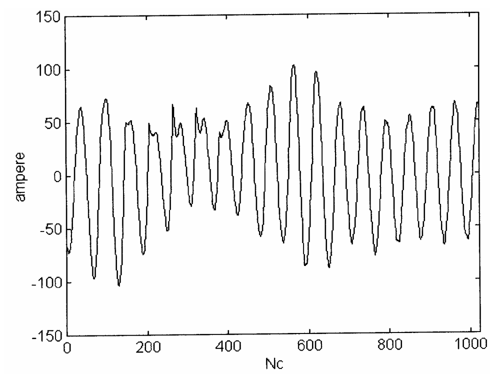


Fig.7. Statoric transient current following a frequency step in the power supply.

The current behaviour underlines the difficulties in finding an immediate balance in the current distortions and how this fact can lead to supply the machine under non-optimal steady state condition. The power losses can consequently influence the final performance of the whole system.

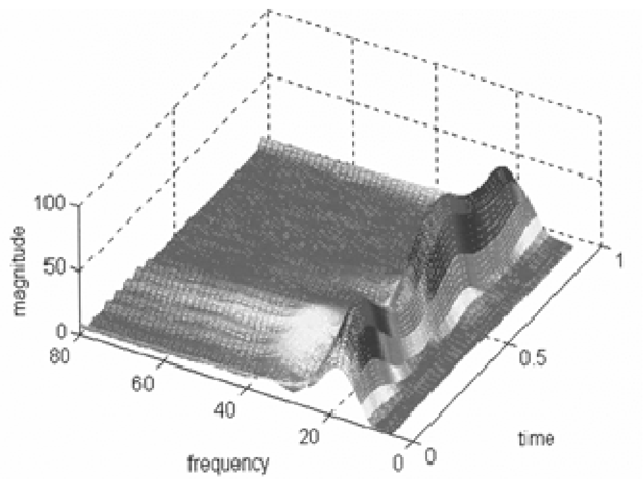


Fig.8. Wavelets analysis of the statoric transient current in case of a frequency step.

## 5 Conclusions

The analysis of an existing driving system for railway locomotives has been conducted with wavelet tools in order to verify the best working condition of the whole system. Following the case study presentation the authors arranged a short-dedicated introduction suited for the specific purposes of this paper and dealing with wavelet approach to time-frequency analysis of modulated signals (as the line-current is in our case).

The proposed technique allows an in-depth analysis of transient conditions for currents flowing in a four-quadrant power converter-induction machine system, constituting a typical electromechanical power converter. The mathematical tool adopted for this study is strictly related to the wavelet analysis of time-frequency representation of evolutionary systems. The consequent results shows how specific behaviours during the transient conditions can be studied in order to define the optimal working conditions of the system and the most suitable control strategies able to reduce losses and critical mechanical behaviours.

The possibility of identifying the harmonic instantaneous content of the power converter current is of great interest from a theoretical and practical point of view. In this way it is possible to identify correctly the proper interaction between the electrical drive and the catenary that powers the system. The results of a similar approach can help in the dynamical parameters definition evidencing the instantaneous harmonic evolution and opening the door to the EMC analysis of line noise provided by such electrical drives.

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