Predictive controllers for non-regular nonlinear systems

I. I. SILLER-ALCALÁ., J. U. LICEAGA-CASTRO[Θ], J. JAIMES-PONCE., AND R. ALCÁNTARA-RAMÍREZ.
Departamento de Electrónica, Grupo Control de Procesos
Universidad Autónoma Metropolitana
Av. San Pablo No. 180, Col. Reynosa Tamaulipas, Del. Azcapotzalco, C. P.0200, México D.F.
MÉXICO
[Θ]Departamento de Ing. Mecatrónica y Automatización
ITESM-CEM
Carretera de Lago de Gpe., Km. 3.5, Col. Margarita Maza de Juárez, Atizapán, Edo. Méx.
MÉXICO

Abstract: - This paper presents the control of non-regular or singular nonlinear systems applying non-linear predictive controllers. The systems treated here have the relative degree not well defined thus feedback linearization is not applicable due to singularities in the input-output linearizing control law. An alternative controller capable of dealing with non-regular systems is the NCGPC (Nonlinear Continuous Time Generalized Predictive Control). The NCGPC is successfully applied to the standard "ball and beam" non-linear non-regular process.

Key-Words: - Nonlinear control, Predictive Control, Switching Control, Singular Systems, Relative Degree, Continuous system.

1 Introduction

Predictive control and feedback linearization constitute two of the most important research lines in non linear control. Geometric linearization theory has permitted the applications of linear control algorithms to nonlinear systems. It is done by using feedback linearization techniques [1], through differential geometry approach to transform a nonlinear input-output system into a linear system; then, a linear controller can be applied to the linearised model. However, the applicability of the linearization algorithm fails when the system has singular points. This happens when $L_g L_f^{r-1} h(x) \neq 0$ for $x \neq x_0$ but $L_g L_f^{r-1} h(x_0) = 0$. This can be viewed as regions where the relative degree r can not be defined. Then x_0 is called a singular point of the nonlinear system and the system is known as a nonlinear system with ill-defined relative degree. These systems are also called singular or non-regular nonlinear systems. The control problem of non-regular nonlinear systems was treated firstly by [2] using a switching control law, unfortunately following this approach the class of outputs that could be tracked is limited. A classical problem of non-regular systems is the non-linear process "beam and ball". The beam and ball problem was treated in [3] by using an approximate system that is input-output linearisable, but exact asymptotic tracking was not possible. Also, in [4] a tracking control law which switches between an approximate tracking control law -close to singularities- and an exact tracking control law -away from singularities- was proposed. Meanwhile, within NMPC approaches [5], [6], [7]-[9], new control schemes are presented, but they can not deal with non-regular nonlinear systems. Moreover, the tracking control law used in [5], becomes unrealizable because the optimal control law given by $b_1(x)^2 - 4b_2(x)kp_1$ becomes imaginary in a vicinity of the singular point x_0.

The NCGPC, [10, 11] is an alternative nonlinear predictive controller capable of dealing with non-regular or singular nonlinear systems. The NCGPC was developed in a different way than conventional nonlinear controllers. The NCGPC [10, 11] is based in the prediction of the system output and due to the fact that it was not derived with the objective of canceling nonlinearities, as feedback linearization techniques do, the NCGPC has three advantages: First, it can constrain the predicted control through N_u -additionally the response becomes slow and the control is not very active-, and second, when $N_u < N_y - r$, there is not zero dynamics cancellation and then the internal stability is preserved. Also, the NCGPC [12] provides a nice way of handling systems with unstable zero dynamics. And the last advantage is the control weight λ, it plays a very important role in the cost function. In this paper, the non-regular nonlinear system is treated by using the last two advantages of the NCGPC. The application of the NCGPC to the standard non-regular nonlinear “beam and ball” system is shown in the following sections.

2 General Formulation

This paper considers nonlinear dynamics systems with the state-space representation:

$$\dot{x}(t) = f(x) + g(x)u \qquad (1)$$
$$y(t) = h(x)$$

where f, g y h are differentiable N_y times with respect to each argument. $x \in R^n$ is the vector of the state variables, $u \in R$ is the manipulated input and $y \in R$ is the output to be controlled.

2.1 Development of the NCGPC

The development of the NCGPC [10, 11]was carried out following the receding horizon strategy of its linear counterpart [13], which principles can be summarised as follows:

1. Predict the output over a range of future times.
2. Assuming that the future setpoint is known, choose a set of future controls which minimize the future errors between the predicted future output and the future setpoint.
3. Use the first element $u(t)$ as a current input and repeat the whole procedure at the next time instant; that is, use a receding horizon strategy.

2.1.1 Prediction of the output

In this section the output prediction is obtained following the idea of CGPC [13]. The output prediction is approximated for a Maclaurin series expansion of the system output as follows.

$$y^*(t,T) = y(t) + \dot{y}(t)T + y^{(2)}(t)\frac{T^2}{2!} + \cdots + y^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \qquad (2)$$

or

$$y^*(t,T) = T_{N_y} Y_{N_y} \qquad (3)$$

where

$$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T \qquad (4)$$

and

$$T_{N_y} = [1 \quad T \quad \frac{T^2}{2!} \quad \cdots \quad \frac{T^{N_y}}{N_y!}] \qquad (5)$$

The predictor order N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u

2.1.2 Prediction of the reference trajectory

The objective of the control is to drive the predicted output along a desired smooth path to a set point. Such a path is called a reference trajectory. The reference trajectory following [13] is given by

$$w_r^*(t,T) = [p_0 + p_1 T + p_2\frac{T^2}{2!} + \cdots + p_r\frac{T^{N_y}}{N_y!}][w - y(t)] + y(t) \qquad (6)$$

where w is the set point, or rewriting this equation

$$w_r^*(t,T) = T_{N_y} w_r + y(t) \qquad (7)$$

where

$$w_r = [p_0 \quad p_1 \quad \cdots \quad p_r]^T (w - y(t)) \qquad (8)$$

and T_{N_y} is given by (5)

2.1.3 Derivative emulation

The NCGPC is based in taking the derivatives of the output, which are obtained as follows

$$\dot{y}(t) = L_f h(x)$$
$$y^{(2)}(t) = L_f^2 h(x)$$
$$\vdots$$
$$y^{(r)}(t) = L_f^r h(x) + L_g L_f^{r-1} h(x) u(t)$$
$$y^{(r+1)}(t) = S_1(x) + J_1(x)u(t) + L_g L_f^{r-1} h(x)\dot{u}(t)$$
$$y^{(r+2)}(t) = S_2(x) + J_2(x)u(t) + I_1(x)\dot{u}(t) + L_g L_f^{r-1} h(x) u^{(2)}(t)$$
$$\vdots$$
$$y^{(N_y)}(t) = S_{(N_y-r)}(x) + J_{(N_y-r)}(x)u(t) + I_{(N_y-r)}(x)\dot{u}(t) + I_{(N_y-r+1)}(x)u^{(2)}(t) + I_{(N_y-r-1)}(x)u^{(N_y-r-1)}(t) + L_g L_f^{r-1} h(x) u^{(N_y-r)}(t) \qquad (9)$$

Where $L_f h(x)$ represents the Lie derivative S_i, J_i and I_i, are some functions of x (and not u). These output derivatives are obtains from the system of equation (1) and N_y is chosen less than the number of the times that the output has to be differentiated in order to obtain terms not linear in u, r is the relative degree. Output and its derivatives can be rewritten by

$$Y_{N_y}(t) = O(x(t)) + H(x(t))u_{N_y} \qquad (10)$$

where

$Y_{N_y} = [y \quad \dot{y} \quad y^{(2)} \quad \cdots \quad y^{(N_y)}]^T$ and $u_{N_y} = [u \quad \dot{u} \quad u^{(2)} \quad \cdots \quad u^{(N_y-r)}]^T$

$$O = \begin{bmatrix} y \\ L_f h(x) \\ L_f^2 h(x) \\ \vdots \\ L_f^r h(x) \\ S_1(x) \\ S_2(x) \\ \vdots \\ S_{(N_y-r)}(x) \end{bmatrix} \quad H = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ L_g L_f^{r-1} h(x) & 0 & \cdots & 0 \\ J_1(x) & L_g L_f^{r-1} h(x) & \cdots & 0 \\ J_2(x) & I_1(x) & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ J_{N_y-r}(x) & I_{N_y-r}(x) & \cdots & L_g L_f^{r-1} h(x) \end{bmatrix} \qquad (11)$$

2.1.4 Cost function minimization

The function is not defined with respect current time, but respect a moving frame, which origin is in time t. Where T is the future variable. Given a predicted output over a time frame the CGPC calculates the future controls. The first element $u(t)$ of the predicted controls is then applied to the system and the same procedure is repeated at the next time instant. This makes the predicted output depend on the input $u(t)$ and its derivatives, and the future controls being function of $u(t)$ and its N_u-derivatives. The cost function is:

$$J(u_{N_y}) = \int_{T_1}^{T_2} [y^*(t,T) - w_r^*(T,t)]^2 \, dT \tag{12}$$

With the substitution of equations (3) and (7) the cost function becomes

$$J(u_{N_y}) = \int_{T_1}^{T_2} [T_{N_y} O + T_{N_y} H u_{N_y} - T_{N_y} w_r)]^2 \, dT \tag{13}$$

and the minimization results in

$$u_{N_y} = K(w_r - O) \tag{14}$$

where

$$T_y = \int_{T_1}^{T_2} T_{N_y}^T T_{N_y} \, dT \quad \text{and} \quad K = [H^T T_y H]^{-1}[H^T T_y] \tag{15}$$

As explained above, just the first element of u_{N_y} is applied. Then, the first row of, which will be called, the control law is given by

$$u(t) = k[w_r - O] \tag{16}$$

3 Special Cases of NCGPC

In this section in order to control the non-regular nonlinear systems, two special cases of NCGPC [11] are developed making use of the three advantages of the NCGPC.

3.1 Case when NCGPC has control weight

The control weight λ plays a very important role in the cost function. In this case, it is assumed that the system described by (1) has stable zero dynamics, $N_u = N_y - r$ and $N_y = r$ and the reference trajectory is the output of the reference model [5] represented by

$$\dot{x}_r = A_r x_r + B_r w \tag{17}$$

$$y_r = C_r x_r$$

where $x_r \in R^{n_r}, A_r \in R^{n_r \times n_r}, B_r \in R^{n_r \times 1}, C_r \in R^{1 \times n_r}, w \in R$. In order to define the predicted output of the reference trajectory, a truncated Taylor series is used, obtaining:

$$y_r(t,T) = y_r(t) + \dot{y}_r(t)T + y_r^{(2)}(t)\frac{T^2}{2!} + \cdots + y_r^{(N_y)}(t)\frac{T^{N_y}}{N_y!} \tag{18}$$

Where the derivatives are easy to obtain from the reference model simulation. Rewriting this equation

$$w_r(t,T) = T_{N_y} w_r \tag{19}$$

where

$$w_r = [y_r \quad \dot{y}_r \quad y_r^{(2)} \quad \cdots \quad y_r^{(N_y)}]^T \tag{20}$$

The cost function is given by

$$J(u^*(t,0)) = [y_r^*(t,T) - y^*(t,T)]^2 + \lambda[u^*(t,0)]^2 \tag{21}$$

where λ is the control weight

$$J(u^*(t,0)) = [y_r(t) - O(x) + H(x)u^*(t,0)]^T T_{N_y}^T T_{N_y} \tag{22}$$

$$[y_r(t) - O(x) + H(x)u^*(t,0)] + \lambda[u^*(t,0)]^2$$

and the minimization results in the following control law:

$$u(t) = \frac{[(y_r - y) + (\dot{y}_r - L_f h)T + \cdots + (y_r^{(r)} - L_f^r h)\frac{T^r}{r!}] L_g L_f^{r-1} h \frac{T^r}{r!}}{\lambda + [L_g L_f^{r-1} h \frac{T^r}{r!}]^2} \tag{23}$$

In order to see, the role of the λ some approximations made by [2] are recalled:

- As $\left| \frac{T_r}{r!} L_g L_f^{r-1} h(x) \right| >> \lambda$, when the system is relatively far from any singular point, then,

$$u(t) = \frac{[(y_r - y) + (\dot{y}_r - L_f h)T + \cdots + (y_r^{(r)} - L_f^r h)\frac{T^r}{r!}]\frac{T^r}{r!}}{L_g L_f^{r-1} h \frac{T^r}{r!}} \tag{24}$$

- But, when $\left| \frac{T_r}{r!} L_g L_f^{r-1} h(x) \right| << \lambda$ when the system is very close to any singular point, we have

$$u(t) \approx 0$$

3.1.1 Simulation results

In order to show the effectiveness of the special case of NCGPC simulation of an example is presented. The following example has singular point when $x_2 = -0.5$

$$\dot{x}_1 = x_3 - x_2^2$$
$$\dot{x}_2 = -x_2 - u$$
$$\dot{x}_3 = x_1^2 - x_3 + u \tag{25}$$

where the output and its derivatives are given by

$$y(t) = x_1$$

$$\dot{y}(t) = x_3 - x_2^2$$

$$\ddot{y} = x_1^2 - x_3 + 2x_2^2 + (1 + 2x_2)u \tag{26}$$

Fig. 1 shows the states and that the output tracks the references. Fig. 2 shows the control input. It is possible to see that when the systems is very close to the singular point $x_2 = -0.5$ the control input is approximately zero, and when is far from this point the input becomes the equation (24), this is done thanks to $\lambda = .001$.

3.2 Case when $N_u < N_y - r$ is considered

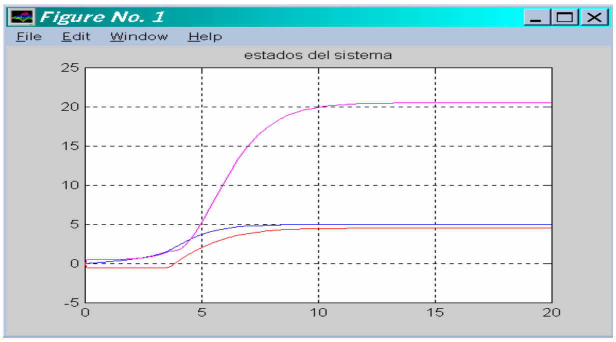

Figure 1. System states

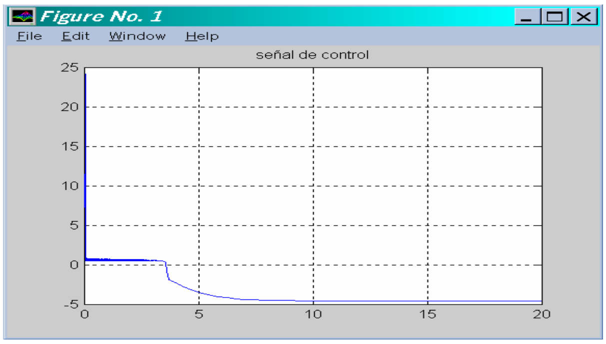

Figure 2. u(t)-note notch when x_2 is close to 0.5

When $N_u < N_y - r$, there is not zero dynamics cancellation and then the internal stability is preserved. Also, the NCGPC [12] provides a nice way of handling systems with unstable zero dynamics. Here, this condition will be used en order to get a special case of NCGPC for non-regular nonlinear "ball and Beam", which was described and treated by [3, 4]

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} x_2 \\ B(x_1x_4^2 - G\sin x_3) \\ x_4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} u \quad (27)$$

where

$x = (x_1, x_2, x_3, x_4)^T = (r, \dot{r}, \theta, \dot{\theta})^T$ and $y = r$

The exact input-output linearization is not defined when $x_1 x_2 = 0$. In order to get the predictive control, the output derivatives are gotten until $N_y = 4$

$$\begin{aligned} y &= x_1 \\ \dot{y} &= x_2 \\ \ddot{y} &= Bx_1x_4^2 - BG\sin x_3 \\ \dddot{y} &= Bx_2x_4^2 + 2Bx_1x_4u - BGx_4\cos x_3 \\ \ddddot{y} &= B^2x_1x_4^4 - BGx_4^2\sin x_3(B-1) + (4Bx_2x_4 - BG\cos x_3)u \\ &+ 2Bx_1u^2 + 2Bx_1x_4\dot{u} \end{aligned} \quad (28)$$

If $2Bx_1x_2$=0 in the third derivative the relative degree is not well defined and exact linearization [1] fails. In [2] this term was neglected and the derivatives proposed were:

$$\begin{aligned} y &= x_1 \\ \dot{y} &= x_2 \\ \ddot{y} &= Bx_1x_4^2 - BGsinx_3 \\ \dddot{y} &= Bx_2x_4^2 - BGx_4\cos x_3 \\ \ddddot{y} &= B^2x_1x_4^2 - BGx_4^2sinx_3(B-1) + (2Bx_2x_4 - BG\cos x_3)u \end{aligned} \quad (29)$$

With this approximation the well known feedback linearization is used.

In the NCGPC the linearization is not achieved, the output is predicted by using the output and its derivatives. Here the term which contains u^2 is neglected as in [3]. The control law obtained by taking account the term u^2 and minimizing the quadratic cost function obtained in [5] is not possible applied in the problem of "beam and ball" because $\sqrt{b_1(x)^2 - 4b_2(x)kp_1}$ becomes imaginary.

$$\begin{aligned} y &= x_1 \\ \dot{y} &= x_2 \\ \ddot{y} &= Bx_1x_4^2 - BGsinx_3 \\ \dddot{y} &= Bx_2x_4^2 + 2Bx_1x_4u - BGx_4\cos x_3 \\ \ddddot{y} &= B^2x_1x_4^2 - BGx_4^2sinx_3(B-1) + (4Bx_2x_4 - BG\cos x_3)u + 2Bx_1x_4\dot{u} \end{aligned} \quad (30)$$

It is possible to see that the two last derivatives have terms of $u(t)$ and $\dot{u}(t)$, but as $N_u < N_y - r$ the contributions of $\dot{u}(t)$ are not considered. The parameters of the algorithm are chosen as $N_y = 4$, $r = 3$, $N_u = 0$ and

$$H = \begin{bmatrix} 0 \\ 0 \\ 2Bx_1x_4 \\ 4Bx_2x_4 - BG\cos x_3 \end{bmatrix} \quad O = \begin{bmatrix} x_2 \\ Bx_1x_4^2 - BGsinx_3 \\ Bx_2x_4^2 - BGx_4\cos x_3 \\ B^2x_1x_4^2 - BGx_4^2sinx_3(B-1) \end{bmatrix} \quad (31)$$

the control law is obtained by substitution in equations (15) and (16).

3.2.1 Simulation results

In order to show the effectiveness of the special case of NCGPC simulation of the "beam and ball" example is presented.

Fig. 3 shows the output system and the reference Fig.4 shows the states. Fig. 5 shows the control input. The parameters of the algorithm are chosen as $N_y = 4$, $r = 3$, $N_u = 0$, $T_1 = 0$, $T_2 = 0.7$ and $R_n/R_d = 2/(s+2)$. As mentioned above $N_u < N_y - r$, then the control law will not have singular points, due to the exact linearization is not achieved.

4 Conclusion

This paper presents the control of non-regular or singular nonlinear systems applying special cases of NCGPC.

The first case was developed by making use of the control weight λ. This case a switching control, which switches between $u(t) \approx 0$ when is close to singularities and an exact linearization when is away from singularities.

As mentioned in [10, 11], when $N_u = N_y - r$ the NCGPC becomes in an exact linearization law as [1]. But, when $N_u < N_y - r$ the linearization is not achieved. The NCGPC can deal with non minimum phase nonlinear systems and non-regular nonlinear systems by using the three advantages mentioned above. The proposed controller exact asymptotic tracking is not possible as in [3, 4]. The NCGPC is an alternative nonlinear predictive controller capable of dealing with non-regular or singular nonlinear systems.

Figure 3. Output System and reference trajectory

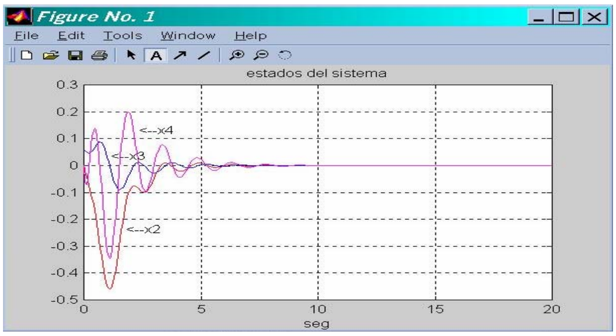

Figure 4. System states

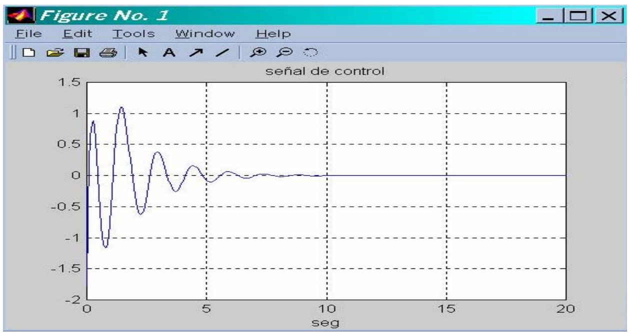

Figure 5. Input u(t)

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