Study of speeds regulators in case of some non-autonomous vibrating mechanisms

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Abstract: - The paper makes an analysis of two speed regulators for an uniform response in the case of some mechanisms with periodic motion. For the first mechanism with one degree of freedom, the conditions for the uniform motion are computed, in the case where the vibrating mass is a rigid coupling with the elastic and damping forces. For the second mechanism with two degree of freedom we have applied the average method and the Van der Pol method. The stability of the solutions in the phases space and the limit cycles for an uniform response of the system.

Key-Words: - Speed regulators, Average method, Van der Pol method, Vibrating mass, Mechanism with one degree of freedom, Mechanism with two degree of freedom, Nonlinear dynamic systems, Inverse problems.

1 Introduction

It is well known the importance of study nonlinear dynamic systems from the field of vibrating mechanisms and machines. Applications of regulators, absorbers and speed control in technique are at every step. The mathematical model leads to nonlinear dynamic systems and so, the study will take place through numeric and approximate methods appealing at results from linear case for stability. Here we mediate the non-stationary nonlinear terms from the dynamic systems that are considered as inverse problem – asking that the system movement should be uniform, the availability conditions for mechanic and geometric parameters are deduced. To find the solutions – similar with the constants variation method, the coefficients and parameters that vary slowly adjusting to Van der Pol method, are considered [1], [2]. To obtain the rotating angular velocities in case of uniform movement regime, graphic-analytical methods are applied, specifying the attraction basins for limit cycles. Generalizations and important applications in transportations and machine-tools are stated [3], [4].

2 Study of dynamic system with rigid coupling

The mechanism from fig. 1 is considered, where r is the radius of the connecting rod and l is the length of the crank.

Kinematic conditions (olonome links):

$$\begin{aligned} x_C &= r\cos\varphi - \frac{l}{2}\sin\beta, \\ y_C &= r\sin\varphi - \frac{l}{2}\cos\beta, \\ y_B &= r\sin\varphi - l\cos\beta, \\ r\cos\varphi &= l\sin\beta . \end{aligned} \tag{1}$$

If the last equation is derived, it is obtained:

$$-r\dot{\varphi}\sin\varphi = l\dot{\beta}\cos\beta , \tag{2}$$

or if we take into account that [4], [5]:

$$\dot{\varphi}=\omega_1 \;\; ; \;\; \dot{\beta}=-\omega_2 \;\; ; \;\; \dot{\omega}_1=\varepsilon_1 \;\; ; \;\; \dot{\omega}_2=\varepsilon_2 , \tag{3}$$

it results:

$$-r\omega_1\sin\varphi = -l\omega_2\cos\beta , \tag{4}$$

or:

$$\omega_2 = \frac{r\sin\varphi}{l\cos\beta}\omega_1 = t\omega_1 , \tag{5}$$

and:

$$-r\varepsilon_1\sin\varphi - r\omega_1^2\cos\varphi = -l\varepsilon_2\cos\beta - l\omega_2^2\sin\beta , \tag{6}$$

where:

$$\varepsilon_2 = \frac{r\sin\varphi}{l\cos\beta}\varepsilon_1 + \left(\frac{r\cos\varphi}{l\cos\beta} - t^2\frac{\sin\beta}{\cos\beta}\right)\omega_1^2 = t\varepsilon_1 + u\omega_1^2 , \tag{7}$$

with:

$$t = \frac{r\sin\varphi}{l\cos\beta} = \lambda\frac{\sin\varphi}{\cos\beta} , \; u = \lambda\frac{\cos\varphi}{\cos\beta} - t^2\frac{\sin\beta}{\cos\beta} . \tag{8}$$

Then, the first derivative relations will give:

$$\begin{Bmatrix} \omega_1 \\ \dot{x}_C \\ \dot{y}_C \\ \omega_2 \\ \dot{y}_B \end{Bmatrix} = \begin{Bmatrix} 1 \\ -r\sin\varphi + \frac{l}{2}t\cos\beta \\ r\cos\varphi - \frac{l}{2}t\sin\beta \\ t \\ r\cos\varphi - lt\sin\beta \end{Bmatrix}\omega_1 = \{A_1\}\omega_1 . \tag{9}$$

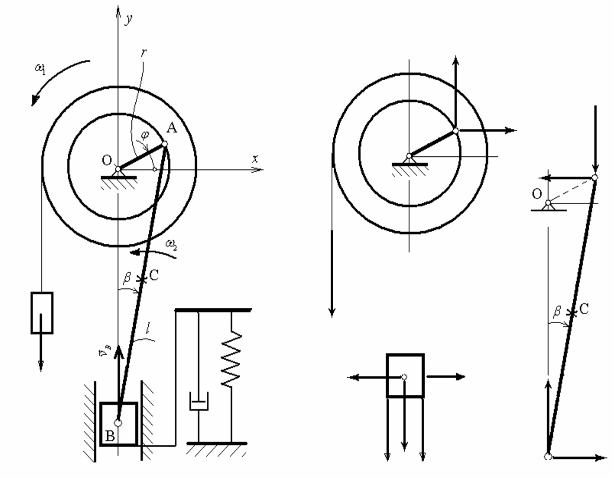

Fig. 1 The crank – connecting rod mechanism

If we derive once more the kinematic conditions, it should be obtain:

$$\begin{Bmatrix} \varepsilon_1 \\ \ddot{x}_C \\ \ddot{y}_C \\ \varepsilon_2 \\ \ddot{y}_B \end{Bmatrix} = \begin{Bmatrix} 1 \\ -r\sin\varphi + \frac{l}{2}t\cos\beta \\ r\cos\varphi - \frac{l}{2}t\sin\beta \\ t \\ r\cos\varphi - lt\sin\beta \end{Bmatrix}\varepsilon_1 + \qquad , \qquad (10)$$

$$+\begin{Bmatrix} 0 \\ -r\cos\varphi + \frac{l}{2}t^2\sin\beta + \frac{l}{2}u\cos\beta \\ -r\sin\varphi + \frac{l}{2}t^2\cos\beta - \frac{l}{2}u\sin\beta \\ u \\ -r\sin\varphi + lt^2\cos\beta - lu\sin\beta \end{Bmatrix}\omega_1^2$$

or in compact form:

$$\{a\} = \{A_1\}\varepsilon_1 + \{A_2\}\omega_1^2 . \qquad (11)$$

2.1 Motion equations

If the basic theorems for the rigid body are applied, considering that the mechanism is composed from three rigid bodies, it can be written:

- for the wheel, the kinetic moment theorem in point O can be written:

$$J_O\varepsilon_1 = MgR - X_A r\sin\varphi + Y_A r\cos\varphi . \qquad (12)$$

- for the rod AB, it can be obtained similar:

$$\begin{aligned} m_b\ddot{x}_C &= -X_A + X_B \\ m_b\ddot{y}_C &= -Y_A + Y_B \end{aligned} , \qquad (13)$$

$$J_C\varepsilon_2 = X_B\frac{l}{2}\cos\beta - Y_B\frac{l}{2}\sin\beta + X_A\frac{l}{2}\cos\beta - Y_A\frac{l}{2}\sin\beta . \qquad (14)$$

- the point C will have a translation motion, so:

$$m\ddot{x}_B = -F_a - F_e - Y_C , \qquad (15)$$

where:

$$F_a = c\,\dot{y}_B = c\omega_1(r\cos\varphi - lt\sin\beta), \qquad (16)$$

$$F_e = k(y_o - y_B) = k(-r - l - r\sin\varphi + l\cos\beta) . \qquad (17)$$

It results the motion equations for the system:

$$\begin{bmatrix} J & & & & \\ & m_b & & & \\ & & m_b & & \\ & & & J_C & \\ & & & & m \end{bmatrix}\begin{Bmatrix} \varepsilon_1 \\ \ddot{x}_C \\ \ddot{y}_C \\ \varepsilon_2 \\ \ddot{y}_B \end{Bmatrix} = \qquad ,(18)$$

$$= \begin{Bmatrix} MgR - X_A r\sin\varphi + Y_A r\cos\varphi \\ -X_A + X_B \\ -Y_A + Y_B \\ X_B\frac{l}{2}\cos\beta - Y_B\frac{l}{2}\sin\beta + X_A\frac{l}{2}\cos\beta - Y_A\frac{l}{2}\sin\beta \\ -F_a - F_e - Y_B \end{Bmatrix}$$

or, if we take into account the kinematic conditions:

$$\begin{bmatrix} J & & & & \\ & m_b & & & \\ & & m_b & & \\ & & & J_C & \\ & & & & m \end{bmatrix}\left(\begin{Bmatrix} 1 \\ -r\sin\varphi + \frac{l}{2}t\cos\beta \\ r\cos\varphi - \frac{l}{2}t\sin\beta \\ t \\ r\cos\varphi - lt\sin\beta \end{Bmatrix}\varepsilon_1 + \right.$$

$$\left. + \begin{Bmatrix} 0 \\ -r\cos\varphi + \frac{l}{2}t^2\sin\beta + \frac{l}{2}u\cos\beta \\ -r\sin\varphi + \frac{l}{2}t^2\cos\beta - \frac{l}{2}u\sin\beta \\ u \\ -r\sin\varphi + lt^2\cos\beta - lu\sin\beta \end{Bmatrix}\omega_1^2 \right) =$$

$$= \begin{Bmatrix} MgR \\ 0 \\ 0 \\ 0 \\ -F_a - F_e \end{Bmatrix} + \begin{bmatrix} -r\sin\varphi & r\cos\varphi & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ \frac{l}{2}\cos\beta & -\frac{l}{2}\sin\beta & \frac{l}{2}\cos\beta & -\frac{l}{2}\sin\beta \\ 0 & 0 & 0 & -1 \end{bmatrix}\begin{Bmatrix} X_A \\ Y_A \\ X_B \\ Y_B \end{Bmatrix} . \qquad (19)$$

With the previous considered notations, we can write in a compact form:

$$[m](\{A_1\}\varepsilon_1 + \{A_2\}\omega_1^2) = \{Q^{ext}\} + \{Q^{leg}\} = \{Q^{ext}\} + [N]\{R\} . \qquad (20)$$

The mechanical work of constraint forces can be written:

$$dL = \{\delta\Delta\}^T\{Q\}^{leg} = \{\dot{\Delta}\}^T\{Q\}^{leg}dt = \{\dot{q}\}^T[A_1]^T\{Q\}^{leg}dt = \qquad , \qquad (21)$$
$$= \omega_1\{A_1\}\{Q\}^{leg}dt = 0$$

from where, the relation:

$$\{A_1\}^T\{Q\}^{leg} = 0 . \qquad (22)$$

The vector $\{Q^{leg}\}$ represents the equivalent force-couple system of the generalized constraint forces according to the considered generalized coordinates.

If we pre-multiply equation (20) with $\{A_1\}^T$ it results the motion equation:

$$J(\varphi)\ddot{\varphi} + \frac{1}{2}J'(\varphi)\dot{\varphi}^2 = M(\varphi), \qquad (23)$$

where:

$$J(\varphi)=\{A_1\}^T[m]\{A_1\} \quad ; \quad J'(\varphi)=\{A_1\}^T[m]\{A_2\}, \qquad (24)$$

$$M(\varphi)=MgR-(r\cos\varphi-lt\sin\beta)(F_a+F_e)=$$
$$=MgR-(r\cos\varphi-lt\sin\beta)[c\omega_1(r\cos\varphi-lt\sin\beta)+$$
$$k(-r-l-r\sin\varphi+l\cos\beta)\,]=$$
$$=MgR+k(r\cos\varphi-lt\sin\beta)(r+l)-$$
$$-k(r\cos\varphi-lt\sin\beta)(-r\sin\varphi+l\cos\beta)-$$
$$-c\omega_1(r\cos\varphi-lt\sin\beta)^2. \qquad (25)$$

In the followings we make the average of the written relations considering a quasi-uniform regime [6], [7], [3]. We can compute:

$$\overline{J'(\varphi)}=\frac{1}{2\pi}\int_0^{2\pi}\{A_1\}^T[m]\{A_2\}d\varphi=0\,, \qquad (26)$$

$$\overline{MgR}=MgR\,,\ \overline{k(r\cos\varphi-lt\sin\beta)(r+l)}=0\,, \qquad (27)$$

$$\overline{k(r\cos\varphi-lt\sin\beta)(-r\sin\varphi+l\cos\beta)}=0\,, \qquad (28)$$

$$\overline{c\omega_1(r\cos\varphi-lt\sin\beta)^2}=cl^2\omega_1(0{,}5\lambda^2+0{,}000204)\,, \qquad (29)$$

because:

$$\overline{\cos^2\varphi}=0{,}5\,,\ \overline{t^2\sin^2\beta}=0{,}000204\,, \qquad (30)$$

$$\overline{t\cos\varphi\sin\beta}=0\,. \qquad (31)$$

The following graphics present the equivalent moment inertia of the system on one rotation, the graphic of the variation of this inertia moment, the force which appears in spring (conservative) and the graphic of the damping force.

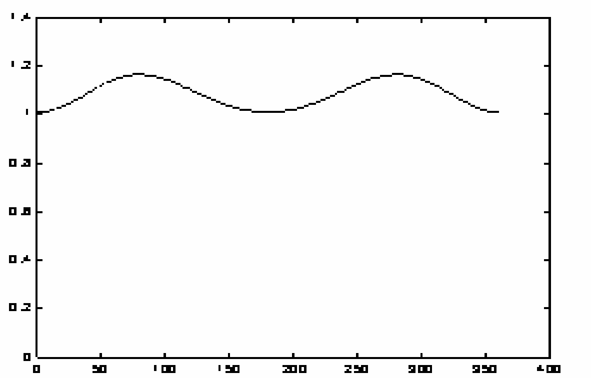

Fig. 2 The equivalent moment of inertia

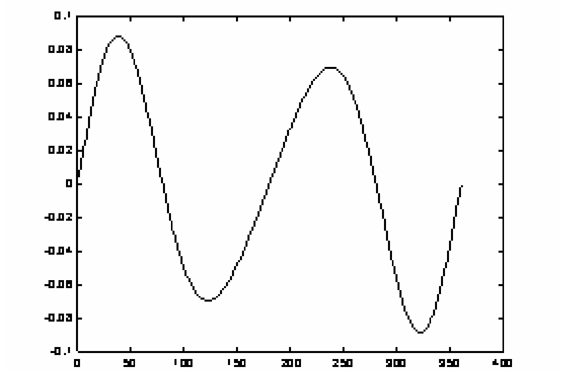

Fig. 3 Variation of the square omega coefficient

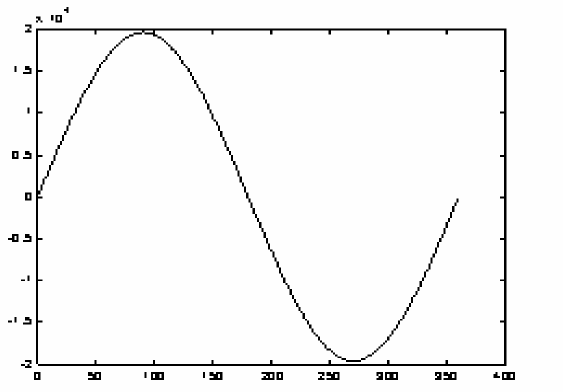

Fig. 4 The graphic of variable component of the spring force

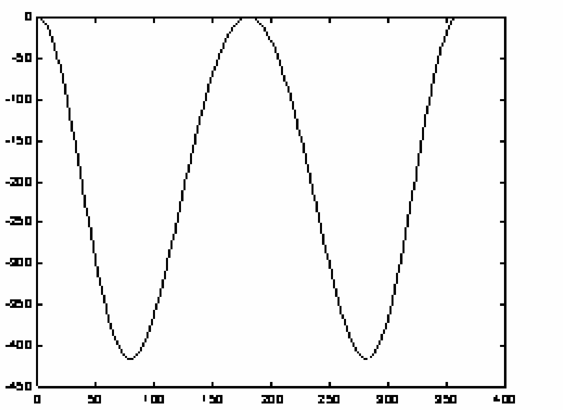

Fig. 5 The graphic of the moment given by the damping force

It results the angular velocity which ensures the quasi-uniform rotation of the system, from:

$$MgR=cl^2\omega_1(0{,}5\lambda^2+0{,}000204)\,, \qquad (32)$$

from where:

$$\omega_1=\frac{MgR}{cl^2(0{,}5\lambda^2+0{,}000204)}\,. \qquad (33)$$

For this working rotation of the system, the energy introduced in the system by the gravitational force is dissipated by the rub which takes place in the damper. In fig. 6 is presented the paths in phases space [6].

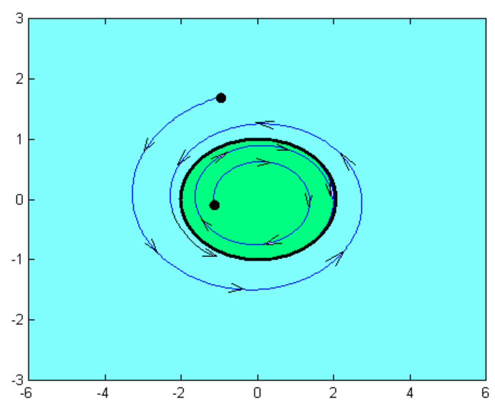

Fig. 6 The paths in the phases space [6]

3 Study of speed regulator with two degrees of freedom

In fig. 7 is presented this system which presents two degrees of freedom.

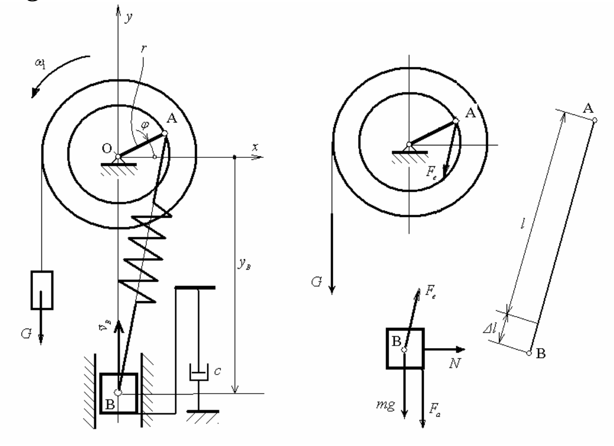

Fig. 7 Mechanical system with two degrees of freedom

In this case the damping force is given by:

$$F_a = c\,\dot{y}_B\,, \tag{34}$$

$$r\cos\varphi=(l+\Delta l)\sin\beta \quad ; \quad r\sin\varphi-(l+\Delta l)\cos\beta=y_B, \tag{35}$$

$$\frac{r\cos\varphi}{r\sin\varphi - y_B} = tg\beta\,, \tag{36}$$

$$\Delta l = \frac{r\cos\varphi}{\sin\beta} - l \cong r\sin\varphi - y_B - l\,, \tag{37}$$

$$F_e = k\Delta l = k(r\sin\varphi - y_B - l)\,. \tag{38}$$

The motion equations for mass B are given by:

$$m\ddot{y}_B + c\dot{y}_B + k(r\sin\varphi - y_B - l)\cos\beta = -mg\,. \tag{39}$$

If we consider $\cos\beta \cong 1$, it results:

$$\ddot{y}_B + 2n\dot{y}_B + p^2 y_B = \frac{k(r\sin\varphi - l)}{m} - g\,, \tag{40}$$

or:

$$\ddot{y}_B + 2n\dot{y}_B + p^2 y_B = p^2 r\sin\varphi - g - p^2 l\,. \tag{41}$$

If we apply the kinetic moment theorem for the wheel, with the same approximation $\cos\beta \cong 1$, it results:

$$(J + MR^2)\ddot{\varphi} - k(r\sin\varphi - y_B - l)r\cos\varphi = MgR\,. \tag{42}$$

We notice that this system has two degrees of freedom, the dispacement y_B and the rotation angle φ. We want to see in what conditions, the wheel rotattes uniform i.e. with constant angular velocity $\dot{\varphi} = \omega = ct$ and how much is that angular velocity which mentain the system in the quasi-stationary regime. We will apply the Van der Pol method – the parameters method that vary slowly or the method of constants variation.

We notice that a solution of equation (41) is $-\frac{g}{p^2} - l$ and because the perturbation force from the right member is $p^2 r\sin\varphi - g - p^2 l$ and $\dot{\varphi} = \omega = ct$, the form of φ may be: $\varphi = \omega t + \alpha$. We search the solution of this equation in the form:

$$y = A\sin(\varphi+\theta) - \frac{g}{p^2} - l \quad ; \quad \dot{\varphi} = \omega\,, \tag{43}$$

where A and θ will be the parameters that varies slowly in time. Derivating:

$$\dot{y} = \dot{A}\sin(\varphi+\theta) + A(\dot{\varphi}+\dot{\theta})\cos(\varphi+\theta) = \\ = \dot{A}\sin(\varphi+\theta) + \dot{\theta}A\cos(\varphi+\theta) + A\omega\cos(\varphi+\theta)\,. \tag{44}$$

We apply the assumption of constants variation and we have:

$$\dot{A}\sin(\varphi+\theta)+\dot{\theta}A\cos(\varphi+\theta)=0; \\ \dot{A}\to 0; \ \ \dot{\theta}\to 0 \ \ slowly \quad , \tag{45}$$

$$\dot{y} = A\omega\cos(\varphi+\theta)\,, \tag{46}$$

$$\ddot{y} = \dot{A}\omega\cos(\varphi+\theta) - A\omega^2\sin(\varphi+\theta) - A\omega\dot{\theta}\sin(\varphi+\theta)\,. \tag{47}$$

We introduce equations (43), (46), (47) in (41) and we have:

$$\dot{A}\omega\cos(\varphi+\theta) - A\omega^2\sin(\varphi+\theta) - A\omega\dot{\theta}\sin(\varphi+\theta) + \\ + 2nA\omega\cos(\varphi+\theta) + p^2 A\sin(\varphi+\theta) - \\ - \frac{g}{p^2} - l = p^2 r\sin\varphi - \frac{g}{p^2} - l, \tag{48}$$

or:

$$\dot{A}\omega\cos(\varphi+\theta) - A\omega^2\sin(\varphi+\theta) = \\ = A\omega\dot{\theta}\sin(\varphi+\theta) - 2nA\omega\cos(\varphi+\theta) - \\ - p^2 A\sin(\varphi+\theta) + p^2 r\sin\varphi\,. \tag{49}$$

With (49) and (45) we can resolve the system in $\dot{A}$ and $\dot{\theta}$, obtaining:

$$\dot{A}\omega\cos(\varphi+\theta) = A(\omega^2 - p^2)\sin(\varphi+\theta)\cos(\varphi+\theta) - \\ - 2nA\omega\cos^2(\varphi+\theta) + p^2 r\sin\varphi\cos(\varphi+\theta), \tag{50}$$

$$A\dot{\theta} = A(p^2 - \omega^2)\sin^2(\varphi+\theta) + \\ + 2nA\omega\sin(\varphi+\theta)\cos(\varphi+\theta) - p^2 r\sin\varphi\sin(\varphi+\theta)\,. \tag{51}$$

Similar, replacing (43) in (42) we obtain:

$$(J + MR^2)\ddot{\varphi} = MgR + k(r\sin\varphi - \\ A\sin(\varphi+\theta) - \frac{g}{p^2})r\cos\varphi\,. \tag{52}$$

Mediate (50), (51), (52) after variable φ with $T = 2\pi$ where:

$$\frac{1}{2\pi}\int_0^{2\pi}\sin^2\varphi\, d\varphi = \frac{1}{2} \quad ; \quad \frac{1}{2\pi}\int_0^{2\pi}\cos^2\varphi\, d\varphi = \frac{1}{2}\,. \tag{53}$$

Integrating all therms, we obtain:

$$\dot{A}\omega = -nA\omega - \frac{rp^2}{2}\sin\theta\,, \tag{54}$$

$$A\dot{\theta} = \frac{A}{2}(p^2 - \omega^2) - \frac{rp^2}{2}\cos\theta\,, \tag{55}$$

$$(J + MR^2)\dot{\omega} = MgR + \frac{kAr}{2}\sin\theta\,. \tag{56}$$

The nonlinear system (54), (55), (56) is nonlinear in the unknowns A, θ, ω.

To find the equilibrium points or uniform static regime we can do directly $\dot{A} \to 0, \dot{\theta} \to 0, \dot{\omega} \to 0$ and it results $A = ct, \theta = \alpha, \omega = ct$ at t=0. In the system we can be replace also:

$$\dot{A} = \frac{\partial A}{\partial \varphi}\omega,\ \dot{\theta} = \frac{\partial \theta}{\partial \varphi}\omega,\ \dot{\omega} = \frac{\partial \omega}{\partial \varphi}\omega, \tag{57}$$

where the variable becomes φ. From the system (57):

$$nA\omega + \frac{rp^2}{2}\sin\theta = 0, \tag{58}$$

$$\frac{A}{2}(p^2 - \omega^2) - \frac{rp^2}{2}\cos\theta = 0, \tag{59}$$

$$MgR + \frac{kAr}{2}\sin\theta = 0, \tag{60}$$

it can be obtained:

$$tg\theta = -\frac{2n\omega}{p^2 - \omega^2}\quad, \tag{61}$$

$$\sin\theta = -\frac{2n\omega}{\sqrt{(p^2 - \omega^2)^2 + 4n^2\omega^2}},$$
$$\cos\theta = \frac{p^2 - \omega^2}{\sqrt{(p^2 - \omega^2)^2 + 4n^2\omega^2}}, \tag{62}$$

$$A = \frac{r p^2}{\sqrt{(p^2 - \omega^2)^2 + 4n^2\omega^2}} = \frac{r p^2}{R}, \tag{63}$$

$$f(\omega) = \frac{r\omega p}{(p^2 - \omega^2)^2 + (2np\omega)^2} = \frac{MgR}{kr^2} = K. \tag{64}$$

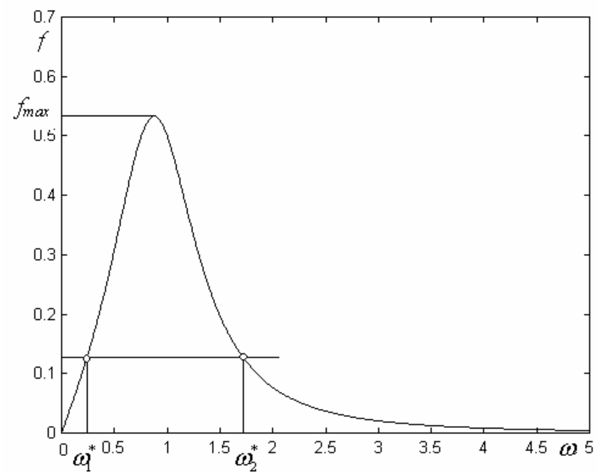

Fig. 8 The dependence $f = f(\omega)$

From the relation (64) we determine the values of ω since we have uniform rotation. It can be seen that in (64) we have an equation of fourth degree which admits two real and two complex roots. This observation can be done drawing the graphic $f = f(\omega)$ intersecting with f=K. If we denote $\omega / p = t$ we have:

$$f(t) = \frac{nt}{(1 - t^2)^2 + (2nt)^2} = K. \tag{65}$$

So, for $K \in (0, f_{max})$ (see fig. 8) we have two roots $\omega \in (0, \omega_1^*)$, $\omega \in (\omega_1^*, \omega_2^*)$, $\omega \in (\omega_2^*, \infty)$. For the values of ω, θ, A we can write the solution of equation (41): $n^2 - p^2 = -\delta^2$ with the particular solution (43) and initial perturbating conditions $y = y_o$, $\dot{y}(t=0) = \dot{y}_o = v_o$:

$$y = e^{-nt}\left(y_o \cos\delta t + \frac{ny_o + \dot{y}_o}{\delta}\sin\delta t\right) +$$
$$+ e^{-nt}\sin(\delta t + \theta_1) + A\sin(\omega t + \alpha) + \frac{g}{p^2}, \tag{66}$$
$$\dot{y} = e^{-nt}H(t, \delta, \theta_1, y_o, \dot{y}_o) + A\omega\cos(\omega t + \alpha)$$

with:

$$\theta_1 = arctg\frac{2n\delta}{n^2 - \delta^2 + \omega^2}. \tag{67}$$

Reconsidering equation (56) since with ω we start in neighborhood of ω_1^*, ω_2^* simulating $\dot{\omega}$ we have $sgn\,\dot{\omega} = sgn(K - f(\omega))$ and for $K < f_{max}$ we can see that ω_1^* is stable atractor and ω_2^* is instable. So, if ω is taken in the neighbourhood of ω_1^* i.e. in the case of first basins, we have motion regimes that stabilizes towards an uniform motion.

Considering the phases space (X,Y) where $(y = X, \dot{y} = Y)$ and the two ellipses E_1, E_2 according to particular solutions:

$$\left(\frac{X - \frac{g}{p^2}}{A(\omega_{1,2}^*)^2}\right)^2 + \frac{Y^2}{(\omega_{1,2}^*)^2 A^2} = 1. \tag{68}$$

These becomes limit cycles $E_1(\omega_1^*)$ stable limit cycle and $E_1(\omega_2^*)$ instable limit cycle, since the paths (X,Y) from (66) are spirals and when $t \to \infty$, the first therms tend to zero approaching at E_1 for the basin α) interior of E_1 and β) between the ellipses E1, E2. The basins β) and γ) give the instable paths for E2. So, from the technical point of view, K must be chosen as low as possible and ω_1^* as small as possible for the stable uniform regime.

In the case when $\omega_1^* = \omega_2^*$, $K = f_{max}$ we have an instable point, the movement becoming accelerated. If $K > f_{max}$ the movement is instable accelerated.

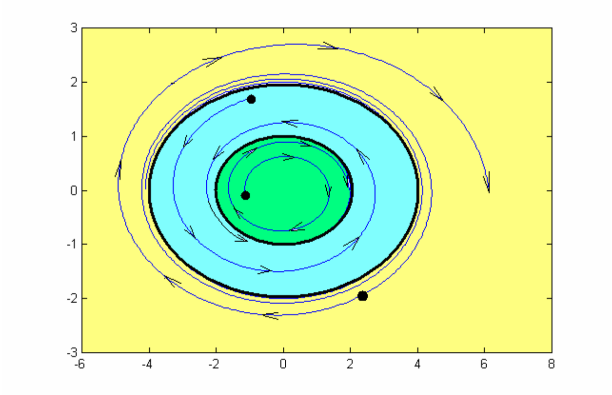

Fig. 9

4 Conclusions

This model of regulator is denoted in technique the Buassa-Sarde model [6]. Observation: another variant of the constants variation – Van der Pol consists in choosing the polar coordinates. Analogous with (43) we can search the solution of equation (41) in the form:

$$y = a(t)\cos\varphi + b(t)\sin\varphi + \frac{g}{p^2}, \qquad (69)$$

$$\dot{\varphi} = \omega$$

where $a(t)$ and $b(t)$ vary slowly:

$$\dot{y} = \dot{a}\cos\varphi + \dot{b}\sin\varphi - a\omega\sin\varphi + b\omega\cos\varphi\,. \qquad (70)$$

According to the constants variation method, we can take:

$$\dot{a}\cos\varphi + \dot{b}\sin\varphi = 0\,, \qquad (71)$$

$$\dot{y} = \omega(-a\sin\varphi + b\cos\varphi)\,, \qquad (72)$$

$$\ddot{y} = \omega(-\dot{a}\sin\varphi + \dot{b}\cos\varphi) + \omega^2(-a\cos\varphi - b\sin\varphi)\,. \qquad (73)$$

Introducing (69), (72), (71) in equation (41) we obtain:

$$\omega(-\dot{a}\sin\varphi + \dot{b}\cos\varphi) + \omega^2(-a\cos\varphi - b\sin\varphi) +$$
$$+ 2n\omega(-a\sin\varphi + b\cos\varphi) +$$
$$+ p^2(a\cos\varphi + b\sin\varphi) + g' = \frac{kr}{m}\sin\varphi + g\,. \qquad (74)$$

The system (71), (74) is resolved, deducing $\dot{a}$, $\dot{b}$:

$$\dot{a}\omega = F(\omega,a,b,\varphi) \quad , \quad \dot{b}\omega = G(\omega,a,b,\varphi)\,. \qquad (75)$$

We make the mediation of relations (75) after $\varphi \in [0,2\pi]$. Similar, in equation (42) is replacing with (69) and making the mediation. The following system is obtained:

$$\dot{a}\omega = \frac{b}{2}(p^2 - \omega^2) - n\omega a - \frac{p^2 k}{2},$$

$$\dot{b}\omega = \frac{a}{2}(p^2 - \omega^2) + n\omega b\,, \qquad (76)$$

$$(J + MR^2)\dot{\omega} = MgR + \frac{kar}{2}\,.$$

In (76) we make $\dot{a} = 0$; $\dot{b} = 0$; $\dot{\omega} = 0$ for the uniform motion and we obtain with $a^2 + b^2 = A^2$, $a = A\sin\theta$, $b = A\cos\theta$ and so $y = a\cos\varphi + b\sin\varphi = A\sin(\varphi + \theta)$ and the formulae for A,θ,ω are the same passing from (a,b,ω) to (A,θ,ω). The paths interpretations are done in palne (a,b) obtaining the same results. The resonance phenomennon puts here also in evidence for $\eta \to 0$ with $\omega \to p$ generating the phenomennon of instability, the amplitude A is increasing. It results that it must be eliminate the coincidence zone $\omega \cong p$ and decreased η.

References:

[1] Teodorescu, P.P., *Sisteme Mecanice*, Ed. Tehnica, 2002 (in Romanian).

[2] Voinea, R., Stroie, I., *Introducere in teoria sistemelor dinamice*, Ed. Academiei Romane, 2000 (in Romanian).

[3] Chiriacescu, S.T., *Stability in the Dynamics of Metal Cutting*, Elsevier, 1990.

[4] Munteanu, M., *Introducere in dinamica masinilor vibratoare*, Ed. Academiei Romane, 1986 (in Romanian).

[5] Chiriacescu, S.T., *Dinamica masinilor unelte. Prolegomene*, Ed. Tehnica, 2004 (in Romanian).

[6] Panovko, Y.G., Gubanova, Y.Y., *Stabilitate si vibratiile sistemelor elastice*,Ed Nauka, 1964 (in Russian).

[7] Lupu, M., Postelnicu, A., Deaconu, A., A study on the Flutter Stability of Dynamic Aeroelastic Systems, *Proceedings Conference CDM*, Vol. 1, 2001, pp. 37-47.