The Application of Hierarchical Linear Model on PISA2003： 15-year-olds students on International Mathematics Cognition Assessment

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Abstract: The purpose of this study was to investigate the PISA2003 database by Hierarchical Linear Model (HLM). This plan of PISA2003 aimed at the evaluation of mathematics and problem-solving ability. The sample included 41 nations, and the task-takers participating in PISA2003 were high school students whose age was about 15-year-olds.There will be estimation errors if all the task-takers were put in the same level when it went to analyze all the casual relation between the variables of nations and students. Therefore, HLM of two levels was used to analyze the PISA2003 database. The unit in level-1 was students. The variables belonging to the level-1 were score of mathematics cognition and mathematics confidence. As to the level-2, the unit was nations and the variable was GCI (Growth Competitiveness Index). The results of HLM analysis showed that the GCI can explain the variance of mathematics cognition for each nation separately. Moreover, the higher this variable was, the higher mathematics cognition will be. Finally, based on the findings of this study, some suggestions for future research were provided.

Key-Words: Growth Competitive Index (GCI), Hierarchical Lineal Model, Mathematics Cognition, PISA2003

1 Background

The PISA2003 database was built by OECD (Organization for Economic Co-operation and Development) under the plan of Programme for International Student Assessment (PISA) in 2003 [5] [6]. It is based on a dynamic model of lifelong learning in which new knowledge and skills necessary for successful adaptation throughout life. PISA2003 is an internationally standardized assessment that was jointly developed by participating 41 countries and administered to 15-year-olds students in educational programme, and the focus of PISA2003 was on mathematics literacy, defined as the capacity of students to identify, understand and engage in mathematics and to make well-founded judgements about the role that mathematics plays in life. The plan of PISA tries to understand the 15-yead-olds students' ability in solving life problems by cognition skills after they finish junior high school education. Hence, mathematics learning and problem solving by using mathematics skills are core capabilities. The contents emphasize the understanding of learning concepts and they could be to deal properly with all kinds of complicated situations. PISA2003 is a paper-and-pencil test, lasting a total of two hours for each student. Test items are a mixture of multiple-choice items and questions requiring students to construct their own responses.

Most of the data analysis methodologies on PISA2003 are basic statistics descriptions. The advanced information for the comparison of relationships between nations and students are limited. Thanks to the features of Hierarchical Linear Model (HLM), it is used to analyze the PISA2003 database in this study, and exploratory study of two levels model will be applied. The unit of level-1 is students and the unit of level-2 is nations.

2 Literature Review

With the unremittingly educational innovation, educational psychology researchers in the world pay more and more attention to emphasize the influential factors on learning achievements. They also concern students how to use the knowledge learned from school to solve the problems in their life. There are more and more institutions to engage in the assessment of international learning achievement and attitude for students. For example, SITES2006, TEDS, PIRLS2006, and TIMSS-R are well-known international assessment. The contents of PISA are different from most of the other international assessments. It is not the test on specific knowledge in the textbooks, but integrates all the field knowledge in reading, mathematics, and science to the life problems. There have been already some research outcomes in PISA, and the research indicates that there are other variables needed to clarify the casual relationships in the international database. In the 1990s, several international data sets were disseminated, demanding more hierarchical data analyses at various organization levels [1]. Statistical analyses of multilevel models provided quite a few information for education outcomes [11]. Nevertheless, family background and school resources are commonly concerns which may have effects on the learning achievement. Many studies also have aimed at the background variables of family and school in order to understand how these variables carry effects on mathematics achievements of students [9] [10]. However, seldom research aimed at the relationship of nation background and mathematics achievement. Moreover, effects of these findings have been less consistent in large cross-national studies. Although there were some research which investigated the PISA database using HLM, these research focused on other achievement test (e.g. reading achievement) and they did not involve all the nations in PISA database [4]. Therefore, it is feasible to investigate the relationship between nations background and students mathematics cognition by HLM.

With proper mathematical presentations, HLM can be specified in both hierarchical and combined forms [3]. If there are J $(j = 1,2,3,\cdots,J)$ nations from a population and n_j students within nation j, nations are units of level-2 and students are units of level-1. An example of hierarchical form for the two levels HLM is exemplified in equation (1), (2) and (3).

Level-1 (e.g., students)

$$Y_{ij} = \beta_{oj} + \beta_{1j} X_{ij} + r_{ij} \qquad (1)$$

Y_{ij} is a level-1 criterion variable (e.g. student mathematics cognition) and X_{ij} is a level-1 predictive variable (e.g. student mathematics confidence. β_{oj} and β_{1j} are level-1 coefficients and r_{ij} is a level-1 random effects. Besides, the centering of X_{ij} here is natural metric. In some cases, grand-mean centering or group-mean centering are chosen for X_{ij}.

Level-2 (e.g., nations)

$$\beta_{0j} = \gamma_{00} + \gamma_{01} W_j + u_{0j} \qquad (2)$$

$$\beta_{1j} = \gamma_{10} + \gamma_{11} W_j + u_{1j} \qquad (3)$$

W_j is a level-2 predictive variable (e.g. nation GCI). γ_{00}, γ_{01}, γ_{10} and γ_{11} are level-2 coefficients and they are also called fixed effects. u_{0j} and u_{1j} are level-2 random effects. The model in combined form will be as it is in equation (4).

$$Y_{ij} = \gamma_{00} + \gamma_{10} X_{ij} + \gamma_{01} W_j + \gamma_{11} X_{ij} W_j + u_{0j} + u_{1j} X_{ij} + r_{ij} \quad (4)$$

The following assumptions are required for the above combined model [7].

$$E(r_{ij}) = 0 \text{ and } Var(r_{ij}) = \sigma^2$$

$$E\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \text{ and } Var\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{bmatrix}$$

$$Cov(u_{0j}, r_{ij}) = Cov(u_{1j}, r_{ij}) = 0$$

σ^2 is the level-1 variance and τ_{00} , τ_{01} , τ_{10} and τ_{10} are level-2 variance-covariance components. The hierarchical form indicates that the level-1 regression coefficients (β_{oj} and β_{1j}) are random outcome variables at level-2. If u_{0j} and u_{1j} were null for every j , the combined model would be equivalent to an ordinary least squares regression model [3]. In this situation, HLM approach is not needed.

3 Method of Data Analysis

Those participating in PISA2003 are 15-year-olds students. The percentages of sample distribution based on area are shown in Table 1. The original data comes from the internet of OECD (http://www.pisa.oecd.org) and there are almost close to 280 thousand students.

Table 1. The Percentages of Sample Distribution based on Area

Area	Sample size	Percentages
Oceania	17062	6.2%
Europe	137998	50.0%
Asia	42705	15.5%
Africa	10556	3.8%
American	67844	24.6%
Total	276165	100.0%

The students are from 41 nations participating in the PISA2003. These nations are depicted in Table 2. All the nations in Table 2 are members of the OECD, except those marked with an asterisk (*).

Table 2. The Nations in PISA2003

Area	Nations	N
Oceania	Australia, New Zealand	2
Europe	Austria, Belgium, Denmark, Finland, France, Germany, Iceland, Hungary, Czech Republic, Italy, Ireland, Latvia*, Luxembourg, Norway, Netherlands, Greece, Poland, Portugal, Serbia and Montenegro*, Spain, Sweden, Switzerland, United Kingdom, Liechtenstein, Slovak Republic	25
Asia	Macao-China*, Indonesia*, Hong Kong-China*, Japan, Korea, Russian Federation*, Turkey, Thailand*	8
Africa	Tunis*	1
American	Brazil*, Canada, Mexico, United States, Uruguay*	5

The variables in level-1 are mathematics cognition and mathematics confidence. The mathematics cognition is the criterion variable and mathematics confidence is the predictive variable. The unit in level-2 is nations. It is assumed that the development of nations will influence the mathematics cognition of students. GCI (Growth Competitive Index) is the popular index applied to indicate nation development in this study. GCI in 2004 is applied and it is built by World Economic Forum (WEF). GCI is based on three broad categories of variables that measure technological advancement and the stages of development.

In this study, the unit in level-1 is the students and the unit in level-2 is nations. HLM 6 software is used to analyze the structural data [8].

4 Research Design and Results

Four submodels will be the exploratory models. These submodels, running from simpler to the more complex, include the one-way ANOVA model with random effects, means-as-outcome regression, random coefficients regression model and model with nonrandomly varying slopes [7].

4.1 One-way ANOVA model with random effects

The leve-1 and level-2 equations are as follows.

$$\text{Level-1:} \quad Y_{ij} = \beta_{oj} + r_{ij} \tag{5}$$

$$\text{Level-2:} \quad \beta_{oj} = \gamma_{00} + u_{0j} \tag{6}$$

Y_{ij} is the mathematics cognition. The result is depicted in Table 3. It shows that there is significant mean difference among nations. The interclass correlation coefficient is ρ = 8.14858 /(8.14858+70.38072)=10.38%. Therefore, the explained variance from nations is 10.38%. On the other hand, certain predictive variables may exist so that the difference of means for nations could be explained. Hence, the following sub-models, means-as-outcome regression, will be investigated further.

Table 3. The Analysis of One-way ANOVA Model with Random Effects

Fixed Effect	Coefficient	Standard Error	p-value		
For β_{oj}					
γ_{00}	12.646269	0.457297	<0.000		
Random Effect	**Standard Deviation**	**Variance Component**	**df**	**Chi-square**	**p-value**
u_{0j}	2.85457	8.14858	37	34991.29375	<0.000
r_{ij}	8.38932	70.38072			

4.2 Means-as-Outcomes Regression

As to the predictive variable GCI in level-2, the intercept is not significant and thus the leve-1 and level-2 equations are as follows.

$$\text{Level-1:} \quad Y_{ij} = \beta_{oj} + r_{ij} \tag{7}$$

$$\text{Level-2:} \quad \beta_{0j} = \gamma_{01} W_j + u_{0j} \tag{8}$$

Y_{ij} is the mathematics cognition of students and W_j is GCI of nations. The analysis result is depicted in Table 4. The coefficient is significant and it shows that predictive variable GCI can explain the mathematics cognition means of nations. Compared to Table 3, it shows that (8.14858-4.77381)/8.14858=41.42%. GCI can explain 41.42% of mathematics cognition means of nations.

Table 4. The Analysis of Means-as-Outcomes Regression (W_j is GCI)

Fixed Effect	Coefficient	Standard Error	p-value		
For β_{oj}					
γ_{01}	2.605925	0.066287	< 0.000		
Random Effect	**Standard Deviation**	**Variance Component**	**df**	**Chi-square**	**p-value**
u_{0j}	2.18490	4.77381	37	17950.03054	< 0.000
r_{ij}	8.38932	70.38072			

4.3 Random Coefficients Regression Model

Mathematics confidence is a factor which may influence mathematics cognition. Hence, mathematics confidence is a predictive variables in level-1. The equations are as follows.

$$\text{Level-1:} \quad Y_{ij} = \beta_{oj} + \beta_{1j} X_{ij} + r_{ij} \tag{9}$$

$$\text{Level-2:} \quad \beta_{0j} = \gamma_{00} + u_{0j} \tag{10}$$

$$\beta_{1j} = \gamma_{10} + u_{1j} \tag{11}$$

X_{ij} is mathematics confidence and analysis result is depicted in Table 5. All the coefficients in level-1 are statistically significant. Therefore, mathematics confidence will influence mathematics cognition. Mathematics confidence is a reverse recording in PISA 2003. That is, the lower the score is, the more confidence the students have. As shown in Table 5, γ_{10} is negative. The more confidence the students have, the higher mathematics cognition they will have. Mathematics confidence can explain 9.87% of mathematics cognition because of (70.38072-63.43663)/70.38072 = 9.87%.

Table 5. The Analysis of Random Coefficients Regression Model

Fixed Effect	Coefficient	Standard Error	p-value
For β_{oj}			
γ_{00}	22.8588994	0.704192	<0.000
For β_{1j}			
γ_{10}	-0.619077	0.021042	<0.000

Random Effect	Standard Deviation	Variance Component	df	Chi-square	p-value
u_{0j}	4.37946	19.17964	37	5177.62957	<0.000
u_{1j}	0.12913	0.01668	37	1435.04263	<0.000
r_{ij}	7.96471	63.43663			

4.4 Model with Nonrandomly Varying Slopes

Since mathematics confidence can influence mathematics cognition in the random coefficients regression model, GCI of nations in level-2 is considered to be predictors for advanced model. It is found that GCI can effectively explain the varying slopes in level-1.

As to the predictive variable GCI in level-2, the best leve-1 and level-2 equations are as follows.

$$\text{Level-1:} Y_{ij} = \beta_{oj} + \beta_{1j} X_{ij} + r_{ij} \quad (12)$$

$$\text{Level-2:} \ \beta_{0j} = \gamma_{01} W_j + u_{0j} \quad (13)$$

$$\beta_{1j} = \gamma_{11} W_j + u_{1j} \quad (14)$$

X_{ij} is mathematics confidence and W_j is GCI. The analysis result is depicted in Table 6. The coefficient γ_{01} shows that GCI provides positive effects on mathematics cognition. The coefficient γ_{11} also shows that the influence of mathematics confidence on mathematics cognition varies with the growth competitive of nations. Moreover, as the growth competitive of nations increases, the influence of mathematics confidence on mathematics cognition will reduce. Furthermore, the GCI can not explain all the variance of varying slopes across nations because both u_{0j} and u_{1j} are statistically significant.

Table 6. Model with Nonrandomly Varying Slopes (W_j is GCI)

Fixed Effect	Coefficient	Standard Error	p-value
For β_{oj}			
γ_{01}	4.688101	0.110045	<0.000
For β_{1j}			
γ_{11}	-0.126405	0.003838	<0.000

Random Effect	Standard Deviation	Variance Component	df	Chi-square	p-value
u_{0j}	3.63979	13.24810	37	3347.43783	<0.000
u_{1j}	0.12573	0.01581	37	1173.47956	<0.000
r_{ij}	7.96471	63.43663			

5 Conclusions

Based on the findings of this research, it shows that mathematics cognition among nations is different. GCI is an effective predictor of mathematics cognition. In addition, the more mathematics confidence the students have, the higher mathematics cognition they have.

The exploratory study of HLM analysis of PISA2003 database demonstrates basic information about international assessment in this study. The step-by-step procedures for data analysis are followed so that the structural relationship of students and nations can be clarified [2]. Although there is only one index of nations applied in this study, the level-2 variables of nations which may carry varying influence on level-1 variables are not completely clear. Advanced research could aim at the findings of level-2 variables. On the other hand, three levels of HLM for PISA2003 and PISA2006 is also a prospective study.

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