A Computational Model of Magnetic Fluid Flow using Maxwell's Equations, Heat Equations and Navier-Stokes Equations

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Abstract: - The magnetic liquids such as ferrofluids, biocompatible magnetic nanocarrier, play an important role as drug carriers in the human body. Based on Maxwell's, Heat and Navier-Stokes equations, a time-dependent analysis of a two- dimensional computational model of magnetic fluid flow is purposed. The FEM is applied to solve Maxwell's equation for a static magnetic field , Navier-Stokes equations for fluid dynamics and heat equation for temperature. The result of magnetic field has some influences to ferrofluid dynamic flow in blood vessel.

Key-Words: - Finite Element Method, Maxwell's Equation, Heat Equation, Navier-Stokes Equation

1 Introduction

In the recent years, the area of Finite Element Method (FEM) has been widely interest in biomedical engineering. Some researchers use FEM as a tool to design polymer-coated targeted drug delivery systems [1]. Based on Laplace-Beltrami operator, the geometry model of human brain is analyzed [2]. In drug delivery system, an integrated system use superparamagnetic, biocompatible nano drug carriers for targeted but non-invasive delivery [3]. Additionally, drug deliver is based on nanotechnology using magnetic guidance of drug loaded magnetic carriers to the targeted site and thereafter released by external ultrasound energy [4]. There are many alternative methods for the treatment of cancer or tumor in human body such as tumor ablation [5] and chemotherapy [6]. To avoid side effect on healthy human cells, which are made by chemotherapy, physician always face with an upper limit in the treatment dose. This limit obstructs the chance of successful treatment of the tumor cells. The objective of modern cancer researches is to focus chemotherapy drugs locally to tumor tissue and to stop the global exposure to the organism. This computational model investigates an external magnetic field that interacts with blood flow with ferrofluids. The magnetic field is described by Maxwell's equation and the dynamic flow is illustrated by Navier-Stoke equations.

In this paper, the velocity field of magnetic fluid and conservation of energy equation in heat transfer is purposed. Based on Partial Differential Equation (PDE), the development of this algorithm comprises with equation of conservation of mass in fluid dynamic, Navier-Stokes equations, and Maxwell's equations. These nonlinear equations are analyzed by FEM. The simulated system is numerically computed to investigate the effect of magnetic potential with magnetic fluid flow and temperature distribution in time dependent model.

2 Navier-Stokes Equations

A dynamic equation describing fluid motion [7] is a momentum equation that can be solved to incompressible Newtonian flow. The most famous equations in fluid dynamics are Navier-Stokes equations. A set of equations composes of three velocity equations in the rectangular coordinates (1-3) and the continuity in an equation (4).

$$\rho\left[\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}\right]=\rho g_x-\frac{\partial p}{\partial x}+\eta\left[\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right] \quad (1)$$

$$\rho\left[\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}\right]=\rho g_y-\frac{\partial p}{\partial y}+\eta\left[\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right] \quad (2)$$

$$\rho\left[\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}\right]=\rho g_z-\frac{\partial p}{\partial z}+\eta\left[\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right] \quad (3)$$

$$\frac{\partial \rho u}{\partial x}+\frac{\partial \rho v}{\partial y}+\frac{\partial \rho w}{\partial z}=\nabla\bullet\rho\,\mathbf{u}=0 \quad (4)$$

$$\rho\frac{\partial \mathbf{u}}{\partial t}=\mathbf{F}+\eta\nabla^2\mathbf{u}-\rho(\mathbf{u}\cdot\nabla)\mathbf{u}-\nabla p \quad (5)$$

The equations are nonlinear partial differential equations where u, v, w, are velocity in x, y, z direction respectively. ρ is density of material. g_x, g_y, g_z are gravitational force field in x, y, z as well. t is time. To observe fluid motion in blood vessel model, Navier-Stokes equation can be resolved and reduced to time dependent as shown in equation (5). Equation (5) is used to compute a fluid flow direction in the model assumed that the fluid is an incompressible flow. The significant parameter, $\mathbf{F}=(F_x, F_y)$, of this section is a volume force field. The relation between **F** and Maxwell's equation is described in equation 9 and 10.

3 Maxwell's Equations

The magnetic vector potential generated by magnet is calculated. The magnetic volume force created by magnetic field is affected the flow field in the blood vessel as shown in equation (6). A magnetic potential vector is A and the magnetization vector is M. Equation 7 and 8 [9] describe magnetization in x and y direction that is used in equation 9 and 10 [10]. To simulate blood flow in model, the x direction, v, follows a sinusoidal expression in time as illustrated in equation 11.

$$\nabla \times \left(\frac{1}{\mu_0 \mu_r} \nabla \times \mathbf{A}\text{-}\mathbf{M}\right) = 0 \quad (6)$$

$$M_x = \alpha \tan^{-1}\left(\frac{\beta}{\mu_r \mu_0} \frac{\partial A_Z}{\partial y}\right) \quad (7)$$

$$M_y = \alpha \tan^{-1}\left(\frac{\beta}{\mu_r \mu_0} \frac{\partial A_Z}{\partial x}\right) \quad (8)$$

$$F_x = \frac{M_x}{\mu_r} \frac{\partial^2 A_Z}{\partial x \partial y} \quad (9)$$

$$F_y = -\frac{M_y}{\mu_r} \frac{\partial^2 A_Z}{\partial y \partial x} \quad (10)$$

$$v = \sin(\omega t) + \sqrt{\sin^2(\omega t)} \quad (11)$$

The magnetic field, magnetic vector potential, generated by magnet is calculated. The computed magnetic field generates a magnetic volume force that affects the flow field in the blood vessel.

Table 1 : Physical Constants [7], [8]

Fluid Physical Constants	Value
Density (ρ)	1,000 kg/m^3
Dynamic Viscosity (η)	5.0×10^{-3} Ns/m^2
Heat Capacity (C)	4,180 Jkg/°C
Thermal Conductivity (W/m°C)	0.492

4 Heat Equations

Based on a conservation of energy, heat transfer analysis is defined as the movement of energy because of temperature difference. The crucial characteristic of heat transfer is divided by the following three mechanisms. Firstly, conduction is heat transfer by diffusion in a stationary medium due to a temperature gradient such as a solid or a liquid. Secondly, convection occurs in moving liquids and gases [11]. Thirdly, radiation is heat transfer between two surfaces via electromagnetic wave. In this paper, two properties, convection and conduction, are described in equation (12) where T is temperature and Q is heat source. The property of material is defined as density, ρ, and thermal conductivity (k).

$$-\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right] - \rho C\left[u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + w\frac{\partial T}{\partial z}\right] + Q = \rho C\frac{\partial T}{\partial t} \quad (12)$$

$$\begin{Bmatrix} q_x \\ q_y \\ q_z \end{Bmatrix} = -\begin{bmatrix} k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & k \end{bmatrix} \begin{bmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{bmatrix} \quad (13)$$

In this paper, an isotropic thermal conductivity is employed as shown in equation (13). For isotropic thermal conductivity, a diagonal of matrix is assumed in same value. As the result, the time dependent version of heat transfer can be written in equation (14) where C_p is heat capacity and t is time.

$$\rho C_p \frac{\partial \boldsymbol{T}}{\partial t} + \nabla \cdot (-k\nabla \boldsymbol{T} + \rho C_p T\mathbf{u}) = Q \quad (14)$$

The PDE equation (5), (6) and (14) must be calculated at the same time. Depending on fluid dynamics, convective temperature $\boldsymbol{T}$ can be distributed by $\mathbf{u}$.

5 Physical Constants

The important physical constants are shown in Table 1 for Navier-Stokes and Heat equation and Table 2 for Maxwell's equations.

Table 2 : Magnetic Constants

Magnetic Constants	Value
Ferrofluid parameter (α)	1×10^{-4}
Ferrofluid parameter (β)	3×10^{-5}
Magnet magnetization	0.5e-5 A/m
Magnet relative permeability	5e3
Tissue relative permeability	0.9998

6 Finite Element Method

Geometry Model and Assumption

To observe magnetic potential and fluid velocity field, the model of blood vessel is created in 3D. The model consists of blood vessel, tissue and magnetic device as shown in Figure 1. The 37 °C inflow of fluid comes from left side and outflow is to the right side. The magnetic device is placed on the tissue to distribute magnetic potential interacting with ferrofluids carrier in blood vessel. To reduce calculating time, 2D model is created in Figure 2 and magnetic device is assumed to be permanent magnet. The 2D geometry model is approximated by many triangles called a mesh model as shown in Figure 3. A model represents the fluid velocity field computed by mesh model. Because the Navier-Stokes equations are computationally arduous, it is considerable to utilize an appropriate mesh. If the mesh is too rough, the solution might not coverage at all or errors might be large. Contrastly, if the mesh is too fine, the solution time for the nonlinear system of equations might be unnecessarily long.

Boundary Condition, Initial Condition

Boundary conditions are most described by considering the magnetostatic, heat transfer and fluid dynamics shown in table 3-5. For time dependent analysis, a set of initial conditions is necessary for the dependent variable. The calculation process comprise with two steps. Firstly, magnetic potential is computed. Secondly, the result of magnetic potential is used as initial condition to calculate fluid velocity field. The wall applies no-slip condition. The out flow is set to pressure condition, p = 0. Finally, temperature is carried by fluid flow. The initial condition of blood vessel is set to 25°C.

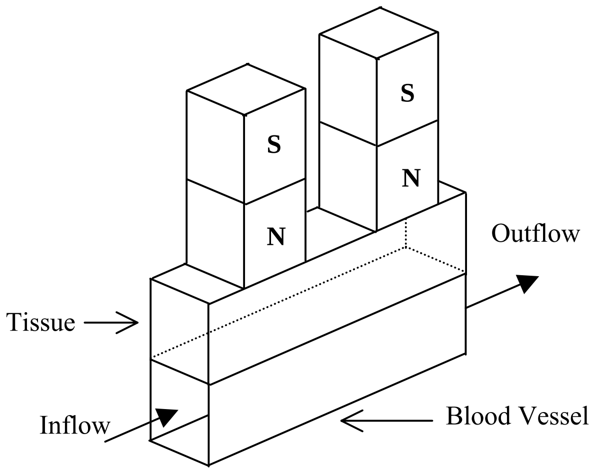

Figure 1 3D geometry of blood vessel

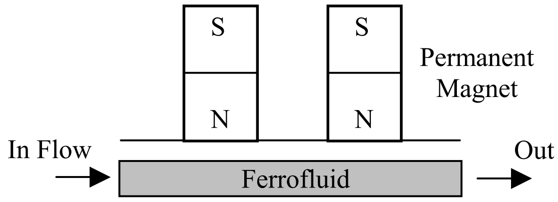

Figure 2 2D geometry of blood vessel

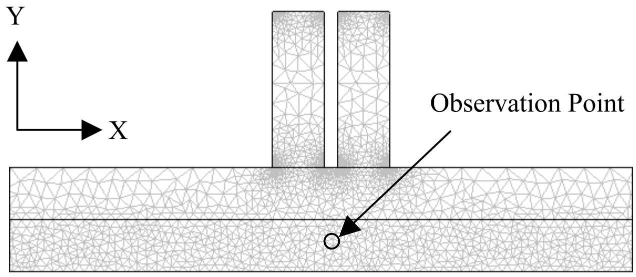

Figure 3 Mesh Model

The mesh model is computed with Pentium® IV 3.2 GHz, DDR RAM 1.5 MB. The calculation time of time-dependent is around 2.5 hours.

Table 3: Boundary condition of magnetostatic

Magnetostatic Equations	Boundary Condition
Magnetic Insulation	$\mathbf{A_Z} = 0$
Constitutive Relation	$\mathbf{B_0}=\mu_0\mu_r\mathbf{H}+\mathbf{B}_r$
Relative Permeability	Isotropic

Table 4: Boundary condition of fluid dynamics

Fluid Dynamics	Boundary Condition
Inflow	**u** = Heart Beat Equation
Outflow	P_0=0
Wall	**u**=0

Table 5 : Boundary condition of heat transfer

Heat Transfer	Boundary Condition
Inflow	Temperature 37°C
Outflow	Convective Flux
Chamber	Thermal Insulation

7 Experimental Results

Based on the FEM given in previous section, the observation point is created as shown in Figure 3. That point places on center of blood vessel model. The fluid velocity at observation point is shown in Figure 6 and 7.

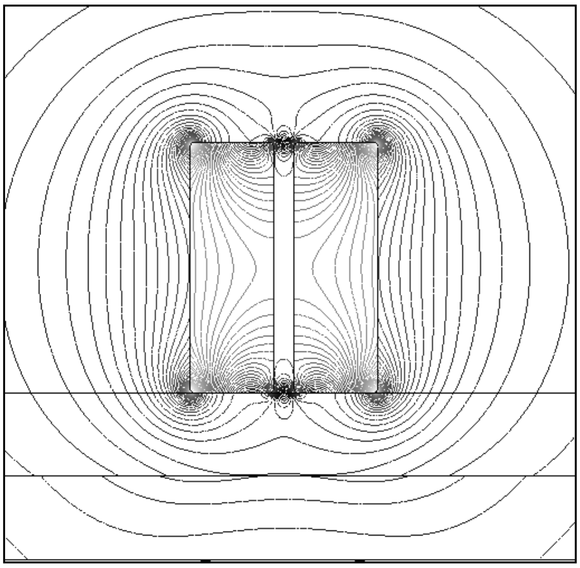

Figure 4 Magnetic flux density

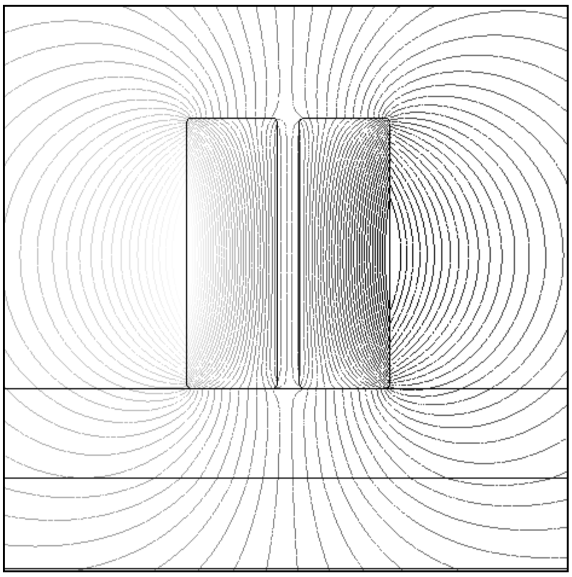

Figure 5 Magnetic potential

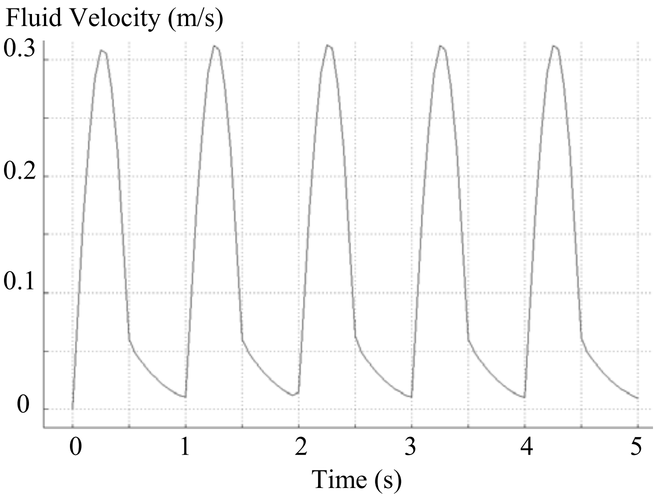

Figure 6 Fluid velocity without magnetic flux density

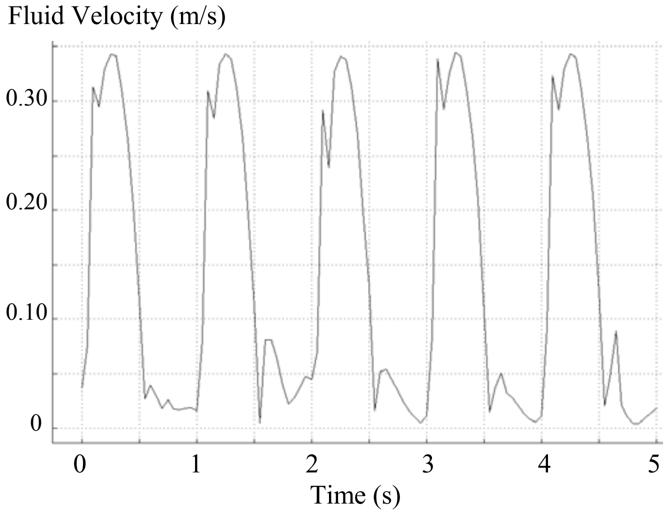

Figure 7 Fluid velocity with magnetic flux density

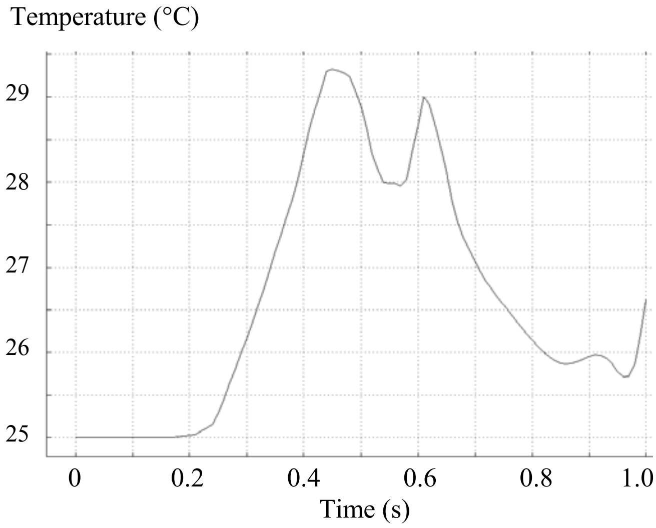

Figure 8 Temperature at observation point

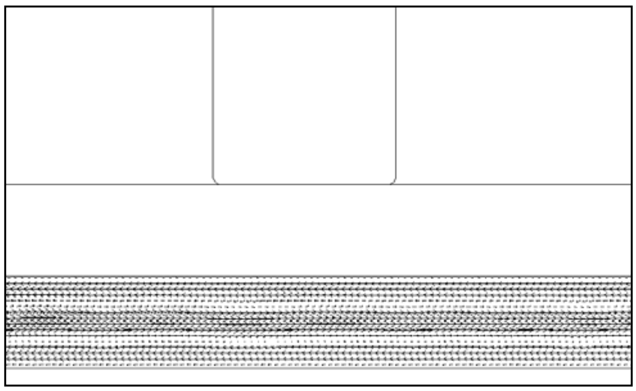

Figure 9 Fluid velocity field without magnetic flux density

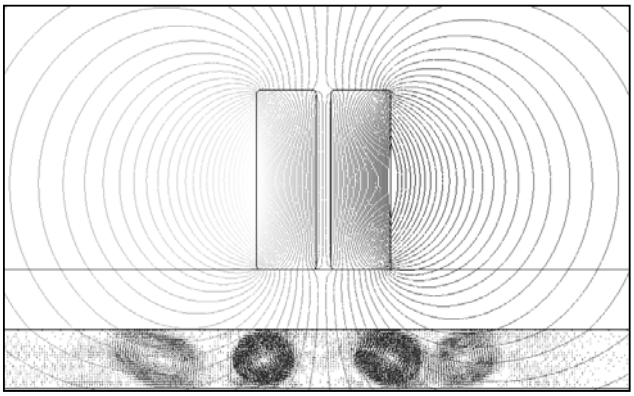

Figure 10 Fluid velocity field with magnetic potential

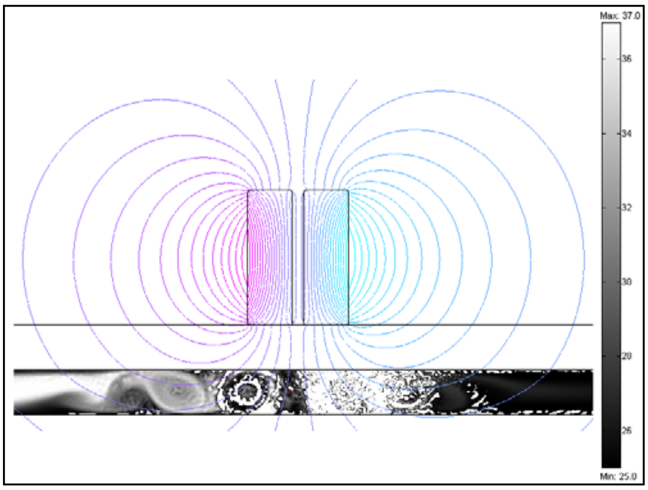

Figure 11 Temperature with Fluid velocity field and magnetic potential

The experimentation is divided in two important parts. Firstly, only magnetic potential is computed with time independent, stationary state. The magnetic flux density is shown in Figure 4 and magnetic potential is shown in Figure 5. Secondly, the results of first step are applied to calculate velocity field by Navier-Stokes equations and heat transfer in time dependent model. In Figure 6, only fluid field velocity is computed without external magnetic field. Because inflow boundary use heart beat equation, fluid velocity field is nearly sinusoidal wave at observation point. The peak velocity of fluid velocity is around 0.31 m/s and not turbulent flow as shown in Figure 9. Then, the magnetic potential field is applied to monitor fluid velocity field. The results in Figure 7 show that peak velocity field increases from 0.31 to 0.34 m/s. The fluid velocity field after applying magnetic potential field is eddy flow. The area of eddy flow is focused only in the area that magnet is place as shown in Figure 10. The result of temperature at observation point is shown in Figure 8. The temperature distribution is shown in Figure 11.

8 Conclusion

In this model, three famous types of PDE equation are coupled together to approximate ferro-hydro-dynamic behavior. A computational algorithm of FEM is applied to compute fluid velocity field with interacted magnetic field. To reduce computational time, 2D model of blood vessel is created. The heart beat equation is created at inlet boundary. Pressure condition boundary is also used at the outlet. The experimental results show the difference of fluid field velocity and fluid flow pattern when apply magnetic potential field. The fluid velocity field is eddied flow from magnetic potential field. This method shows the potential to target chemotherapy drug in the human body locally. The future work will increase the complexity that use 3D geometry model that require more calculation time.

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