Antimonotonicity in Chua's Canonical Circuit

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Abstract: Antimonotonicity, i.e. forward period-doubling bifurcation sequences followed by reverse period-doubling sequences, as a parameter is varied in a monotone way, is observed for the first time in Chua's canonical circuit, with a nonlinear resistor, which has a symmetric piecewise linear (PWL) υ characteristic of type – N. Antimonotonicity is observed, when the well-known double scroll attractor of Chua's circuit family can not be created. Dynamics of the circuit, in the parameter space where the reverse period – doubling cascades are observed, is very sensitive to initial conditions, as chaotic attractors coexist with period-1 limit cycles.

Key-Words: Chua's canonical circuit, chaos, antimonotonicity, bubbles, crisis-induced intermittency, coexisting attractors.

1 Introduction

Electric circuits with a nonlinear resistor, which is characterized by a piecewise linear (PWL) υ characteristic, have emerged as a simple, yet−i powerful experimental and analytical tool in studying chaotic behavior in nonlinear dynamics. Among the piecewise linear circuits that have been studied, the members of the Chua's circuit family [1], particularly, have been investigated in depth. Each member of this family consists of linear resistors, three linear dynamic elements (capacitors and/or inductors), and a nonlinear resistor characterized by a piecewise linear (PWL), odd symmetric, υ characteristic with at least one segment having a negative slope [2].

Chua's circuit family is a paradigm for chaos [3]. Spiral, double scroll, torus, and other interesting attractors among other dynamical phenomena have been observed at different members of this family by computer simulation and by experiment [4-7]. Among the members of Chua's circuit family, the autonomous canonical Chua's circuit introduced by Chua and Lin [8] is of considerable importance. This is because it is capable of realizing the behavior of every member of the Chua's circuit family [9,10]. It consists of two active elements, one linear negative conductor, and one nonlinear resistor with odd-symmetric piecewise linear υ characteristic of type – N.

For a piecewise-linear nonlinearity, very extensive literature exists on theoretical, numerical and experimental aspects of the dynamics of the members of Chua's circuit family. The reasons for the previous choice of a piecewise-linear nonlinearity were the following:

- The corresponding circuits can be easily built with off-the-shelf components.
- Explicit Poincaré map can be derived which allows a *rigorous* mathematical proof, that Chua's circuit is chaotic in the sense of Shil'nikov's theorem.

Cascades of period-doubling bifurcations have long been recognized to be one of the most common routes to chaos, as exemplified e.g. by the one-dimensional (1–D) logistic map

$$x_{n+1} = \lambda x_n \left(1 - x_n\right) \qquad (1)$$

As the parameter λ in such a map is increased, it is known that periodic orbits are only created but never destroyed. Unlike the monotone bifurcation behavior of the logistic map, however it has been shown that, in many common nonlinear dynamical systems, forward period-doubling bifurcation sequences are followed by reverse period-doubling sequences, as a parameter is varied in a monotone way. Dawson et al., [11], named this type of creation and annihilation of periodic orbits *antimonotonicity*.

Reversals of period-doubling cascades have been observed in various nonlinear physical systems both numerically and experimentally. In one of the first studies of this phenomenon [12], the occurrence of such reverse sequences was connected to the dynamics of a cubic 1–D map. As examples of numerical simulations, we cite the van der Pol equation [13], Duffing's oscillator [14], a RC-ladder chaos generator [15], an autonomous 4^{th}-order nonlinear electric circuit [16], and on Chua's canonical circuit, with a cubic υ–i characteristic [17]. Experimental manifestations of antimonotonicity have also been observed on the driven R,L,p-n junction nonlinear circuit [18-20], on Chua's circuit with an asymmetric υ–i characteristic [21]. In this paper, we have studied the dynamics of Chua's canonical circuit [8,22,23] focusing on the phenomenon of antimonotonicity, which has never observed in the members of Chua's circuit family with a piecewise linear odd symmetric υ–i characteristic until now.

2 The Canonical Chua's Circuit

Chua's canonical circuit is a nonlinear autonomous 3^{rd}-order electric circuit (Fig.1). The nonlinear element is a nonlinear resistor, N_R, with an odd symmetric $\upsilon - i$ characteristic of type-N (Fig.2), while G_n is a linear negative conductance.

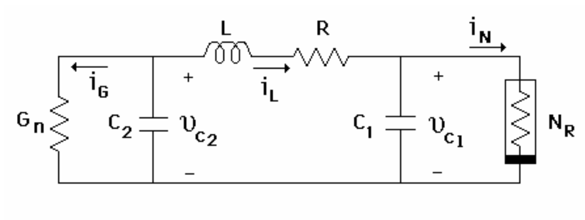

Fig.1. Chua's canonical circuit

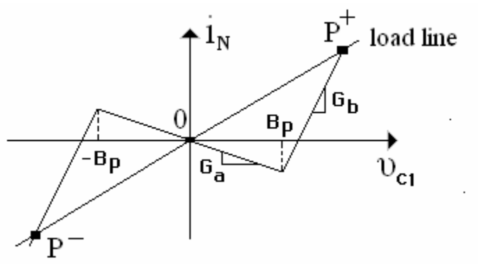

Fig.2. The $\upsilon - i$ characteristic of the nonlinear resistor, and the equilibrium points.

The state equations of the circuit are the following:

$$\frac{d\upsilon_{C1}}{dt} = \frac{1}{C_1}\left(i_L - i_N\right) \tag{2}$$

$$\frac{d\upsilon_{C2}}{dt} = -\frac{1}{C_2}\left(i_L + G_n\upsilon_{C2}\right) \tag{3}$$

$$\frac{di_L}{dt} = \frac{1}{L}\left(-\upsilon_{C1} + \upsilon_{C2} - Ri_L\right) \tag{4}$$

The current i_N through the nonlinear resistor is

$$\begin{aligned} i_N = g\left(\upsilon_{C1}\right) &= G_b\upsilon_{C1} + \\ &+0.5\left(G_a - G_b\right)\left\{\left|\upsilon_{C1} + B_p\right| - \left|\upsilon_{C1} - B_p\right|\right\} \end{aligned} \tag{5}$$

3 Dynamics of the Circuit

Chua's circuit main result is the well-known double scroll attractor [1-5]. In all published papers on Chua's circuits the double scroll attractor is always present (Fig.3). The system follows the period doubling route to chaos and the spiral chaotic attractor is created (Fig.4). The transition from the spiral to the double scroll attractor, as C_2 is further decreased, follows the crisis-induced intermittency scenario, [23]. Finally, the double scroll chaotic attractor transits to a period-1 limit cycle by intermittency, [24]. Reverse period doubling sequences have not been observed for these values of the circuit elements.

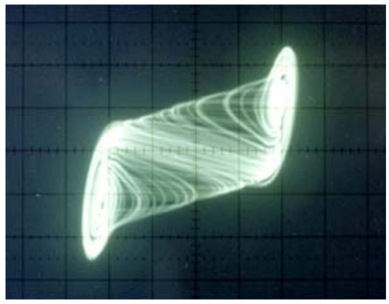

Fig.3. The double scroll attractor [23].

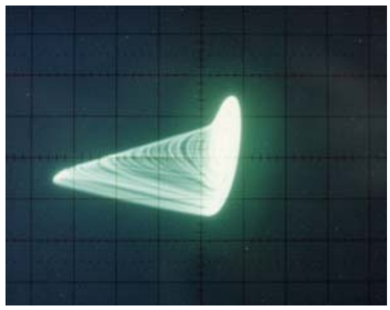

Fig.4. The spiral chaotic attractor [23].

In the present paper, we have chosen the following values for the circuit parameters: $L = 100$ mH, $R = 330\ \Omega$, and $G_n = -0.40$ mS, while $B_p = 1.0$ V, $G_a = -0.30$ mS, and $G_b = 4.0$ mS. Giving constant values to capacitance C_1, we have plotted the bifurcation diagrams υ_{C1} vs. C_2. The comparative study of the bifurcation diagrams gives the qualitative changes of the dynamics of the system, as C_1 takes different discrete values.

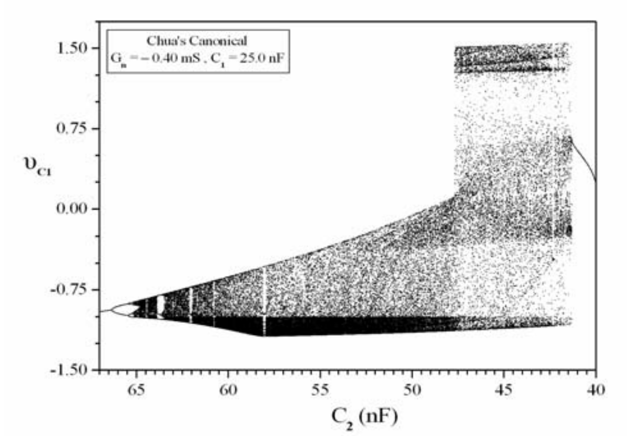

Fig.5. The bifurcation diagram, υ_{C1} vs. C_2 for $C_1 = 25.0$ nF.

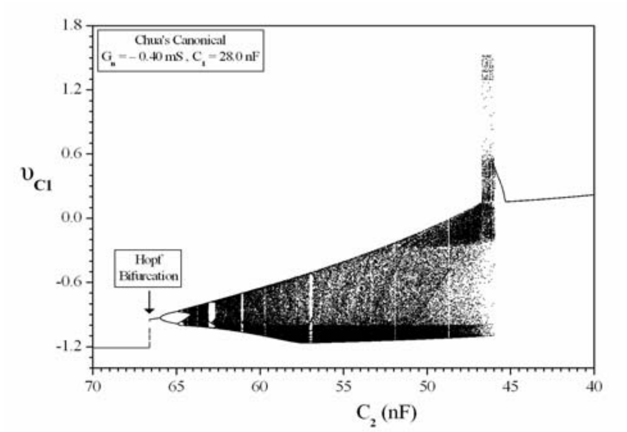

Fig.6. The bifurcation diagram, υ_{C1} vs. C_2 for $C_1 = 28.0$ nF.

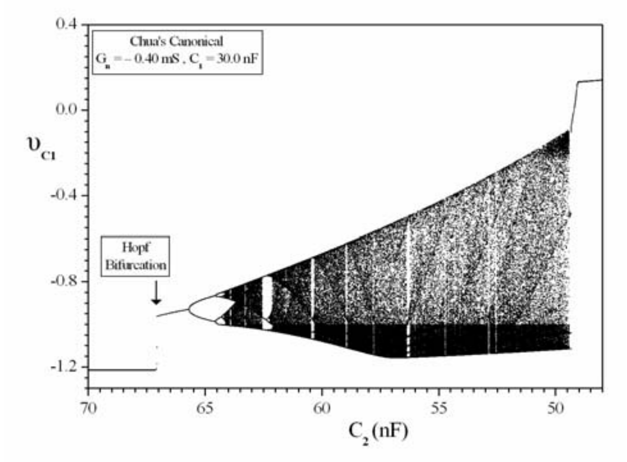

Fig.7. The bifurcation diagram, υ_{C1} vs. C_2 for $C_1 = 30.0$ nF.

The bifurcation diagram for $C_1 = 25.0$ nF is shown in Fig.5. For $C_2 < 47.7$ nF, we can observe an expansion across the υ_{C1} – axis, which corresponds to the creation of the chaotic double scroll attractor. Double scroll attractors are created for $C_1 \leq 28.0$ nF, as we can observe in the bifurcation diagrams of Figs.6 and 7. The system follows the period-doubling route to chaos, after a Hopf bifurcation.

3.1 Period – N bubbles

In many common nonlinear dynamical systems, forward period-doubling bifurcation sequences are followed by reverse period-doubling sequences, as a parameter is varied in a monotone way, a phenomenon called antimonotonicity. The general form of the bifurcation diagram in the case of antimonotonicity, is shown in Fig.8. The system, starting from a period–N state, following the period doubling route, enters the chaotic state. The system, as the control parameter "b" varies in a monotone way, leaves the chaotic regime via a reverse period doubling cascade ending in period–N state. This configuration is known as "period–N chaotic bubble".

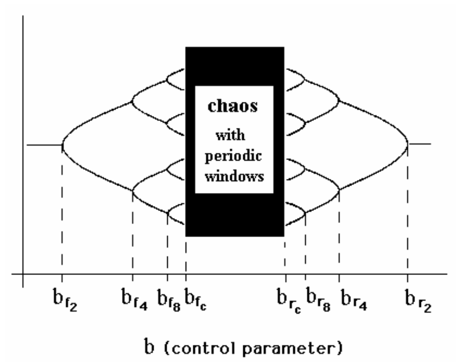

Fig.8. The general scheme of a period – 1 chaotic bubble.

Antimonotonicity is observed for $C_1 > 50.0$ nF. The bifurcation diagrams, υ_{C1} vs. C_2, for $C_1 = 51.0$ nF, $C_1 = 55.0$ nF, and $C_1 = 57.0$ nF are shown in Figs.9-11 respectively, and they follow the general scheme of the period – 1 chaotic bubble.

In the bifurcation diagrams of Figs.12-14, the circuit always remains in a periodic state, as C_2 is decreased. For $C_1 = 63.0$ nF, Fig.14, the bifurcation diagram follows the scheme: period–1

→ period–2 → period–1 (or p–1 → p–2 → p–1). Bier and Bountis, [12], named this scheme "primary bubble".

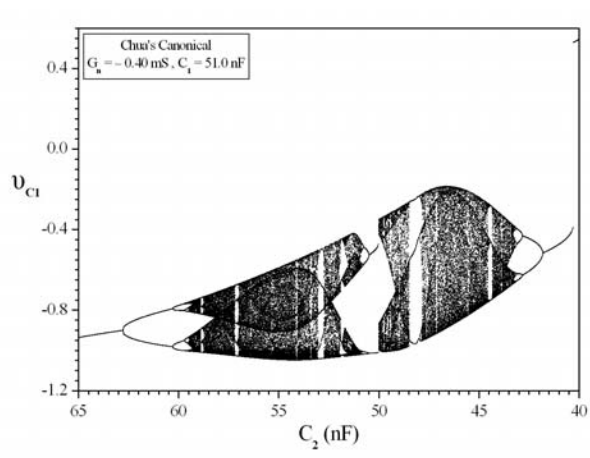

Fig.9. The bifurcation diagram, υ_{C1} vs. C_2, for $C_1 = 51.0$ nF.

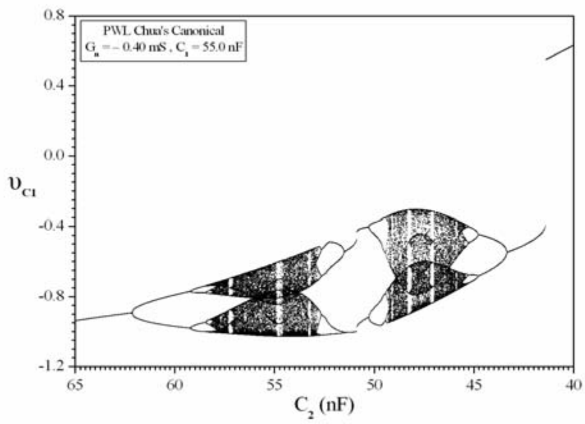

Fig.10. The bifurcation diagram, υ_{C1} vs. C_2, for $C_1 = 55.0$ nF.

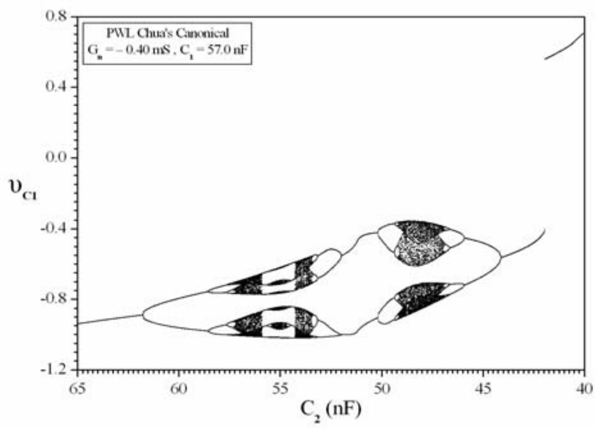

Fig.11. The bifurcation diagram, υ_{C1} vs. C_2, for $C_1 = 57.0$ nF.

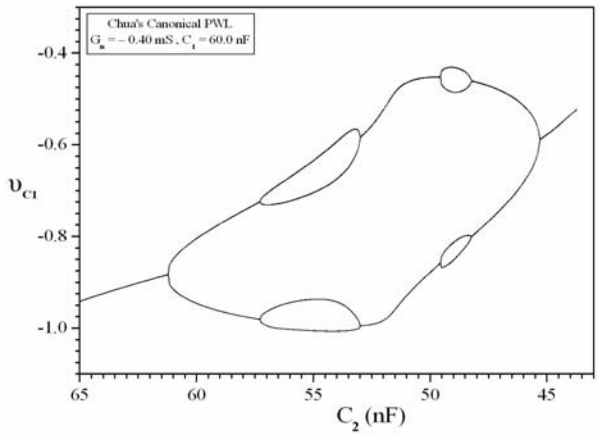

Fig.12. The bifurcation diagram υ_{C1} vs. C_2, for $C_1 = 60.0$ nF.

The bifurcation diagram υ_{C1} vs. C_2, for $C_1 = 61.0$ nF is shown in Fig.13. The system remains again in a periodic state, but a period–4 state has now been formed.

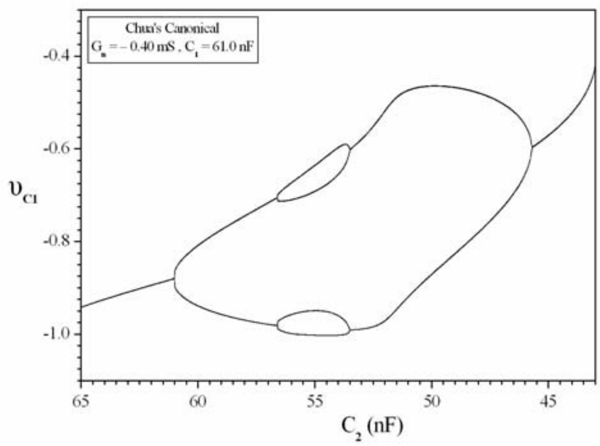

Fig.13. The bifurcation diagram υ_{C1} vs. C_2, for $C_1 = 61.0$ nF.

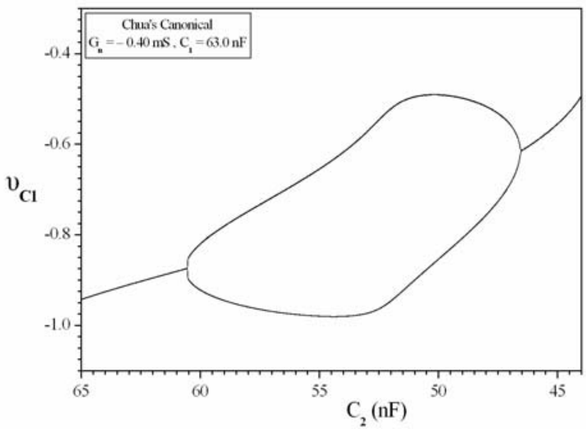

Fig.14. The bifurcation diagram υ_{C1} vs. C_2, for $C_1 = 63.0$ nF.

4 Discussion and Conclusions

Bier and Bountis, [12], demonstrated that reverse period doubling sequences are expected to occur, when a minimum number of conditions is fulfilled. Their main result was, that a reverse period doubling sequence is likely to occur in any nonlinear system, where there is a symmetry transformation, under which the state equation remains invariant.

Indeed our system of differential equations (2-4) under the transformation

$$\upsilon_{C1} \to -\upsilon_{C1} \ , \ \upsilon_{C2} \to -\upsilon_{C2} \ , \ i_L \to -i_L \qquad (6)$$

remains invariant. In addition, it has also been demonstrated in the literature, [12,25], that reverse period doubling commonly arises in nonlinear dynamical systems involving the variation of two parameters. These parameters in our circuit are the two capacitances C_1 and C_2. It is important, however, that the period doubling "trees" develop symmetrically *towards* each other along some line in parameter space. This would allow them to terminate, by joining their "branches" to form "bubbles", thus exhibiting the phenomenon of antimonotonicity.

We have to notice, that antimonotonicity is present, when the well-known "double-scroll" Chua's attractor is *absent*. This is an explanation, that antimonotonicity has not been observed in Chua's circuit until now.

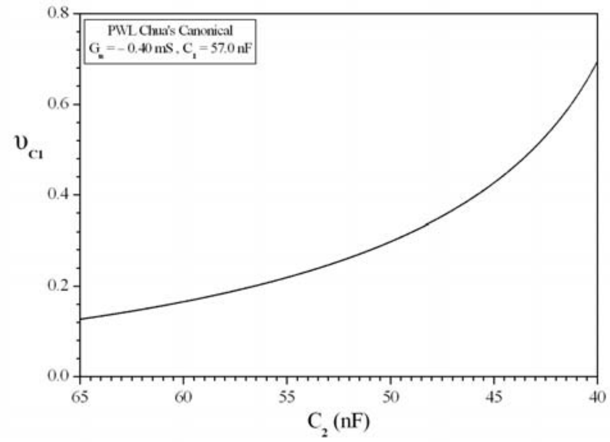

Fig.15. The bifurcation diagram υ_{C1} vs. C_2, for $C_1 = 57.0$ nF, and initial conditions $(\upsilon_{C1})_0 = 0.12$V, $(\upsilon_{C2})_0 = 0.14$V, and $(i_L)_0 = 1.2$mA.

The creation of bubbles is also very sensitive to initial conditions. The spiral attractors coexist with limit cycles of period-1, so the circuit can be driven to two quite different states, depending on the initial condition. This becomes obvious by comparing the bifurcation diagrams of Figs.11 and 15, which correspond to the same values of circuit parameters but different initial conditions.

As an example, in Fig.16, the chaotic spiral attractor is shown for $C_1 = 57.0$ nF and $C_2 = 48.0$ nF and initial conditions $(\upsilon_{C1})_0 = -0.82$V, $(\upsilon_{C2})_0 = -0.95$V, and $(i_L)_0 = 0.5$mA, while in Fig.17, a period-1 limit cycle is shown for $C_1 = 57.0$ nF and $C_2 = 48.0$ nF and initial conditions $(\upsilon_{C1})_0 = 0.12$V, $(\upsilon_{C2})_0 = 0.14$V, and $(i_L)_0 = 1.2$mA.

Coexisting attractors play an important role in dynamics of identical coupled nonlinear systems, especially in the case of synchronization.

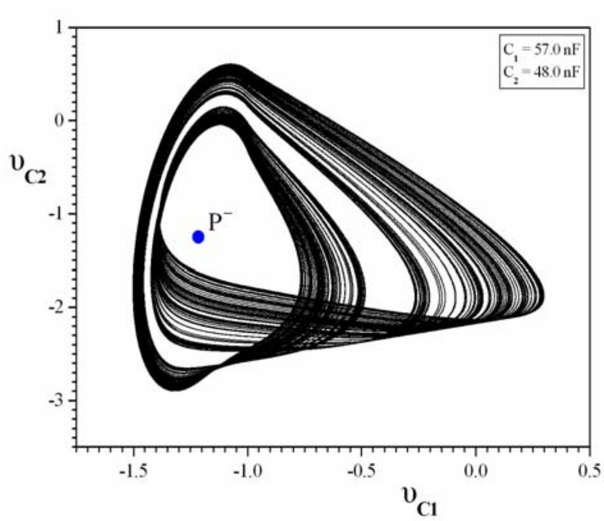

Fig.16. Chaotic spiral attractor for $C_1 = 57.0$ nF, $C_2 = 48.0$ nF and initial conditions $(\upsilon_{C1})_0 = -0.82$V, $(\upsilon_{C2})_0 = -0.95$V, and $(i_L)_0 = 0.5$mA

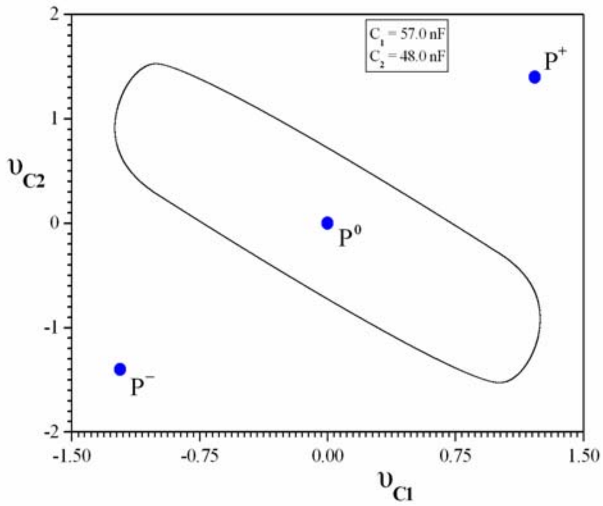

Fig.17. Period-1 limit cycle for $C_1 = 57.0$ nF, $C_2 = 48.0$ nF, and initial conditions $(\upsilon_{C1})_0 = 0.12$V, $(\upsilon_{C2})_0 = 0.14$V, and $(i_L)_0 = 1.2$mA.

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