Wavelets Packets Analysis for Doppler Effect Detection

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Abstract - In this article a method for the parameter estimation of Doppler radar signals is proposed. It is based on the discrete wavelet transform, spline orthogonal wavelets are used from the projection of the mixed signal; wavelet packets analysis play the role of bandpass filter banks with narrowing frequencies ranges. An experimental case is analyzed and the results obtained are discussed.

Key-Words: - Wavelets packets, multiresolution, Doppler radar signals.

1 Introduction

Assume that $\psi(\tau)$ is an orthogonal wavelet with Fourier transform

$$\hat{\psi}(f) = |\hat{\psi}(f)| e^{-i\pi f}; \quad (1)$$

where $\hat{\psi}$ is a bandpass filter on $1/2 \le |f| \le 1$.

In this article we use the orthogonal *Lemarié-Battle* cubic spline (OCS) wavelet ψ ([1],[8]) depicted in Fig. 1.

Figure 2 shows the module of the frequency response $\mathcal{F}(\psi)(f) \equiv \hat{\psi}(f)$ corresponding to the OCS wavelet for $f > 0$.

We remark that $|\hat{\psi}|$ is almost constant on the band; that is, it is not localized at any frequency. For these reasons the wavelet transform give us a time-scale representation of the signal rather than a time-frequency one.

To overcome this disadvantage we can use wavelet packets ([1],[2],[3],[6]), as waveforms well localized both in the time and the frequency domains. They become from lineal combination of finite wavelets over common intervals and scales.

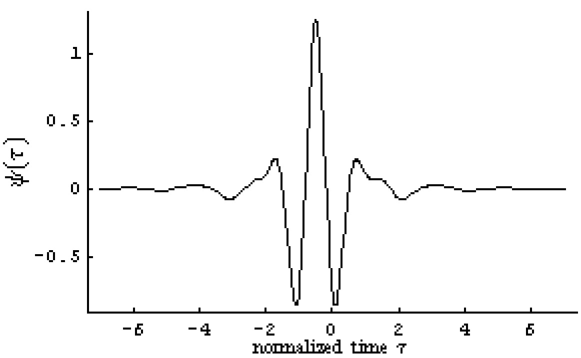

Fig. 1: Orthogonal cubic spline (OCS) wavelet.

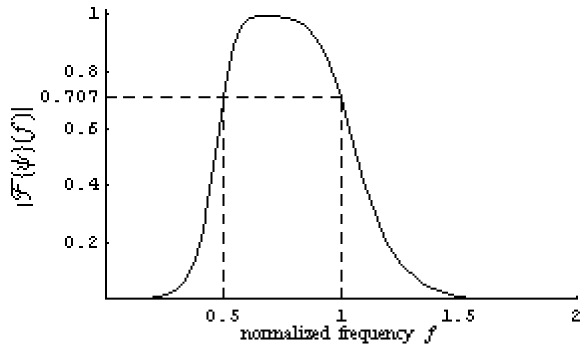

Fig. 2: OCS-bandpass filter with wavelet function.

Here we will propose a new family of wavelets packets. Following simple ideas, we will implement a localized Fourier transform computed from the wavelet coefficients

$$c_{j,k} = \langle s, \psi_{j,k} \rangle . \quad (2)$$

In order to our purposes, assume that $j = 0$, we also denote

$$W_0 = \text{span}\{\psi(\tau - k); k \in \mathbb{Z}\} \quad (3)$$

and we recall that if $s_0 \in W_0$, then $\hat{s}_0$ is localized on $1/2 \le |f| \le 1$.

We observe that for a pure monochromatic signal $e^{i2\pi f_0 \tau}$, the wavelet transform is

$$\langle e^{i2\pi f_0 \tau}, \psi(\tau - k) \rangle = |\hat{\psi}(f_0)| e^{-i\pi f_0 (2k+1)} \quad (4)$$

for any $f_0 \in [1/2, 1]$ and $k \in \mathbb{Z}$.

Of course, since $|\hat{\psi}(f)| \simeq cte$ in the passband of the filter, the specific information of the frequency parameter f_0 must be contained in the sequence $e^{-i\pi f_0(2k+1)}$.

Moreover, for particular choices of f_0 these sequences will be periodic and this gives the key for the detection.

We use the following notation:

t	:	the time.
Δt	:	the sampling rate interval.
$\tau = t/\Delta t$	:	the normalized time, with t and Δt in the same unit (e.g.: sec.).
ν	:	the frequency.
$\nu_s = 1/\Delta t$	:	the sampling rate frequency.
$f = \nu/\nu_s$	:	the normalized frequency, with ν and ν_s in the same unit (e.g.: Hz).

2 Method Description

Let $N \geq 2$ an even integer and define the frequencies

$$f_n = \begin{cases} 1 + n/N & \text{if } 1 - N/2 \leq n \leq 0 \\ n/N - 1 & \text{if } 1 \leq n \leq N/2 \end{cases} ; \quad (5)$$

we remark that

$$\begin{matrix} -1 < f_n \leq -1/2 & \text{if } n > 0 \\ 1/2 < f_n \leq 1 & \text{if } n \leq 0 \end{matrix} ,$$

that is, they are localized on $1/2 \leq |f| \leq 1$. Also note that $f_{-n} = -f_n$.

We denote the basic functions

$$e_{N,n} = \left(\frac{1}{\sqrt{N}} e^{-i\pi f_n(2k+1)} \right)_{0 \leq k < N} \quad (6)$$

for $1 - N/2 \leq n \leq N/2$.

We easily check that the family

$$\{e_{N,n} ; 1 - N/2 \leq n \leq N/2\}$$

is an orthogonal basis (ONB) for $\mathbb{C}^N$.

Finally, fixed N, we define the wavelets packets as

$$\Psi_{N,n}(\tau) = \sum_{k=0}^{N-1} e_{N,n}[k] \psi(\tau - k) \quad (7)$$

with $1 - N/2 \leq n \leq N/2$; this is an ONB for the subspace

$$\text{span}\{\psi(\tau - k) ; 0 \leq k < N\} \subset W_0 ;$$

and the family

$$\{\Psi_{N,n}(\tau - rN) ; r \in \mathbb{Z}\}$$

is a ONB for W_0.

In general, if q divides N,

$$\{\Psi_{N,n}(\tau - rN/q) ; r \in \mathbb{Z}\}$$

is a frame for W_0.

First we remark that these are complex wavelet packet functions. The wavelet coefficients for a given signal s are

$$c_{0,k} = \langle s(\tau), \psi(\tau - k) \rangle \quad (8)$$

The orthogonal *wavelet packet transform* in W_0 is defined as

$$d_{0,n,r} = \sum_{k=0}^{N-1} e_{N,n}[k] \, c_{0,k+rN} \quad (9)$$

for $1 - N/2 \leq n \leq N/2$ and $r \in \mathbb{Z}$.

For real signals, the values $|d_{0,n,r}|^2 + |d_{0,-n,r}|^2$ represent the contribution of the frequency $|f_n|$ on the interval $(rN, (r+1)N)$.

Note that, if $s(\tau) = e^{i2\pi f_p \tau}$, then

$$d_{0,n,r} = \begin{cases} a & \text{if } n = p \\ 0 & \text{if } n \neq p \end{cases} ; \ a = \text{constant}.$$

The extension to other subspaces

$$W_j = \text{span}\left\{ \psi(2^j \tau - k) ; k \in \mathbb{Z} \right\}$$

is analogous; here we remark that we can choice a different integers N_j for each scale j.

In summary, the values

$$(d_{j,n,r})_{j \in \mathbb{Z}, n \in [1 - N_j/2, N_j/2], r \in \mathbb{Z}}$$

give us a time-frequency transform.

3 Implementation

In this article we applied the method described in the previous section to calculate the radial velocity v of a target, detected by the frequency shift (Doppler effect) between the delayed received signal (echo) s_r and the emitted signal s_e for a continuous wave (CW) radar system.

In the theorical case that s_e and s_r (without noise) are sinusoidal signals, this shift ν_0 is the frequency (beat note) of the sinusoidal signal s_B obtained when the radar receiver mixes s_r with s_e and passes the output through a lowpass filter.

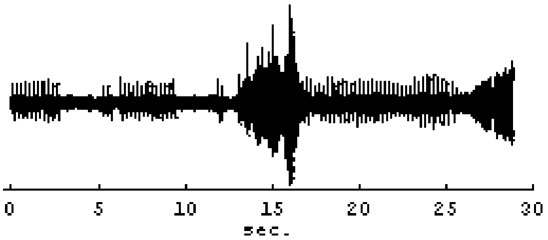

Fig. 3: Signal $s_B(t)$; $0 \le t \le 28.8$ sec.

It can be demonstrated ([4],[5],[7]) that the absolute value of the radial velocity v of the target result

$$|v| \approx \frac{c\,\nu_0}{2\,\nu}\,; \qquad (10)$$

with relative error $|v|/(c - |v|)$.

We use a CW radar with $\nu = 10^{10}$ Hz. Then, knowing ν_0 and assuming $c = 3 \cdot 10^8$ m/sec, with (10) we can estimate $|v|$ in Km/h using

$$|v| \approx 0.054\,\nu_0\,; \qquad (11)$$

with relative error less than $9.26 \cdot 10^{-8}$ for velocities less than 100 Km/h.

So, the precision of the obtained $|v|$ value will depend on the frequency approach.

In this work we considered, specially, small shifts ν_0 produced by velocities between $40 - 100$ Km/h of a car moving along a linear trajectory toward the radar.

Figure 3 shows the signal $S_B(t)$ to be analyzed; it was sampled with $\nu_s = 44100$ Hz.

The magnitude of the signal spectrum, obtained with FFT, is depicted in Fig. 4; $|\hat{s}_B(\nu)|$ is maximum for $\nu_0 = 1247.5$ Hz corresponding to a velocity $|v| \approx 67.4$ Km/h along an interval of 28.8 sec. of the signal.

In this work, we propose to analyze the signal S_B in three steps:

1. First step: in the multiresolution framework with OCS, we proceed with the wavelets analysis using the Mallat algorithm ([6],[9]), for the *resolution levels* $j = -1, -2, \cdots, -10$.

Table 1 shows for each resolution level, the frequency bands

$$[2^{j-1}\nu_s\,;\,2^{j}\nu_s]\,;\quad j = -1, \cdots, -10$$

and the corresponding –using (11)– velocity intervals

$$[0.054\,2^{j-1}\nu_s\,;\,0.054\,2^{j}\nu_s]\,;\quad j = -1, \cdots, -10.$$

level	**frequency**		**velocity**	
j	Hz		Km/h	
$\cdots$	min	max	min	max
-1	11025.00	22050.00	595.35	1190.70
-2	5512.50	11025.00	297.68	5953.50
-3	2756.25	5512.50	148.34	297.68
-4	1378.13	2756.25	74.42	148.84
-5	689.06	1378.13	37.21	74.42
-6	344.53	689.06	18.60	37.21
-7	172.27	344.53	9.30	18.60
-8	86.13	172.27	4.65	9.30
-9	43.07	86.13	2.33	4.65
-10	21.53	43.07	1.16	2.33

Table 1: Frequency bands and velocity intervals for each multiresolution level j.

2. Second step: We compute the energy of the signal in each multiresolution frequency band; this energy results

$$E_j = \sum_k |c_{j,k}|^2\,;\quad j = -1, \cdots, -10. \qquad (12)$$

Table 2 shows the greatest values $100\,E_j/E_t$ observed, where $E_t = \sum_j E_j$; the more important (38%) corresponds to $j = -5$, in accord with the real velocity of the car.

level	**energy**	**velocity**	
j	%	min –Km/h– max	
-4	6	74.42	148.84
-5	38	37.21	74.42
-6	10	18.60	37.21
-9	26	9.30	18.60

Table 2: Percentual values of energy.

3. Third step: We applied wavelet packets in the level $j = -5$, the more significant energy level (see Table 2).

In this level we analyze 77 consecutive groups of 512 coefficients $c_{-5,k}$, corresponding to 77 nonoverlapping adjacent intervals with $512 \cdot 2^5 = 16384$ samples ($16384/44100\text{Hz} \approx 0.37$ sec.) of the signal.

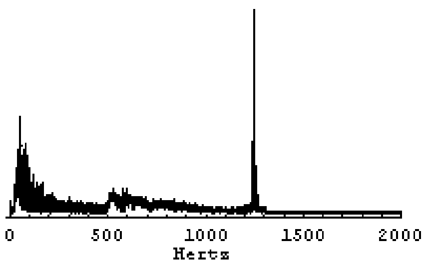

Fig. 4: Magnitude of $s_B(t)$ spectrum, $|\hat{s}_B(\nu)|$.

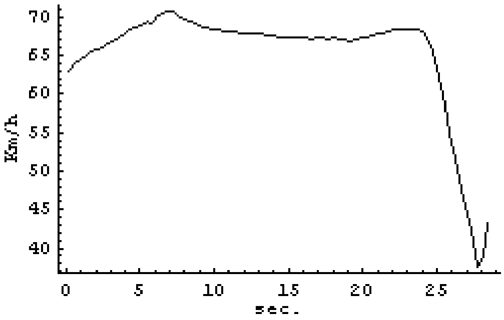

Fig. 5: Car velocity v function of time t.

Working with 512 coefficients in the frequency band $[689.06, 1378.13]$ Hz, we can estimate ν_0 with

$$|\Delta \nu_0| = \frac{1378.13 - 689.06}{512/2} \text{Hz} \approx 2.7 \text{ Hz} ,$$

corresponding to a velocity precision

$$|\Delta v| \approx 0.15 \text{ Km/h} .$$

Then, we can estimate $|v|$ each 0.37 sec.; Fig. 5 shows the car velocity as function of time. The mean velocity in the interval $[0, 28.8]$ sec. results

$$|\bar{v}| = 65.2 \text{ Km/h}.$$

The advantage of methods based on wavelets and wavelet packets, in comparison with standard spectral methods (which give, in this case, similar results; $|\bar{v}| = 65.4$ Km/h) are two. In the first place, to adjust the accuracy in time and frequency; in the second, the use of minimum number of entries (the wavelet coefficients) diminish the computational burden.

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