Wavelets Packets Analysis for Doppler Effect Detection

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Abstract - In this article a method for the parameter estimation of Doppler radar signals is proposed. It is based on the discrete wavelet transform, spline orthogonal wavelets are used from the projection of the mixed signal; wavelet packets analysis play the role of bandpass filter banks with narrowing frequencies ranges. An experimental case is analyzed and the results obtained are discussed.

Key-Words: - Wavelets packets, multiresolution, Doppler radar signals.

1 Introduction

Assume that $\mathcal{A}(\zeta)$ is an orthogonal wavelet with Fourier transform

$$\hat{\mathcal{A}}(f) = j\mathcal{A}(f) e^{j\pi f};$$

where $\mathcal{A}$ is a bandpass filter on $1=2 \cdot jf \cdot 1$.

In this article we use the orthogonal Lemarié-Battle cubic spline (OCS) wavelet $\hat{\mathcal{A}}$ depicted in Fig. 1.

Figure 2 shows the module of the frequency response $F(\hat{\mathcal{A}}(f)) \cdot \mathcal{A}(f)$ corresponding to the OCS wavelet for $f > 0$.

We remark that $j\mathcal{A}$ is almost constant on the band; that is, it is not localized at any frequency. For these reasons the wavelet transform give us a time-scale representation of the signal rather than a time-frequency one.

To overcome this disadvantage we can use wavelet packets ([1],[2],[3],[6]), as waveforms well localized both in the time and the frequency domains. They become from lineal combination of finite wavelets over common intervals and scales.

Here we will propose a new family of wavelets packets. Following simple ideas, we will implement a localized Fourier transform computed from the wavelet coefficients

$$q_{j;k} = < s; \hat{\mathcal{A}}_{j;k} > :$$

In order to our purposes, assume that $j = 0$, we also denote

$$W_0 = \text{span} f\hat{\mathcal{A}}(\zeta) ; k \in \mathbb{Z} g$$

and we recall that if $s_0 \notin W_0$, then $s_0$ is localized on $1=2 \cdot jf \cdot 1$.

We observe that for a pure monochromatic signal $e^{2\pi i f_0 \zeta}$, the wavelet transform is

$$< e^{2\pi i f_0 \zeta}; \hat{\mathcal{A}}(\zeta) > = j\mathcal{A}(0) e^{i\pi f_0(2k+1)}$$
for any \( f_0 \in \mathbb{Z} \) and \( k \in \mathbb{Z} \).

Of course, since \( j \hat{A}(f) = \text{cte} \) in the passband of the filter, the specific information of the frequency parameter \( f_0 \) must be contained in the sequence \( e^{j\pi f_0 (2k+1)} \).

Moreover, for particular choices of \( f_0 \) these sequences will be periodic and this gives the key for the detection.

We use the following notation:

- \( t \) : the time.
- \( \xi t \) : the sampling rate interval.
- \( \xi t = t - \xi t \) : the normalized time, with \( t \) and \( \xi t \) in the same unit (e.g.: sec.).
- \( \xi = \xi t \) : the frequency.
- \( \xi_s = 1 - \xi t \) : the sampling rate frequency.
- \( f = \xi_s \) : the normalized frequency, with \( \xi \) and \( \xi_s \) in the same unit (e.g.: Hz).

### 2 Method Description

Let \( N \), \( 2 \) an even integer and define the frequencies

\[
\xi_n = \begin{cases} 
1 + n & \text{if } 1 \leq n < 0 \\
N & \text{if } n = 0 \\
1 + n & \text{if } n > 0 \\
1 & \text{if } n < 0 
\end{cases}, \quad n \in \mathbb{N}
\]  

(5)

we remark that

\[
\begin{align*}
1 & < \xi_n < 1 & \text{if } n > 0 \\
1 & < \xi_n & \text{if } n < 0 \\
1 & < \xi_n < 1 & \text{if } n = 0
\end{align*}
\]

that is, they are localized on \( 1 < \xi_n < 1 \) if \( n > 0 \) and \( 1 < \xi_n < 1 \) if \( n < 0 \).

We denote the basic functions

\[
\mathbf{e}_{\xi_n} = \mathbf{e}_n = \frac{1}{\sqrt{N}} e^{j\pi \xi_n (2k+1)}
\]

(6)

for \( 1 \leq \xi_n \leq 1 \).

We easily check that the family

\[
\{ \mathbf{e}_{\xi_n} \} \quad 1 \leq \xi_n \leq 1
\]

is an orthogonal basis (ONB) for \( \mathbb{C}^N \).

Finally, fixed \( N \), we define the wavelets packets as

\[
\mathbf{E}_{\xi_n} = \sum_{k=0}^{N} \mathbf{e}_{\xi_n} \hat{A}(\xi_n, k)
\]

(7)

with \( 1 \leq \xi_n \leq 1 \); this is an ONB for the subspace

\[
\text{span} \{ \mathbf{e}_{\xi_n} \} ; \quad 0 < \xi_n < N \quad \xi_s W_0
\]

and the family

\[
\{ \mathbf{E}_{\xi_n} \} ; \quad 1 \leq \xi_n \leq 1 \quad \xi_s W_0
\]

is an ONB for \( W_0 \).

In general, if \( q \) divides \( N \),

\[
f \in W_0 \cup \xi_s \quad 1 \leq \xi_n \leq 1 \quad \xi_s W_0
\]

(6)

is a frame for \( W_0 \).

First we remark that these are complex wavelet packet functions. The wavelet coefficients for a given signal \( s \) are

\[
\mathbf{c}_{0,k} = < s(\xi_n) \quad \xi_s \quad \xi_s W_0
\]

(8)

The orthogonal wavelet packet transform in \( W_0 \) is defined as

\[
d_{\xi_n} = \sum_{k=0}^{N} \mathbf{e}_{\xi_n} c_{0,k} + \xi_s \quad \xi_s W_0
\]

(9)

for \( 1 \leq \xi_n \leq 1 \) and \( r \in \mathbb{Z} \).

For real signals, the values \( j \mathbf{d}_{\xi_n} + j \mathbf{d}_{\xi_n} \) represent the contribution of the frequency \( f \) on the interval \( (rN; r + 1)N \).

Note that, if \( s(\xi_n) = e^{j2\pi f \xi_n} \), then

\[
\mathbf{d}_{\xi_n} = \mathbf{a} \quad \text{if } n = p \quad \text{a = constant}:
\]

The extension to other subspaces

\[
\mathbf{W}_j = \text{span} \{ \mathbf{A}(\xi_n, k) \} ; \quad 1 \leq \xi_n \leq 1 \quad \xi_s W_0
\]

is analogous; here we remark that we can choose a different integers \( N_j \) for each scale \( j \).

In summary, the values

\[
\{ \mathbf{d}_{\xi_n} \} ; \quad 1 \leq \xi_n \leq 1 \quad \xi_s W_0
\]

give us a time-frequency transform.

### 3 Implementation

In this article we applied the method described in the previous section to calculate the radial velocity \( v \) of a target, detected by the frequency shift (Doppler effect) between the delayed received signal (echo) \( s_r \) and the emitted signal \( s_e \) for a continuous wave (CW) radar system.

In the theoretical case that \( s_0 \) and \( s_r \) (without noise) are sinusoidal signals, this shift \( \xi_0 \) is the frequency (beat note) of the sinusoidal signal \( s_0 \) obtained when the radar receiver mixes \( s_0 \) with \( s_e \) and passes the output through a lowpass filter.
with OCS, we proceed with the wavelets analysis in three steps:

1. First step: in the multiresolution framework with FFT, is depicted in Fig. 4; it was sampled with relative error less than 100 Km/h.

2. Second step: We compute the energy of the signal. The energy result

\[ E_j = \sum_{k} |S_j[k]|^2 \]  

for velocities between 28.8 sec. of a car moving along a linear trajectory toward the radar.

Figure 3 shows the signal \( S_B(t) \) to be analyzed; it was sampled with \( \theta_s = 44100 \) Hz.

The magnitude of the signal spectrum, obtained with FFT, is depicted in Fig. 4; \( |S_B(\omega)| \) is maximum for \( \omega = 1247.5 \) Hz corresponding to a velocity \( |v| = 67.4 \) Km/h along an interval of 28.8 sec. of the signal.

In this work, we propose to analyze the signal \( S_B(t) \) in three steps:

1. First step: in the multiresolution framework with OCS, we proceed with the wavelets analysis using the Mallat algorithm ([6],[9]), for the resolution levels \( j = i; j = i+1; j = i+2; \ldots; j = 10 \).

Table 1 shows for each resolution level, the frequency bands

\[ 2^j \omega_0; 2^{j+1} \omega_0 \]  

and the corresponding velocity intervals

\[ 0.054 \omega_0; 0.054 \omega_0 \]  

for velocities between

with relative error less than 9.26 \( 10^8 \) for velocities less than 100 Km/h.

So, the precision of the obtained \( |v| \) value will depend on the frequency approach.

In this work we considered, specially, small shifts \( \omega_0 \) produced by velocities between \( 40 \) i \( 100 \) Km/h of a car moving along a linear trajectory toward the radar.

Table 2 shows the greatest values \( 100 E_j = E_1 \) observed, where \( E_j = \sum_{k} |E_k| \); the more important (38%) corresponds to \( j = 5 \), in accord with the real velocity of the car.

3. Third step: We applied wavelet packets in the level \( j = 5 \), the more significant energy level (see Table 2).

In this level we analyze 77 consecutive groups of 512 coefficients \( c_{j,k} \), corresponding to 77 nonoverlapping adjacent intervals with 512 samples (16384=44100Hz = 0.37 sec.) of the signal.
Working with 512 coefficients in the frequency band $[689.06; 1378.13]$ Hz, we can estimate $\omega_0$ with
$$j \omega_0 = \frac{1378.13 - 689.06}{512} \text{Hz} < 2.7 \text{Hz};$$
corresponding to a velocity precision
$$j v_j < 0.15 \text{Km/h}.$$

Then, we can estimate $|v_j|$ each 0:37 sec.; Fig. 5 shows the car velocity as function of time. The mean velocity in the interval $[0; 28.8]$ sec. results
$$|v| = 65.2 \text{Km/h};$$

The advantage of methods based on wavelets and wavelet packets, in comparison with standard spectral methods (which give, in this case, similar results; $|v| = 65.4 \text{Km/h}$) are two. In the first place, to adjust the accuracy in time and frequency; in the second, the use of minimum number of entries (the wavelet coefficients) diminish the computational burden.

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