Real Time Nonlinear Indoor Positioning with Trilateration Based on Microwave Backscatter

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***Abstract:* —** We present two techniques for estimating accurately the position of a mobile base station in industrial environment with Trilateration. By eliminating the need to measure angles, trilateration uses only distance measurements to estimate position, which facilitate the implementation of real-time positioning systems like the Global Positioning System (GPS). Iterative linear least squares and nonlinear least squares estimators are implemented through simulation. We consider the minimization of the squares of errors, where the main difference depends on the linearity of the measurement equation which relates the coordinates to be estimated to the measured distance. The results obtained show that the nonlinear squares technique results in more accurate position estimates than linear least squares. Error distribution functions and histograms in both cases are also derived and analysed.

***Key Words*: -** Nonlinear Systems, Indoor Local Positioning, Microwave Sensing, Statistical Analysis, Radar

1 Introduction

The triangulation location sensing technique uses the geometric properties of triangles to compute object locations. Triangulation includes two subcategories lateration, using distance measurement and angulation using primarily angle or bearing measurements.

Lateration computes the position of an object by measuring its distance from multiple reference positions. Calculating an *x*,*y* object's position requires distance measurements from 3 non-collinear points. In three dimensions, distance measurements from four non-coplanar points are required.

Angulation is similar to lateration except, instead of distances, angles are used for determining the position of an object. In general, two dimensional angulation requires two angle measurements and one length measurement such as the distance between the reference points. In three dimensions, one length, one azimuth, and two angle measurements are needed to specify a precise position. Angulations implementations choose to designate a constant reference vector (e.g. magnetic north) as 0. Phased antenna arrays are an excellent enabling technology for the angulation technique. Multiple antennas with known separation measure the time of arrival of a signal. Given the differences in arrival times and the geometry of the receiving array, it is then possible to compute the angle from which the emission originated. If there are enough elements in the array and large enough separations, the angulation calculation can be performed. Standard surveying techniques are based on measurement of angles and base line distance to determine the unknown position components of appoint relative to a fixed coordinate system, such triangulation methods are expensive and slow. Trilateration surveying techniques replace angular measurements with range measurements. In practice, distances are measured with error, and statistical methods can quantify the uncertainty in the estimate of the unknown location. The usefulness of statistical methods for the three dimensional trilateration positioning can be utilized in any system that involves distance measurements obtained with radio, lasers, or manual measurements. Trilateration facilitates the implementation of fully automated real time positioning systems similar to the global positioning system GPS[2] without angle measurements. The techniques presented in this paper are tested in the context of real time local positioning radar. This work applies a system of radar transponders at known locations on the boundaries of an indoor working area as shown in Fig.1. The maximum distance between two facing rows of transponder can be up to 60m. The transponders used have a directive antenna with 160° beam angle in the horizontal plane and 20° in the vertical plane (3dB attenuation limit).This needs that the base station and transponders should be installed almost on the same level with a height tolerance up to 30cm.The coordinates of the transponders are carefully determined with conventional surveying methods to nearly perfect accuracy. To determine the position of a mobile vehicle, each transponder reflects frequency modulated continuous wave radar signal from the base station at 5.6 GHz. The distance between a transponder and the target point can be computed as a function of the measured time between transmission and reception. The spatial coordinates of the target would then be computed from the measured distances to various transponders [1]. In this work two techniques for estimating the two dimensional position of a mobile station via trilateration are presented. Linear least squares and non linear least squares.

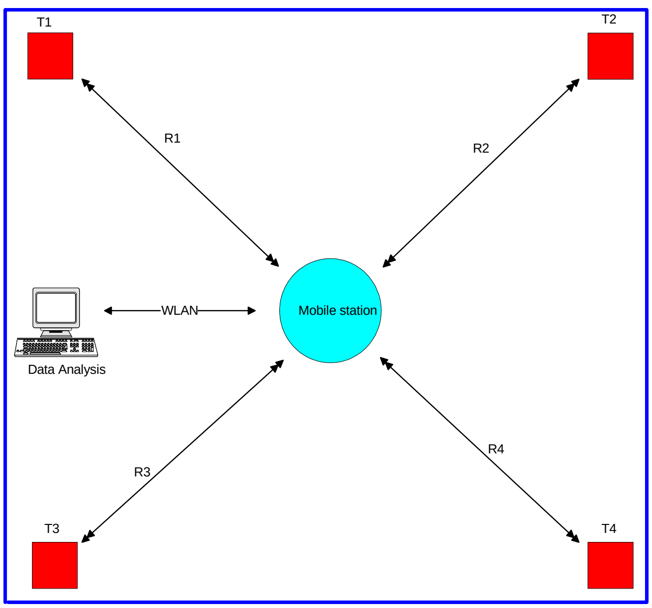

Fig.1. Measurement system setup (mobile unit, transponders (T1…T4), and data processing unit)

2 Sources of Errors

There are three major sources of errors in time based location system: the receiver system delay, the wireless multi-path fading channel, and the non-line-of-sight (NLOS) transmission as illustrated in Fig.2. The receiver system delay is the time duration from which the signal hits the receiver antenna until the signal is decoded accurately by the receiver. This time delay is determined by the receiver electronics. Usually it is constant or varies in very small scale when the receiver and the channel are free from interference. This system delay can he predetermined and be used to calibrate the measurements. For example, Transponders T1 and T3 can always eliminate the system delay before they conveyed in their reply messages to base station. Meanwhile, as time delay is measured by one sensor, the effect of receiver system delay may cancel out, if the base station can provide precise a prior*i* information on receiver system time delay. The wireless multi-path fading channel will greatly influence the location accuracy of any location detection system. Major factors influencing multi-path fading include multi-path propagation, speed of the receiver, speed of the surrounding objects, and the transmission signal bandwidth. Multi-path propagation refers to the fact that a signal transmitted from the sender can follow a multiple number of propagation paths to the receiving antenna. In our system, the performance is not affected by the speed of the receivers since all sensors, excepting the base station antenna, are stationary. However, a moving object in the surrounding area can cause interference. There are two important characteristics of multi-path signals. First, the multiple non-direct path signals will always arrive at the receiver antennae latter than the direct path signal, as they must travel a longer distance. Second, in LOS transmission model, non-direct multi-path signals will normally be weaker than the direct path signal, as some signal power will be lost from scattering. If NLOS exists, the non direct multi-path signal may be stronger, as the direct path is hindered in some way. Based on these characteristics, scientists can always design more sensitive receivers to lock and track the direct path signal. For example, multi-path signals using a pseudo-random code arriving at the receiver later than the direct path signal will have negligible effects on a high-resolution receiver. Another factor related to wireless channels that causes location detection errors is NLOS transmission. To mitigate NLOS effects, base stations can be placed well above the surrounding objects such that there are line-of-sight transmission paths among all base stations and from base stations to microwave sensors [3].

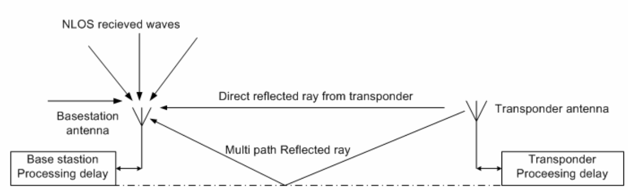

Fig.2. Illustration of main sources of errors

3 Solution Approaches

The obvious approach in solving the positioning problem is to treat the coordinates of the mobile vehicle $p(x, y, z)$ as the point of intersection of several spheres, whose centres are the locations of the n Transponders $T(x_i, y_i, z_i)$ for $i = 1,2,3.....,n.$ the exact distances between the transponders and the vehicle are the radii of the individual spheres. The equation for any of these spheres is

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = r_i^2 \quad (1)$$

The point of intersection of n of these spheres is obtained by letting $i = 1,2,3....,n,$ and solving the resulting n nonlinear equations simultaneously to eliminate two coordinates. The solution technique is not feasible because it produces a nonlinear equation of high degree. Moreover, since the equation is quadratic , many cases of the signs would have to be considered. Linearizing the system of equation geometrically converts the problem into one of finding the point of intersection of several planes. When the exact distances from four beacons are available, the solution of the linear system of equations is completely determined. There are three equations, three unknowns, and exactly one solution. As a result, the theoretical minimum number of beacons is four. When approximate distances are used, the position that is obtained by direct solution of the linear equations is no longer acceptable [4].

3.1 Linear least square

The solution of the linear system $A\vec{x} = \vec{s}$ is an improvement over solving for the intersection off spheres. However, it is sometimes unacceptable because it doesn't determine the locations with a good tolerance when used with approximate distances. The linear system that will be presented in this section can be used with exact distances and four arbitrary selected beacons to accurately calculate an unknown location by finding the intersection of three planes. The constrains are the equations of the spheres with radii r_i,

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = r_i^2$$
$$(i = 1,2,........,n) \quad (2)$$

The j^{th} constrain is used as a linearizing tool. Adding and subtracting x_j, y_j and z_j in (2) yields:

$$(x - x_j + x_j - x_i)^2 + (y - y_j + y_j - y_i)^2 + (z - z_j + z_j - z_i)^2 = r_i^2 \quad (3)$$

expanding and reordering the terms yields:

$$(x-x_j)(x_i-x_j)+(y-y_j)(y_i-y_j)+(z-z_j)(z_i-z_j)=$$
$$\frac{1}{2}[(x-x_j)^2-(y-y_j)^2+(z-z_j)^2-r_i^2+(x_i-x_j)^2+$$
$$(y_i-y_j)^2+(z_i-z_j)^2]=\frac{1}{2}[r_j^2-r_i^2+d_{ij}^2]=b_{ij}$$

Where:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

is the distance between transponders T_i *and* T_j.

Arbitrarily we select the first constrain ($j = 1$) as a linearizing constrain. This is analogous to select the first transponder. Since $i = 2,3,.........,n$, this leads to a linear system of ($n - 1$) equations with 3 unknowns as follows:

$$(x - x_1)(x_2 - x_1) + (y - y_1)(y_2 - y_1) + (z - z_1)(z_2 - z_1)$$
$$= \frac{1}{2}[r_1^2 - r_2^2 + d_{21}^2] = s_{21}$$

$$(x - x_1)(x_3 - x_1) + (y - y_1)(y_3 - y_1) + (z - z_1)(z_3 - z_1)$$
$$= \frac{1}{2}[r_1^2 - r_3^2 + d_{31}^2] = s_{31}$$

$$(x - x_1)(x_n - x_1) + (y - y_1)(y_n - y_1) + (z - z_1)(z_n - z_1)$$
$$= \frac{1}{2}[r_1^2 - r_n^2 + d_{n1}^2] = s_{n1}$$

The linear system can be written in matrix form as: $A\vec{x}' = \vec{s}$ with :

$$A = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_n - x_1 & y_n - y_1 & z_n - z_1 \end{bmatrix}, \vec{x} = \begin{bmatrix} x - x_1 \\ y - y_1 \\ z - z_1 \end{bmatrix},$$

$$\vec{s} = \begin{bmatrix} \vec{s}_{21} & \vec{s}_{31} & \vec{s}_{n1} \end{bmatrix}^{-1}$$

The system described above has $n - 1$ equations with three unknowns. Therefore, theoretically only four transponders are needed to determine the unique position of the vehicle, provided no more than two transponders are co-linear.

The coordinates of positions obtained by applying the linear least squares method to the linear system in derived above are more accurate than the coordinates obtained by solving four equations from the linearized system of equations directly. Since the measured distances are only approximate, the problem requires the determination of $\vec{x}$ such that $A\vec{x} \approx \vec{s}$. Minimizing the sum of squares of residuals:

$$\vec{r}^{\,T}\vec{r} = (\vec{s} - A\vec{x})^T(\vec{s} - A\vec{x}) \quad (4)$$

this leads to the standard equation:

$$A^T A\vec{x} = A^T\vec{s} \quad (5)$$

If $A^T A$ is non-singular and well conditioned then:

$$x = (A^T A)^{-1} A^T\vec{s} \quad (6)$$

If $A^T A$ is singular, then the normalized QR-decomposition of A is generally used. In this method, $A = QR$, where Q is an orthonormal matrix and R is an upper-triangular matrix. The solution for $\vec{x}$ in the normalized QR-decomposition is found from:

$$R\vec{x} = Q^T\vec{s} \quad (7)$$

By back substitution when A is full rank. If the matrix $A^T A$ is close to singular, QR decomposition may overcome the problem. Otherwise, singular value decomposition (SVD) can be used to solve the least squares problem fairly accurately [5].

3.2 Nonlinear least square

Nonlinear least square method gives the most accurate results of all methods developed and examined when approximate distances are involved in the calculations. The use of this technique is restricted to situations where the vehicle is inside the perimeter of the transponders, and below the common plane of the transponders. The accuracy of the solution decreases as the elevation of the vehicle increases and the vehicle moves out farther outside the perimeter of the transponders.

The sum of the squares of the errors on the distances is minimized in this least squares method. To do this, one must minimize the function:

$$F(x,y,z) = \sum_{i=1}^{n}(\bar{r}_i - r_i)^2 = \sum_{i=1}^{n} f_i(x,y,z)^2 \quad (8)$$

Where:

$$f_i(x,y,z) = \bar{r}_i - r_i = \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2} - r_i$$

$\bar{r}_i$: is the exact distance. Different approaches can be used to minimize the sum of errors. Newton Raphson iterative technique is one of the most common for this purpose and will be considered later to find the optimal solution $p(x,y,z)$. A good initial guess can be obtained from linear least square method developed before [7]. Differentiating (12) with respect to *x* gives:

$$\frac{\partial F}{\partial x} = 2\sum_{i=1}^{n} f_i \frac{\partial f_i}{\partial x} \quad (9)$$

The equations for the partials for y and z are the same. Introducing the vectors, $\vec{f}$, $\vec{h}$ and the jacobian matrix **J,** leads to:

$$\vec{h} = 2J^T\vec{f} \quad (10)$$

Where: $$J = \begin{bmatrix} \frac{\partial f_1}{\partial x} & \frac{\partial f_1}{\partial y} & \frac{\partial f_1}{\partial z} \\ \frac{\partial f_2}{\partial x} & \frac{\partial f_2}{\partial y} & \frac{\partial f_2}{\partial z} \\ \vdots & \vdots & \vdots \\ \frac{\partial f_n}{\partial x} & \frac{\partial f_n}{\partial y} & \frac{\partial f_n}{\partial z} \end{bmatrix}$$

$$f = \begin{bmatrix} f_1 & f_2 & f_n \end{bmatrix}^{-1} \qquad \vec{h} = \begin{bmatrix} \frac{\partial F}{\partial x} & \frac{\partial F}{\partial y} & \frac{\partial F}{\partial z} \end{bmatrix}^{-1}$$

Using the Vector $\vec{p} = \begin{bmatrix} x & y & z \end{bmatrix}^{-1}$ to indicate the estimated position vector, Newton method gives:

$$\vec{p}_{k+1} = \vec{p}_k - (J_k^T J_k)^{-1} J_k^T \vec{f}_k \quad (11)$$

$$J^T\vec{f} = \begin{bmatrix} \sum_{i=1}^{n}\frac{(x-x_i)f_i}{(f_i+r_i)} & \sum_{i=1}^{n}\frac{(y-y_i)f_i}{(f_i+r_i)} & \sum_{i=1}^{n}\frac{(z-z_i)f_i}{(f_i+r_i)} \end{bmatrix}^{-1}$$

Where: $\vec{P}_k$ is the *kth* approximate solution, using the explicit form of function f gives :

$$J^T\vec{f} = \begin{bmatrix} \sum_{i=1}^{n}\frac{(x-x_i)f_i}{(f_i+r_i)} & \sum_{i=1}^{n}\frac{(y-y_i)f_i}{(f_i+r_i)} & \sum_{i=1}^{n}\frac{(z-z_i)f_i}{(f_i+r_i)} \end{bmatrix}^{-1} \quad (12)$$

Practically, this technique works fast, especially when the matrix $J^T J$ is augmented by a diagonal matrix which effectively biases the search direction towards that of steepest decent.

To test the performance of these algorithms, we have built a simulation program in matlab that incorporate a real distance measurements offline from the mobile vehicle while at the same time provide a comprehensive and systematic procedure to evaluate the different solution approaches that described above and compare the results. The simulation program make use of the matlab functions *lsqnonlin* , *lsqr, fsolve* and the predefined fixed positions of the transponders. The effectiveness of each of the solution algorithms is tested by statistically analysing the errors between the estimated and the measured distances. The accuracy of the results is degraded when the unknown position is outside the perimeter of transponders, and when the vehicle changes its speed and direction rapidly (edge conditions) [6].

4 Simulation Results

Our experimental test bed is a local positioning radar system that offers a real-time contact free tracking of a mobile vehicle in a harsh industrial environment. The system was developed in cooperation with the control and measurement department at Symeo GmbH in Munich, Germany [1].Our sample installation is used to track a sample Eisenbahn in an area 11 m long and 10 wide. Four Transponders are installed along boundaries of the working area while the base station is installed on the sample Eisenbahn. The transponders are frequency multiplexed, this means they transmit simultaneously and are therefore recorded at the same time. Each transponder is equipped with 160°/10° dipole antenna. The base station is implemented as a real-time embedded system with Omni directional antenna which measures the distance to all transponders within range of about three times per second. It transmits a frequency modulated signal which is received, amplified, and modulated by each transponder and reflected back. The signal round trip time is calculated by comparing the transmitted and received signals. The base station communicates via WLAN with a centralised PC database, where the received raw data are processed and analysed. The system measures the ranges between transponders and base station within a 10 cm accuracy assuming line of sight between the transponders and base station with an update time of about 5 ms. A major advantage of this system is that it doesn't suffer from high temperature, dirt and vibration like laser-based systems [9]. Several simulation scenarios have been done, based on the radians measurements that we obtained from the LPR system. We considered different data sets; each consists of 5000 measured values and corresponds to a certain distribution of the transponders around the working environment [8]. The data sets are saved in matrix form with four columns, where the entries of each row are the radians measured from transponders during one time cycle. The trilateration algorithm is run to calculate the *x,y* coordinates of the mobile vehicle and to find the average and mean squared localization errors corresponds to each estimated position. Moreover, the total probability distribution of the error for all data sets is modelled. The initial position is chosen as the geometrical centre of the working area and subjects to changes in order to get faster convergence. The z-coordinate is set to zero where further investigations have to be done to elaborate this issue in the next phase. Because of the presence of many metallic objects inside the indoor environment, where the real measurements have been taken, the channels were subjected to severe multi-path interferences. However, LOS has been guaranteed at all positions of the mobile vehicle. The mobile vehicle was moved at constant speed with a height difference of 0.30 m from transponders level. The position mean error is regarded as the Euclidian distance between the position estimate and the real position measured by the FMCW radar. The root mean squared error serves as additional performance measure and defined by:

$$e_{rms} = \sqrt{\frac{\sum_{i=1}^{n} e_i^2}{n}} . \qquad (13)$$

Three illustrations of error distribution as function of the measured distance are shown in Fig.3. Transponders locations and coverage with the detected position of mobile station are also illustrated. The maximum localization error obtained is about 75 cm under line of site conditions. Fig.4. shows the total error probability distribution function with respect to estimated distance using linear and nonlinear least squares. The distribution of the Non LLS errors is a Nakagami with mean 0.3514 and variance of 0.04798, where the LLS errors form Rayleigh distribution with mean of 0.3912 and variance of 0.04317. The maximum error obtained from NLLS is 0.74 m compared with 0.81 m from LLS.

5 Conclusion

The position coordinates obtained by applying the linear least square method to the linear system of equation described above is more accurate than solving for *x,y* directly, where singular value decomposition is required if matrix A is singular. The non linear least square method give more accurate position coordinates than the linear least square method, where the sum of the squares of the errors on the distances is minimized. Analysis of the results and the details of individual points that are out of 0.5 m tolerance indicates that the accuracy is inversely affected by relative positions of the transponders and the elevation of the mobile station with respect to transponders level. The accuracy is decreased when the vehicle is outside the coverage of two transponders. Moreover, the positions of the transponders a round the perimeter of the working space has large effect on the multipath signal distribution and soon the final measured radiuses. The results obtained in this work are with out any type of prefiltering or post- processing techniques like kalman or particle filters, this indicates a potential for further accuracy improvement by applying a suitable filtering technique.

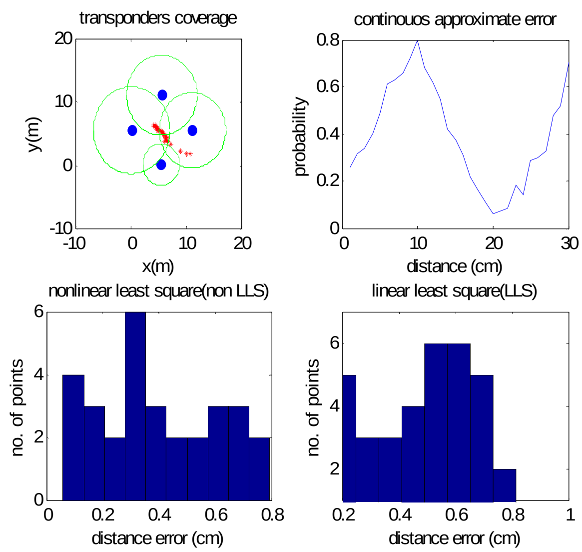

Fig.3. Localization accuracy with error distribution

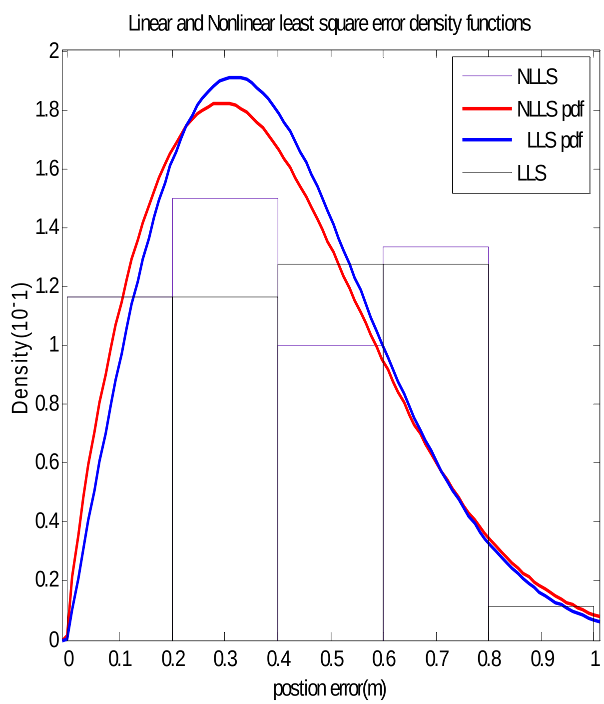

Fig.4. Total error density functions with linear and nonlinear least square techniques

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