Initial Conditions and Coexisting Attractors in an Autonomous Circuit

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Abstract: - In this paper we have studied the dynamics of a non driven circuit of fourth-order. The circuit constitutes of two active elements, one linear negative conductance and one non linear resistor exhibiting a symmetrical piecewise linear v-i characteristic. The resistor R_1, which couples the negative conductance and the nonlinear element, serves as the control parameter of the system. We have observed formation of "bubbles" for some initial conditions. In a narrow region of R_1 values, we have observed antimonotonicity, different routes to chaos via period doubling sequences and reverse period doubling and transition from periodic to quasi-periodic and finally to chaos. We have also studied the dependence of circuit's behavior on initial conditions.

Key-Words: - Coexisting attractors, Initial conditions, Control parameter, Period doubling, Bubbles, Antimonotonicity, Chaos.

1 Introduction

The last twenty years have been characterized by the huge development in the study of phenomena with complicated or chaotic behavior. Chaos is a noise like phenomenon that is a product of the inevitable nonlinearity of physical systems. The science of chaos is relatively new and many scientists have observed chaotic behaviors in different areas of science. Firstly, physicians and mathematicians mainly, dealt theoretically with the study of chaotic phenomena. Theoretical study or simulation of these chaotic phenomena was much easier than study in experimental circuits because of the difficulty in searching of chaotic behaviors.

The main advantage of using electric circuits in studying dissipative systems is the easy implementation using low cost materials. Chaotic phenomena, such as bifurcation, antimonotonicity [1], period doubling and reverse period doubling have all been observed in electric circuits e.g., the period-doubling route to chaos [2-4], quasiperiodicity route to chaos [5]. Cascades of period-doubling bifurcations have long been recognized to be one of the common routes to chaos, as exemplified by the one-dimensional logistic map $x_{n+1} = \lambda x_n(1 - x_n)$. As the parameter λ, in the logistic map, is increased, it is known that periodic orbits are only created but never destroyed. Unlike the monotone bifurcation behavior of the logistic map, it has been seen [6] that in many common nonlinear dynamical systems periodic orbits must be both created and destroyed infinitely often, as the parameter is increased near certain common parameter values. Dawson et al. [6] named this concurrent creation and annihilation of periodic orbits antimonotonicity.

Chua's circuit family, which was created during the effort to make real Lorenz's model, is the most classical example route to chaos. In previous studies we have realized many modified Chua's circuits [3, 7]. What makes these circuits different is that any changes in the values of capacitances, resistance and inductors cause many different dynamic behaviors.

Moreover, the behavior of the systems is very sensitive to initial conditions (specially the autonomous circuits). The study of Chua's circuit family presents inexhaustible wealth of dynamical behavior and their study has not been completed.

In a recent paper [8] we have studied the dynamics of a fourth-order, non autonomous, nonlinear electric circuit (driven by a sinusoidal voltage source). In this paper we study the dynamics of a fourth-order, autonomous, nonlinear electric circuit (Fig.1) with two active elements, one nonlinear resistor R_N with a symmetrical piecewise linear v-i characteristic (Fig.2) and one linear negative conductance G_n (Fig.3).

2 The Autonomous Circuit

The fourth-order autonomous circuit that we have studied is shown in Fig.1, while in Figs.2 and 3 we can see the v-i characteristics of the nonlinear resistor R_N and negative conductance, respectively.

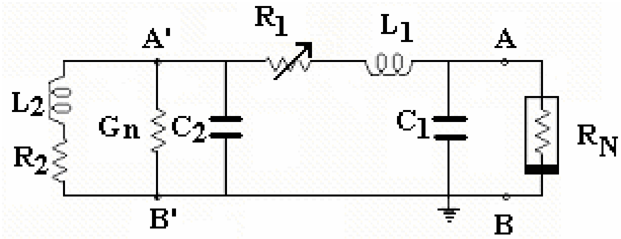


Fig.1. The fourth-order autonomous electric circuit.

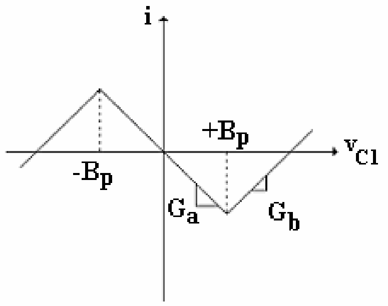


Fig.2. The v-i characteristic of nonlinear resistor R_N.

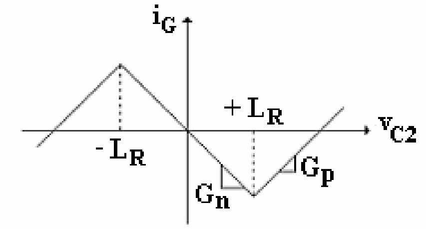


Fig.3. The v-i characteristic of negative conductance.

The state equations of the circuit are:

$$\frac{dv_{C1}}{dt} = \frac{1}{C_1}\left(i_{L1} - i\right) \tag{1}$$

$$\frac{dv_{C2}}{dt} = -\frac{1}{C_2}\left(G \cdot v_{C2} + i_{L1} + i_{L2}\right) \tag{2}$$

$$\frac{di_{L1}}{dt} = \frac{1}{L_1}\left(v_{C2} - v_{C1} - R_1 i_{L1}\right) \tag{3}$$

$$\frac{di_{L2}}{dt} = \frac{1}{L_2}\left(v_{C2} - R_2 i_{L2}\right) \tag{4}$$

where

$$i = g(v_{C1}) = G_b v_{C1} + 0.5(G_a - G_b)(|v_{C1} + B_p| - |v_{C1} - B_p|) \tag{5}$$

The circuit's parameters are: L_1=33.0mH, L_2=100.0mH, C_1=6.60nF, C_2=15.00nF, R_2=90Ω, G_n= -0.45mS, G_p=0.45mS, L_R=7.5V, G_a= -0.105mS, G_b= 7mS and B_p= 0.68V, while R_1 varies from 400Ω to 1100Ω.

3 The Effect of Initial Conditions in Circuit's Dynamics

Systems that exhibit chaotic behavior are very sensitive to the changes of the initial conditions. Different initial conditions will probably create totally different dynamic behavior. In our case, we have studied the dynamics of the system for selected values of R_1, plotting bifurcation diagrams, phase portraits and Lyapunov exponents for two different sets of initial conditions: i.c.1/ v_{C1}=0V, v_{C2}=0V, I_{L1}=0.5mA, I_{L2}= -0.5mA and i.c.2/ v_{C1}= -0.706V, v_{C2}=0V, I_{L1}= -0.08mA, I_{L2}= -0.48mA.

The bifurcation diagrams i_{L2} vs. R_1 for C_1=6.6nF and initial conditions 1 (i.c.1) is shown in Fig.4 and the same one for initial conditions 2 (i.c.2) is shown in Fig.5. We observe that for R_1=800Ω to 712Ω the behavior of the system is completely different for the two sets of initial conditions, while for R_1=712Ω to 600Ω the system appears exactly the same behavior, without any external influence.

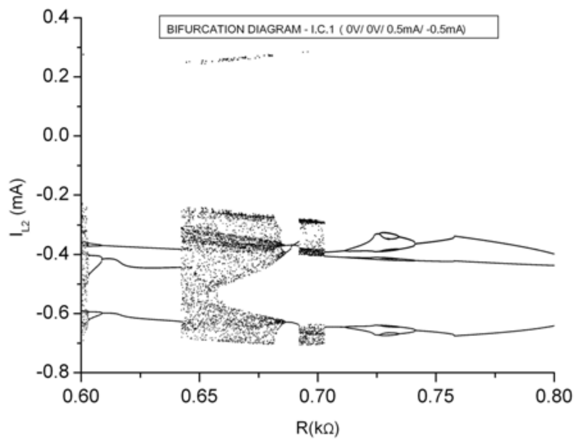


Fig.4. Bifurcation diagram i_{L2} vs. R_1 for initial conditions 1 (i.c.1).

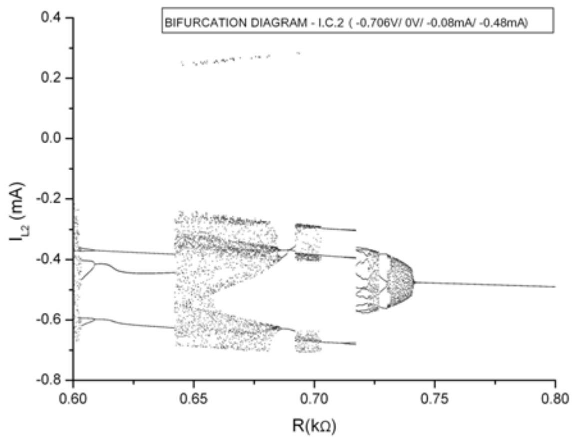
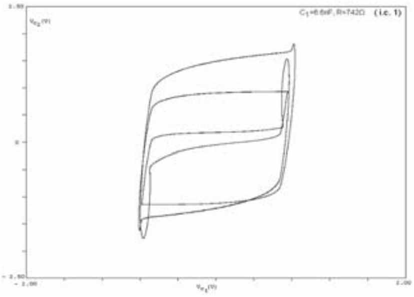
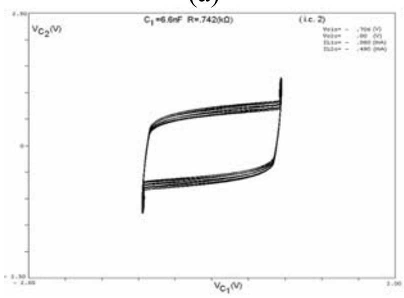


Fig.5. Bifurcation diagram i_{L2} vs. R_1 for initial conditions 2 (i.c.2).

In Fig.6 the phase portraits v_{C2} vs. v_{C1} are presented for R_1=742Ω and initial conditions 1 and 2 (i.c.1 and i.c.2), while in Fig.7 the phase portraits v_{C2} vs. i_{L2} are shown for R_1=737Ω and initial conditions 1 and 2 (i.c.1 and i.c.2).The state of the circuit is periodic or quasiperiodic, depending on the initial conditions.

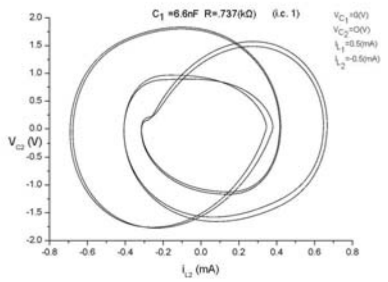


(a)

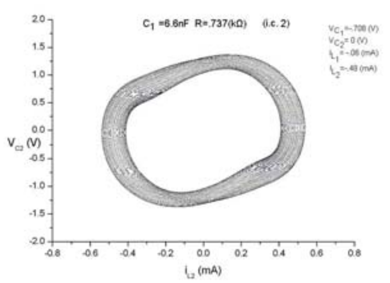


(b)

Fig.6. Phase portrait v_{C2} vs. v_{C1} for R_1=742Ω and initial conditions (a) i.c.1--periodic (LE1=0, LE2<0, LE3<0, LE4<0) (b) i.c.2--quasiperiodic (LE1=0, LE2=0, LE3<0, LE4<0).



(a)



(b)

Fig.7. Phase portrait v_{C2} vs. i_{L2} for R_1=737Ω and initial conditions (a) i.c.1--periodic (LE1=0, LE2<0, LE3<0, LE4<0) (b) i.c.2--quasiperiodic (LE1=0, LE2=0, LE3<0, LE4<0).

In Fig.8 we focus on the bifurcation diagram for i.c.1. As R_1 is increased, the circuit remains in a periodic state according to the following scheme: period-3 (741.5Ω<R_1< 800.0Ω) → period-6 (734.7Ω <R_1<741.5Ω) → period-12 (726.6Ω<R_1<734.7Ω) → period-6 (712.2Ω<R_1<726.6Ω) → period-3 (703.0Ω <R_1<712.2Ω). This scheme is named “triple primary bubble”. The triple primary bubble disappears for other initial conditions (like i.c.2) and the behavior of the system is completely different (see Fig.5).

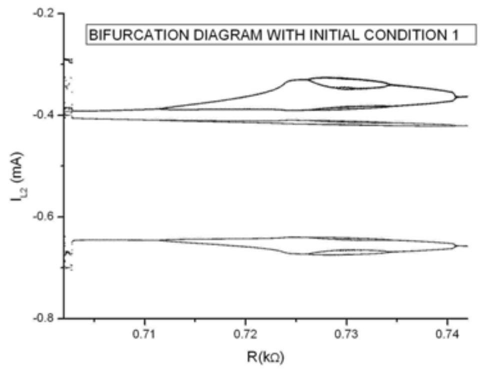


Fig.8. Bifurcation diagram i_{L2} vs. R_1 for initial conditions 1 (i.c.1)--The triple primary bubble.

4 Period Doubling Route to Chaos

In the following figures, we present the phase portraits for a route to chaos via period doubling in another resistor R_1 range. This route is the same for both initial conditions (i.c.1. and i.c.2).

4.1 Theoretical Phase Portraits

In Figs.9-13 we can see the theoretical phase portraits v_{C2} vs. v_{C1} for various values of resistance R_1. We can observe the scheme: R_1=600.0Ω → chaos, R_1=603.3Ω → period-12, R_1=604.0Ω → period-6, R_1=620.0Ω → period-3, R_1=650.0Ω → chaos.

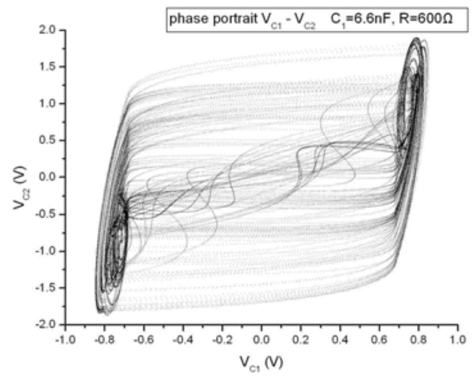


Fig.9. Phase portrait v_{C2} vs. v_{C1} for R_1=600.0Ω (LE1=0.034>0, LE2=0,LE3=-0.10<0--chaos).

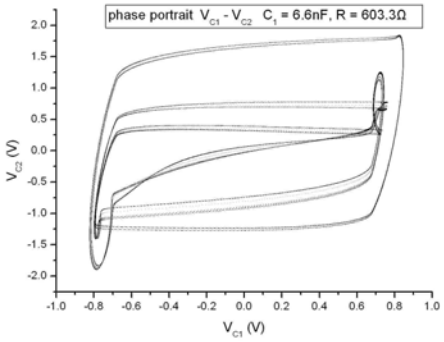


Fig.10. Phase portrait v_{C2} vs. v_{C1} for R_1=603.3Ω (LE1=0, LE2=-0.031<0, LE3=-0.121<0--period-12).

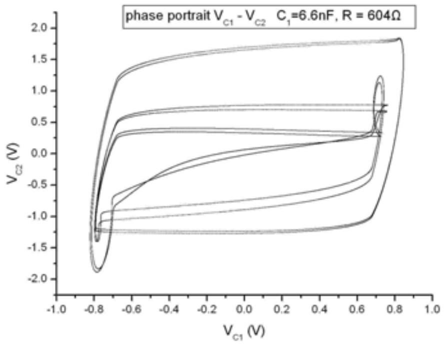


Fig.11. Phase portrait v_{C2} vs. v_{C1} for R_1=604.0Ω (LE1=0, LE2=-0.0306<0, LE3=-0.116<0--period-6).

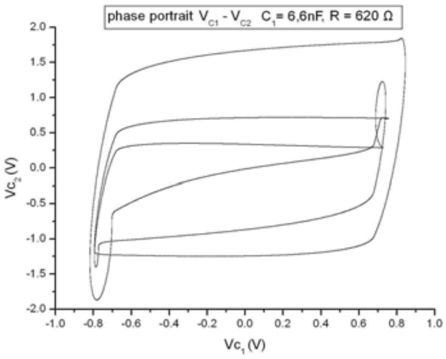


Fig.12. Phase portrait v_{C2} vs. v_{C1} for R_1=620.0Ω (LE1=0, LE2=-0.069<0, LE3=-0.079<0--period-3).

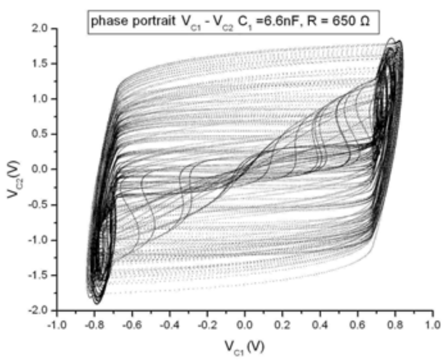


Fig.13. Phase portrait v_{C2} vs. v_{C1} for R_1=650.0Ω (LE1=0.0466>0, LE2= 0, LE3=-0.112<0--chaos).

4.2 Experimental Phase Portraits

In Figs.14-17 we observe the experimental phase portraits v_{C2} vs. v_{C1} for corresponding values of resistance R_1, as in Figs.9-13, respectively.

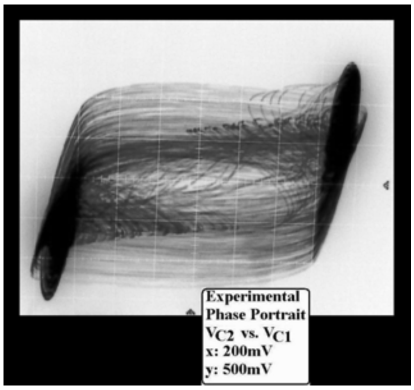


Fig.14. Experimental diagram v_{C2} vs. v_{C1} (chaos).

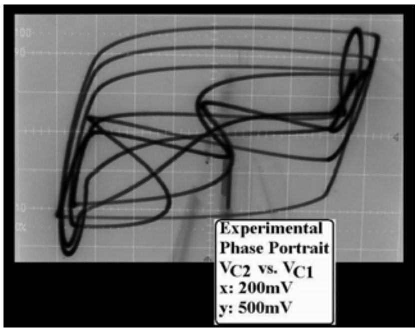


Fig.15. Experimental diagram v_{C2} vs. v_{C1} (period-6).

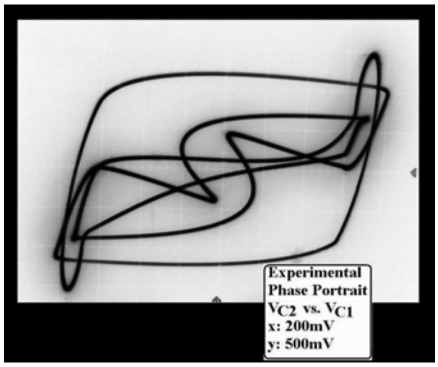


Fig.16. Experimental diagram v_{C2} vs. v_{C1} (period-3).

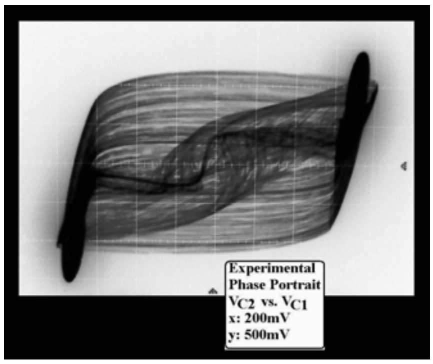


Fig.17. Experimental diagram v_{C2} vs. v_{C1} (chaos).

5 Conclusion

In this paper we have studied an autonomous fourth-order electric circuit [9-11], which constitutes of a non linear resistor R_N and a negative conductance G_n. We have observed the dynamics of the circuit for various values of the coupling resistance R_1 and different initial conditions. Using the resistance R_1 as the control parameter, we have observed antimonotonicity in a narrow region of R_1 values.

In the bifurcation diagram of Fig.4 formation of "bubbles" has been observed for some initial conditions (i.c.1). This scheme called "period-3-bubble". As the value of R_1 is increased the system remains in periodic state and no chaotic state appears. For other initial conditions (i.c.2) the behavior of the system is completely different. In the same region of R_1 values we observe (Fig.5) alternation between period and chaotic states. The creation of the "bubbles" is very sensitive to initial conditions.

Finally, we have studied the dependence of circuit's behavior on the initial conditions. The system is in periodic state for value of R_1 equal to 0.742kΩ and initial conditions 1 (Fig.6(a)), but the state of the circuit varies to quasiperiodic (Fig.6(b)), while the initial conditions are changing (i.c.2.). The coexistence at least two different attractors, drives the circuit in two different ways to chaos, but for smaller values of R_1 we have convocation in the same way.

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