On a bivariate interpolation formula

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Abstract: The aim of this paper is to introduce an interpolation bivariate formula, which generalizes the univariate Hermite interpolation formula for an arbitrary set of points. This formula is obtained using a generalization of the tensorial product method. Two expressions for the remainder are given too. The main steps of the method are presented using a numerical case. The comparatione between this method and other methods which generalize Hermite univariate interpolation formula emphasizes that one of the main advantage of our method is the constructibility.

Key–Words: Hermite interpolation, interpolation remainder, functionals, parallel calculus

1 Introduction

The real processes and phenomena can almost always be described using functions of two or three variables. The expression of these functions is not known. We know only some information about these functions and the modelling process requires the construction of some approximations of these functions using the known information. One of the approximation method is represented by the polynomial interpolation method. The polynomial interpolation of functions in several variables is more difficult than in the univariate case be cause the dimensions of the polynomial subspaces does not cover the entire set of natural numbers, N.

$$dim\,\Pi_n^d = \begin{pmatrix} n+d \\ d \end{pmatrix}$$

We denoted by Π_n^d the space of d-variate polynomials of degree n.

Let $\mathcal{F}$ be a space of functions which includes polynomials, $f \in \mathcal{F}$ an arbitrary function and let $\Lambda = \{\lambda_i | i = 1, \ldots, m\}$ be a set of linear functionals, linear independent, which represent the interpolation conditions. The general problem of the polynomial interpolation is to find a polynomial which matches $f \in \mathcal{F}$ on Λ, that is

$$\lambda_i(f) = \lambda_i(p),\ \forall i = 1, \ldots, m. \qquad (1)$$

Another problem is to find an interpolation space, that is a polynomial subspace, $\mathcal{P}$, such that there is an unique polynomial in this subspace with the property (1). The algebraic method, which consists in finding the coefficients of the interpolation polynomial by using the interpolation conditions can not be generally applied directly, be cause the interpolation space is not known apriori.

Very important cases in interpolation are the Lagrange and Hermite-Birkhoff ones. In the first case the interpolation conditions are given by the evaluation functionals on a set of distinct points and in the second one we use some derivatives of function.

The aim of this paper is to present a method for obtaining the polynomial interpolation in the case of a set of Hermite type conditions, given on a set of arbitrary points in $\mathbb{R}^2$.

This method is a generalization of the tensorial product method introduced by Gordon in [6]. In the classical form, this method works for the rectangular and triangular domains.

We also briefly presented two other methods which can be applied for the conditions that we considerate, in order to make a comparatione with our method. One of these method is the least interpolation introduced by C. de Boor and A. Ron in [2] and the other one was introduced by T. Sauer and Y. Xu in [7].

2 A generalization of univariate Hermite interpolation

2.1 The interpolation formula

In the space of bivariate functions, we introduce an interpolation problem which generalize Hermite univariate interpolation problem.

Let be $\Theta = \{\theta_i = (x_i, y_i) | i = 0, \ldots, n\} \subset \mathbb{R}^2$ a set of arbitrary points. We take the set, Λ, of interpolation conditions associated to these points:

$$\Lambda = \bigcup_{k=0}^{n} \Lambda_{\theta_k} \tag{2}$$

$$\Lambda_{\theta_k} = \left\{ \delta_{\theta_k} \circ D^{\alpha} | \alpha = (\alpha_1, \alpha_2) \in I_{\theta_k} \subset N^2 \right\} \tag{3}$$

with I_{θ_k} being a lower set. We denoted by δ_{θ_k} the evaluation functional at the point θ_k. In order to generalize the univariate Hermite interpolation conditions, we consider I_{θ_k} as a lower set, that is, if $\alpha \in I_{\theta_k}$, then $\beta \in I_{\theta_k}, \forall \beta < \alpha$.

We organize the points from the set Θ in a way which allows us to apply a generalization of the tensorial product method, for a set of arbitrary points.

We group the points with the same abscise, from Θ, and obtain the following sets of points:

$$M_k = \{(x_k, y_{k,i}) \mid i = 0, \ldots, n_k\}, \tag{4}$$

$k = 0, \ldots, m$.

Obviously, $\sum_{k=0}^{m} (n_k + 1) = n + 1$.

Remark 1 *Any arbitrary set of points, $\Theta \subset \mathbb{R}^2$, can be represented as $\Theta = \bigcup_{k=0}^{m} M_k$, with M_k given in (4) and $M_i \neq M_k$, $\forall i \neq k$.*

We will use the notations:

$$\gamma_{k,j} = \max\{\alpha_1 \mid (\alpha_1, \alpha_2) \in I_{(x_k, y_{k,j})}\} \tag{5}$$

$$\beta_{k,j} = \max\{\alpha_2 \mid (\alpha_1, \alpha_2) \in I_{(x_k, y_{k,j})}\} \tag{6}$$

$$r_k = \max\{\gamma_{k,i} \mid i = 0, \ldots, n_k\} \tag{7}$$

$$\mu_{k,j,a} = \begin{cases} \max\{\alpha_2 \mid (a, \alpha_2) \in I_{(x_k, y_{k,j})}\}, \\ \quad \text{for } j \in \{0, \ldots, \gamma_{k,j}\} \\ -1, \\ \quad \text{for } j \in \{\gamma_{k,j} + 1, \ldots, r_k\} \end{cases} \tag{8}$$

$a \in \{0 \ldots, \gamma_{k,j}\}$.

We use two levels of approximation. In the first level we make an approximation with respect to the variable x, using as knots of interpolation, the $m + 1$ distinct abscises of the points from Θ. The multiplicity order of the knot x_k is $r_k + 1$, with

$$r_k = \max\{\alpha_1 \mid (\alpha_1, \alpha_2) \in I_{\theta_i}; \theta_i \in M_k\} = \tag{9}$$

$$= max\{\gamma_{k,j} \mid j = 0, \ldots, n_k\}$$

We obtain a number of $q + 1$ interpolation conditions:

$$q + 1 = \sum_{k=0}^{m} (r_k + 1) \tag{10}$$

The univariate Hermite interpolation operator H_q^x applied to f, with respect to x gets:

$$f(x, y) = \sum_{k=0}^{m} \sum_{j=0}^{r_k} h_{k,j}(x) \cdot f^{(j,0)}(x_k, y) + (R_q^x f)(x, y) \tag{11}$$

with

$$h_{k,j}(x) = \tag{12}$$
$$= \frac{(x - x_k)^j}{j!} u_k(x) \sum_{l=0}^{r_k - j} \frac{(x - x_k)^l}{l!} \left(\frac{1}{u_k(x)} \right)_{x = x_k}$$

$$u(x) = \prod_{k=0}^{m} (x - x_k)^{r_k + 1} \tag{13}$$

$$u_k(x) = \frac{u(x)}{(x - x_k)^{r_k + 1}} \tag{14}$$

For the functions $f^{(j,0)}(x_k, y)$, $j = 0, \ldots, r_k$, we will apply another interpolation formula, with respect to y. Taking into account that I_{θ_i} are lower sets, the interpolation conditions will include for the point $(x_k, y_{k,i})$ the derivatives $D^{(a,b)}$, with $a \leq \gamma_{k,i}$ and $b \leq \mu_{k,i,a}$.

We construct the sets:

$$Y_{k,j} = \{y_{k,i} \in M_k \mid \gamma_{k,i \geq j}\}; \tag{15}$$

$j \in \{0, \ldots, r_k\}$, $k \in \{0, \ldots, m\}$.

We approximate the function $f^{(j,0)}(x_k, y)$ using an univariate Hermite operator on the knots $Y_{k,j}$, having the orders of multiplicity $\mu_{k,i,j} + 1$, with $\mu_{k,i,j}$ given in (8).

Let denote by:

$$Z_{k,j} = \{i \mid y_{k,i} \in Y_{k,j}\} \tag{16}$$

The number of interpolation conditions is $p_{k,j} + 1$, with

$$p_{k,j} + 1 = \sum_{i \in Z_{k,j}} (\mu_{k,i,j} + 1) \tag{17}$$

Thus, we obtain the formula:

$$f^{(j,0)}(x_k, y) = \tag{18}$$

$$= \sum_{i\in Z_{k,j}} \sum_{s=0}^{\mu_{k,i,j}} g_{k,j,i,s}(y) f^{(j,s)}(x_k, y_{k,i}) + (R^y_{p_{k,j}} f)(x, y),$$

with

$$g_{k,j,i,s}(y) = \frac{(y-y_{k,i})^s}{s!} v_{k,j,k,i}(y)\cdot \tag{19}$$

$$\cdot \sum_{l=0}^{\mu_{k,i,j}} \frac{(y-y_{k,i})^l}{l!} \left(\frac{1}{v_{k,j,k,i}(y)}\right)^{(l)}_{y=y_{k,i}}$$

$$v_{k,j}(y) = \prod_{i\in Z_{k,j}} (y-y_{k,i})^{\mu_{k,i,j}+1} \tag{20}$$

$$v_{k,j,k,i}(y) = \frac{v_{k,j}(y)}{(y-y_{k,i})^{\mu_{k,i,j}+1}} \tag{21}$$

From (11) and (18) the following formula holds

$$f(x, y) = \tag{22}$$

$$= \sum_{k=0}^{m} \sum_{j=0}^{r_k} \sum_{i\in Z_{k,j}} \sum_{s=0}^{\mu_{k,i,j}} h_{k,j}(x) g_{k,j,i,s}(y) f^{(j,s)}(x_k, y_{k,i}) +$$

$$+ (Rf)(x, y)$$

The remainder in the formula (22) is given by

$$(Rf)(x, y) = (R^x_q f)(x, y) + \tag{23}$$

$$+ \sum_{k=0}^{m} \sum_{j=0}^{r_k} h_{k,j}(x) (R^y_{p_{k,j}} f)(x, y)$$

Theorem 1 *The formula (22) solves the general Hermite interpolation problem given by the conditions (3) for an arbitrary set of points from* $\mathbb{R}^2$.

Next, we will give two expressions of the remainder. To do this we need the following notations:

$$a = min\{x, x_0, \ldots, x_m\},\ b = max\{x, x_0, \ldots, x_m\},$$

$$c = min\{y, y_0, \ldots, y_n\},\ d = max\{y, y_0, \ldots, y_n\}.$$

We supposed that $x_0, \ldots, x_m$ are the distinct abscises of the points from Θ.

We will use the values q and $p_{k,j}$ given in (10) and (17). We will denote $p_k = \max\{p_{k,j} \mid j = 0, \ldots, r_k\}$.

We supposed that $y_{k_0} < y_{k_1} < \ldots < y_{k_{n_k}}$.

Theorem 2 *If* $f^{(q,0)}(\cdot, y) \in C[a, b]$ *and exists* $f^{(q+1,0)}(\cdot, y)$ *on* (a, b)*, for every* $y \in [c, d]$ *and if* $f^{(0,p_k)}(x_k, \cdot) \in C[c, d]$ *and exists* $f^{(0,p_k+1)}(x_k, \cdot)$ *on* (c, d) *for every* $k \in \{0, \ldots, m\}$*, then the remainder in the interpolation formula (22) is given by*

$$(Rf)(x, y) = \frac{u(x)}{(q+1)!} f^{(q+1,0)}(\xi, y) + \tag{24}$$

$$+ \sum_{k=0}^{m} \sum_{j=0}^{r_k} h_{k,j}(x) \frac{v_{k,j}(y)}{(p_{k,j}+1)!} f^{(0,p_{k,j}+1)}(x_k, \eta_k)$$

with $\xi \in (a, b)$ *and* $\eta_k \in (y_{k,0}, \ldots, y_{k,n_k})$.

Proof: We use the formula for the remainder in Hermite univariate interpolation formula (see [9]) and the expression (23). □

Theorem 3 *In the hypothesis from the theorem 2 the remainder in the interpolation formula (22) is given by*

$$(Rf)(x, y) = \int_a^b \varphi_q(x, s) f^{(q+1,0)}(s, y) ds + \tag{25}$$

$$+ \sum_{k=0}^{m} \sum_{j=0}^{r_k} h_{k,j}(x) \int_{y_{k,0}}^{y_{k,n_k}} \xi_{p_{k,j}}(y, t) f^{(0,p_{k,j}+1)}(x_k, t) dt,$$

with

$$\varphi_q(x, s) = \frac{1}{q!} \{(x-s)^q_+ - \tag{26}$$

$$- \sum_{k=0}^{m} \sum_{j=0}^{r_k} h_{k,j}(x) \left[(x_k - s)^q_+\right]^{(j)}\}$$

and

$$\xi_{p_{k,j}}(y, t) = \frac{1}{p_{k,j}!} \left\{(y-t)^{p_{k,j}}_+ - \right. \tag{27}$$

$$\left. - \sum_{i\in Z_{k,j}} \sum_{s=0}^{\mu_{k,i,j}} g_{k,j,i,s}(y) [(y_{k,i} - t)^{p_{k,j}}_+]^{(s)} \right\}$$

Proof: We use the Peano theorem in order to obtain the remainder in Hermite univariate interpolation formula (see [9]) and then we replace it in (23). □

Proposition 1 *Let be* Hf *the interpolation polynomial in formula (22). The degree of this polynomial is*

$$deg(Hf) = q + \tag{28}$$

$$+ \max\{p_{k,j} \mid k = 0, \ldots, m;\ j = 0, \ldots, r_k\},$$

with q *given in (10) and* $p_{k,j}$ *given in (17).*

Proof: From (22) we obtain:

$$deg(Hf) = \max\{deg(h_{k,j}) + deg(g_{k,j,i,s})\},$$

$k \in \{0, \ldots, m\},\ j \in \{0, \ldots, r_k\},\ i \in Z_{k,j}$,
$s \in \{0, \ldots, \mu_{k,i,j}\}$.

The polynomials $h_{k,j}(x)$ are Hermite fundamental polynomials for an univariate interpolation problem with $q+1$ conditions and hence they all have the degree q. Similarly, for given k and j, $g_{k,j,i,s}(y)$ are Hermite univariate fundamental polynomials for an univariate interpolation problem with $p_{k,j}+1$ conditions and hence they have the degree $p_{k,j}$. □

We can give a graphical representation of our method, using a triorthogonal system of axes (see fig. 1). The points from the set Θ are situated in the xOy plan. The parallels to Oy through the points $(x_k, 0, 0)$ represent the sets M_k. In the plan xOz, on the parallels to Oz through the points $(x_k, 0, 0)$ are represented the values α_1, from the derivatives $D^{(\alpha_1,\alpha_2)}(x_k, y_{k,j})$, $j \in \{0, \ldots, n_j\}$, that is, the values $\{0, \ldots, r_k\}$.

The conditions for the first level of approximation can be seen in the plan xOz. The second level of approximation is represented by the plans parallel with yOz, through the points $(x_k, 0, 0)$. In these plans, on the parallels to Oz through the points $(x_k, y_{k,j}, 0)$, $j \in \{0, \ldots, n_j\}$ are represented the values α_2, from the derivatives $D^{(\alpha_1,\alpha_2)}(x_k, y_{k,j})$, $j \in \{0, \ldots, n_j\}$, which represents the conditions for the second level of approximation.

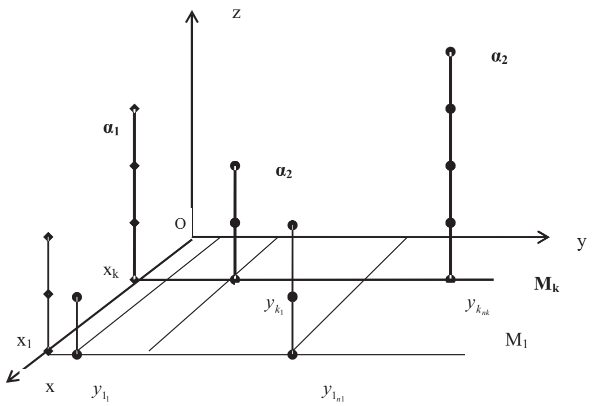

fig. 1

2.2 Particular cases

Some particular cases are enumerated below.

1. The points from Θ are situated on a rectangular grid.
   In this case $\#M_0 = \ldots = \#M_m$, that is $n_1 = \ldots = n_m$.

2. The points from Θ are situated on a triangular grid.
   In this case $n_{k+1} = n_k + 1$, $\forall k = 1, \ldots, m-1$ and $n_0 = 1$.

3. All abscises x_k, $k = 0, \ldots, m$ are distinct.
   In this case, $n_0 = \ldots = n_m = 1$ and in the second level of approximation, only operators of Taylor type are used, that is, we use for $f^{(j,0)}(x_k, y)$ the approximation

$$f^{(j,0)}(x_k, y) \simeq \sum_{s=0}^{\mu_{k,j}} \frac{(y-y_k)^s}{s!} f^{(j,s)}(x_k, y_k)$$

   We denoted $\mu_{k,1,j} = \mu_{k,j}$.

4. There are many sets M_k with $\#M_k >> 1$.
   In this case, taking into account the graphical representation of the approximation process, it is efficient to use a parallel algorithm for the calculus of the interpolation polynomial. Let suppose that we have $m+2$ processors. One processor works in the plan xOz and computes the $(r_k+1)(m+1)$ univariate polynomials $h_{k,j}(x)$, using a software which can make symbolical calculus with polynomials (MATLAB for example). In every plan parallel with yOz, which corresponds to a set M_k, works one processor which calculates the polynomials $g_{k,j,i,s}(y)$, for the given value of k.

5. The set of conditions is $\Lambda = \{\delta_{\theta_k} \mid k = 0, \ldots, n\}$.
   This is the Lagrange case and we obtain the formula given in [9].

2.3 Application

Let be $\Theta = \{\theta_i \mid i = 0, \ldots, 4\}$, with $\theta_0 = (0,0)$, $\theta_1 = (0,1)$, $\theta_2 = (1,0)$, $\theta_3 = (2,1)$, $\theta_4 = (2,2)$ and the interpolation conditions given by the sets I_{θ_i}: $I_{\theta_0} = \{(0,0), (1,0), (0,1)\}$, $I_{\theta_1} = \{(0,0), (1,0), (2,0), (0,1)\}$, $I_{\theta_2} = \{(0,0)\}$, $I_{\theta_3} = \{(0,0), (1,0), (0,1), (1,1)\}$, $I_{\theta_4} = \{(0,0), (1,0)\}$.

We will identify and compute the main elements in the formula (22).

First, we form the sets M_k: $M_0 = \{\theta_0, \theta_1\}$, $M_1 = \{\theta_2\}$, $M_2 = \{\theta_3, \theta_4\}$. Therefore we have: $n = 4$, $m = 2$, $r_0 = 2$, $r_1 = 0$, $r_2 = 1$, $q = 5$ and $x_0 = 0$, $n_0 = 1$, $y_{0,0} = 0$, $y_{0,1} = 1$, $\gamma_{0,0} = 1$, $\beta_{0,0} = 1$, $\gamma_{0,1} = 2$, $\beta_{0,1} = 1$, $\mu_{0,0,0} = 1$, $\mu_{0,0,1} = 0$, $\mu_{0,0,2} = -1$, $\mu_{0,1,0} = 1$, $\mu_{0,1,1} = 0$, $\mu_{0,1,2} = 0$;

$x_1 = 1$, $n_1 = 0$, $y_{1,0} = 0$, $\gamma_{1,0} = 0$, $\beta_{1,0} = 0$, $\mu_{1,0,0} = 0$;
$x_2 = 2$, $n_2 = 1$, $y_{2,0} = 1$, $y_{2,1} = 2$, $\gamma_{2,0} = 1$, $\beta_{2,0} = 1$, $\gamma_{2,1} = 1$, $\beta_{2,1} = 0$, $\mu_{2,0,0} = 1$, $\mu_{2,0,1} = 1$, $\mu_{2,1,0} = 0$, $\mu_{2,1,1} = 0$.

The degree of the fundamental Hermite polynomials $h_{k,j}(x)$, $k \in \{0,1,2\}$, $j \in \{0,\ldots,r_k\}$ is $q = 5$.

In the second level we have to approximate the functions: $f^{(0,0)}(x_0,y)$, $f^{(1,0)}(x_0,y)$, $f^{(2,0)}(x_0,y)$, $f^{(0,0)}(x_1,y)$, $f^{(0,0)}(x_2,y)$, $f^{(1,0)}(x_2,y)$. In order to do this we must define the sets $Y_{k,j}$, $Z_{k,j}$ and compute the values $p_{k,j}$.
$Y_{0,0} = \{y_{0,i} \mid \gamma_{0,i} \geq 0\} = \{y_{0,0}, y_{0,1}\}$, $Z_{0,0} = \{0,1\}$, $p_{0,0} = 3$. Hence, the function $f^{(0,0)}(x_0,y)$ will be approximated using a polynomial of degree 3:
$f^{(0,0)}(x_0,y) \simeq g_{0,0,0,0}(y)f^{(0,0)}(x_0,y_{0,0}) + g_{0,0,0,1}(y)f^{(0,1)}(x_0,y_{0,0}) + g_{0,0,1,0}(y)f^{(0,0)}(x_0,y_{0,1}) + g_{0,0,1,1}(y)f^{(0,1)}(x_0,y_{0,1})$
Similarly, $Y_{0,1} = \{y_{0,i} \mid \gamma_{0,i} \geq 1\} = \{y_{0,0}, y_{0,1}\}$, $Z_{0,1} = \{0,1\}$, $p_{0,0} = 1$. Hence, the function $f^{(1,0)}(x_0,y)$ will be approximated using a polynomial of degree 1:
$f^{(1,0)}(x_0,y) \simeq g_{0,1,0,0}(y)f^{(1,0)}(x_0,y_{0,0}) + g_{0,1,1,0}(y)f^{(1,0)}(x_0,y_{0,1})$
$Y_{0,2} = \{y_{0,i} \mid \gamma_{0,i} \geq 2\} = \{y_{0,1}\}$, $Z_{0,2} = \{1\}$, $p_{0,2} = 0$. In this case, we will use for approximation the Taylor polynomial of degree 0, that is
$f^{(2,0)}(x_0,y) \simeq f^{(2,0)}(x_0,y_{0,1})$
The other approximations from the second level will be found in the same way.

2.4 Other generalization of Hermite univariate interpolation

In the field of interpolation of functions in several variables, there are many directions regarding the generalization of Hermite univariate interpolation. We will present here two of them, in order to make a comparatione with our method.

In [2] is introduced an interpolation scheme named "least interpolation", which supplies a minimal polynomial interpolation space for an arbitrary set of conditions, given by a set Λ of linear functionals, linear independent. We will briefly present this method.

Let be $\lambda \in \Lambda$ and λ^ν the generating function of λ.

$$\lambda^\nu(z) = \lambda(e_z(x)), \tag{29}$$

with $e_z(x) = e^{z\cdot x}$, $z, x \in R^d$ and $z \cdot x$ being the euclidian inner product between x and z.

For an analytical function f, we will denote by $f\!\downarrow$ the least term of f, that is the nonzero homogeneous component of least degree in the Taylor series of f.

We will use the following notations

$$H_\Lambda = span\{\lambda^\nu;\ \lambda \in \Lambda\};\ H_\Lambda\!\downarrow = span\{g\!\downarrow; g \in H_\Lambda\} \tag{30}$$

In [2] is proved that $H_\Lambda\!\downarrow$ is a minimal interpolation space for the conditions Λ. For the generalization of Hermite interpolation conditions, in [1] is used the set of functionals

$$\Lambda = \{\lambda_{q,\theta}\,|\lambda_{q,\theta}(p) = (q(D)p)\,(\theta)\,\} \tag{31}$$

$q \in \mathcal{P}_\theta$; $\theta \in \Theta \subset R^2$; $\mathcal{P}_\theta \subset \Pi^2$.
We denoted by $q(D)$ the constant coefficients operator associated to the polynomial q. If $q = \sum\limits_{|\alpha|\leq deg(q)} c_\alpha x^\alpha$, $\alpha \in N^d$, $|\alpha| = \alpha_1 + \ldots + \alpha_d$, then $q(D) = \sum\limits_{|\alpha|\leq deg(q)} c_\alpha D^\alpha$. If the polynomial spaces $\mathcal{P}_\theta$ are scalar and D-invariant, then, according [1], the conditions (31) represent a generalization of Hermite interpolation conditions. Using (30) it is easy to obtain that

$$H_\Lambda = \sum_{\theta\in\Theta} e_\theta \mathcal{P}_\theta;\ H_\Lambda\!\downarrow = \left(\sum_{\theta\in\Theta} e_\theta \mathcal{P}_\theta\right)\!\downarrow \tag{32}$$

The conditions Λ given in (3) represent a particular case of (31). In order to prove that we choose

$$\mathcal{P}_\theta = span\{x^\alpha \mid \alpha \in I_{\theta_k}\}.$$

We proved in [8] that if I_{θ_k} is a lower set than the space $\mathcal{P}_\theta$ is scalar and D-invariant. Therefore our interpolation conditions given in (3) represent a particular case of Hermite interpolation problem introduced by C. de Boor and A. Ron.

A very important property of an interpolation scheme is the constructibility, that is the possibility of obtaining, using numerical methods, the interpolation polynomial and the interpolation space. A method used in least interpolation for obtaining the interpolation space and the interpolation polynomial is the Gauss elimination by segments method. More details about this method can be found in [3]. This method was implemented only in the Lagrange case, be cause many difficulties appear in the case of a set of arbitrary functionals. The main difficulty consists of the segmentation of the matrix used in elimination process, such that a recurrence relation, which allows the crossing from one segment to the next one, could be establish.

In [7] is taking into account the general case of Hermite multivariate interpolation, consisting of interpolation of consecutive chains of directional derivatives. The definition of Hermite interpolation, introduced in [7] is very general and includes the case presented in subsection 2.1, but theoretical results regarding the form of the interpolation polynomial and of the remainder are given only for a particular case, named ”blockwise Hermite interpolation” and the constructibility aspects were not be discussed. Be cause the method introduced in [7] involves a complex terminology and notation we can not give more details about it here. A study of the interpolation problem given by the conditions (2) using the terminology from the article [7], can be found in [8].

3 Conclusion

The main advantage of the interpolation method presented in this article is the constructibility, that is the possibility of implementing an algorithm for the construction of the interpolation polynomial. In the particular case 4, presented in subsection 2.2 the execution time of the algorithm can be improved by using parallel calculus.

The method can be generalize for the conditions of Birkhoff type. This generalization suppose the replacing of Hermite fundamental polynomials $h_{k,j}$ with Birkhoff fundamental polynomials and the reconstruction of the sets $Y_{k,j}$ in a more general way.

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