Algebraic Implementation of CTL Model Checker

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Abstract: In this work, is presented the methodology and supporting sets of tools, what permitted the possibility automatically generate of the algorithms verify the models for temporal logics from a set of algebraic specifications. This fact is of great deal to the research of suggested aim. The development of temporal logics and the model checking the algorithms can be used to the verification property of system. These are used in to specify model checkers as the mappings of the form $\mathcal{L}_s \rightarrow \mathcal{L}_t$ where $\mathcal{L}_s$ is a temporal logic source language and $\mathcal{L}_t$ is a target language represents sets a state of the model $M$ as $\mathcal{L}_s (f \in \mathcal{L}_s) = \{s \in M \mid s \models f\}$. Is noticed how this algebraic framework can be used to specify of the model checking algorithms for CTL (Computation Tree Logic). The paper [10] is the base of results obtained.

Key-Words: CTL, algebraic specification, model checkers, temporal logics, algebraic structure, directed graph, implementation

1 Introduction

Temporal logic is used to express qualitative [2] the properties of the system. The model checkers are tools which can be used to verify that a given system satisfies a given temporal logic formula. The certified system may be a physical system, or concurrent either an interactive program as behave is described of the Kripke model [6]. The model is a directed graph where the nodes represent the states of the system and the edges represents the state transitions. The nodes and the edges can be labeled with atomic propositions what describe the states and the transitions of the system. A property were a critical section, and to executed in the critical section. The atomic propositions $T_i, N_i$ and $C_i$ denote, respectively, process $i, 1 \leq i \leq 2$, try to enter into critical section, not to enter into critical section, and to executed in the critical section.

The CTL formulas are defined by the following rules [2]:

1. The logical constants $true$ and $false$ are CTL formulas.
2. Every atomic proposion, $ap \in AP$ is a CTL formula.
3. If $f_1$ and $f_2$ are CTL formulas, then so are $\neg f_1$, $f_1 \land f_2$, $f_1 \lor f_2$, $EX f_1$, $AX f_1$, $E[f_1 U f_2]$, and $A[f_1 U f_2]$.

A model is defined [2] as a directed graph $M=\langle S,E, P:AP \rightarrow 2^S \rangle$ where $S$ is a finite sets of states also called nodes, $E$ is a finite sets of directed edges, and $P$ represents proposition labeling function which labels each nodes with logical proposition. For each $s \in S$, use the notation $\text{succ}(s) = \{s' \in S \mid (s,s') \in E\}$. Each state in $E$ must have at least one successor, is $\forall s \in S$, $\text{succ}(s) \neq \emptyset$. A path in $M$ is a infinite sequence of states $(s_0, s_1, s_2, ...)$ such that $\forall i, i \geq 0$, we have $(s_i, s_{i+1}) \in E$. The labeling function $P$ maps an atomic proposition in $AP$ to the set of states in $S$ on which sentences is $true$. The Figure 1 exhibits a model [2] the behavior two processes competing for the in an entrance the critical section. The atomic propositions $T_i, N_i$ and $C_i$ denote, respectively, process $i, 1 \leq i \leq 2$, try to enter into critical section, not to enter into critical section, and to executed in the critical section.

In this section is delivered a traditional definition of CTL and then given an algebraic description of CTL.

The CTL formulas are defined by the following rules [2]:

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Just as defined in [2], the satisfaction relation, $\models$, of a CTL formula, $f$ on a state $s$ in a model $M$ is denoted $M,s \models f$ and is read "$s$ satisfies $f$". The satisfied set of a formula $f$ across a model $M$ is the set $\{s \in S | M,s \models f\}$. The satisfaction of logical constants is defined as $\forall s \in S, s \models \text{true}$ and $\exists s \in S, s \models \text{false}$. The relation $\models$ is defined as follows:

$$
\begin{align*}
&\text{true} \models \text{true}, \text{false} \models \text{false}, \neg f \models \neg s \models \text{true}, \\
f_1 \land f_2 \models s \models f_1, s \models f_2, \\
f_1 \lor f_2 \models s \models f_1 \lor f_2, \\
\exists t . f \models s \models \exists t . f, \\
\forall t . f \models s \models \forall t . f, \\
f \models \langle o \rangle t . f \models s \models \langle o \rangle t . f, \\
f \models \langle o \rangle \lor f \models s \models \langle o \rangle \lor f, \\
f \models \langle o \rangle \land f \models s \models \langle o \rangle \land f, \\
\exists i . f \models s \models \exists i . f, \\
\forall i . f \models s \models \forall i . f.
\end{align*}
$$

Many model checking algorithms were developed for different temporal logics [2], thus in this paper is presented a simple universal algorithm based on the algorithm of homeomorphism computation which is used in an algebraic compiler [8]. The generic homeomorphism algorithm is customized by a set of specifications to implement a model checkers, implemented as an algebraic compiler $C : L_c \rightarrow L_t$, the source language $L_c$ is the language of temporal logic and the target language $L_t$ is the language of sets of states of a given models $M$. The algebraic compiler translates temporal logic formulas their in of satisfy $C(f) = \{s \in S | M, s \models f\}$. The primary advantage of this approach is that the model checking algorithm can be automatically generated by the tools from its specifications. These specifications consist of a finite set of rules in which each the rule defines the syntax of a class of constructs in the source language and the semantic values these constructs as the expressions in the syntax of the target language.

## 2 Algebraic description of CTL

The source and the target language used in an algebraic compiler $C : L_c \rightarrow L_t$ are defined using heterogeneous $\Sigma$-algebras and $\Sigma$-homeomorphisms [5]. The operator scheme of a $\Sigma$-algebras is a tuple $\Sigma=(S,O,\sigma)$ where $S$ is a finite set of sorts, $O$ is a finite set of operator names, and $\sigma : O \rightarrow \Sigma$ is a function which defines the signature of the operators. These signatures are denote as $\sigma(o) = s_1 \times s_2 \times \ldots \times s_n$ to, where $s_i \in S$, $1 \leq i \leq n$. A $\Sigma$-algebra is a tuple $A_2 = (\langle A_1 \rangle_{s \in S}, \sigma(O))$, where $\langle A_1 \rangle_{s \in S}$ is a family of non-empty sets indexed by the sorts $S$ of $\Sigma$, called the carrier sets of the algebra, and $\sigma(O)$ is a set of operations across the sets in $\langle A_1 \rangle_{s \in S}$ such that for each $o \in O$ with signature $\sigma(o) = s_1 \times s_2 \times \ldots \times s_n$ to, $\sigma(o)$ is a function $\sigma(o) : A_1 \times A_2 \times \ldots \times A_n$, $A_1$ is identified $\sigma(o)$ with $O$. A $\Sigma$-algebra is a tuple $L=(\Sigma,M,\sigma,S)$ [8], where $\Sigma$ is a $\Sigma$-algebra called the language semantics, $S$ is a $\Sigma$ word or a term algebra called the language syntax and $\sigma$ is a partial mapping called the language learning function. Is the maps semantic objects in $\Sigma$ to their expressions as the valid constructs in $\Sigma$. Defined $\Sigma$ and $\sigma$, define an operator scheme $\Sigma_{ct}$ as the tuple $(\Sigma_{ct}, \sigma_{ct}, \Sigma_{ct})$, where the sorts $\Sigma_{ct}=\{F\}$ represents the formulas: $O_{ct}=\{true, false, not, and, or, AX, EX, AU, EU\}$, and $\sigma_{ct}$ is represented in Figure 2.

### 2.1 Algebraic structure of CTL language

Defined CTL as a $\Sigma$ - language, define an operator scheme $\Sigma_{ct}$ as the tuple $(\Sigma_{ct}, \sigma_{ct}, \Sigma_{ct})$, where the sorts $\Sigma_{ct}=\{F\}$ represents the formulas: $O_{ct}=\{true, false, not, and, or, AX, EX, AU, EU\}$, and $\sigma_{ct}$ is represented in Figure 2.

<table>
<thead>
<tr>
<th>Operator</th>
<th>Description in $A_{ct}^w$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$true$</td>
<td>$true \in A_{ct}^w$</td>
</tr>
<tr>
<td>$false$</td>
<td>$false \in A_{ct}^w$</td>
</tr>
<tr>
<td>$not$</td>
<td>$not(F) \rightarrow F$</td>
</tr>
<tr>
<td>$and$</td>
<td>$f \land g \rightarrow F$</td>
</tr>
<tr>
<td>$or$</td>
<td>$f \lor g</td>
</tr>
</tbody>
</table><p>ightarrow F$ |
| $AX$    | $\forall i . f \rightarrow F$ |
| $EX$    | $\exists i . f \rightarrow F$ |
| $AU$    | $\forall i . U f \rightarrow F$ |
| $EU$    | $\exists i . U f \rightarrow F$ |</p>

Fig. 1: Model example
\begin{table}[h]
\centering
\begin{tabular}{|c|c|}
\hline
Operator & Description in $A_{\text{ctl}}^w$ \\
\hline
$EU:F \times F \rightarrow F$ & if $f_1, f_2 \in A_{\text{ctl}}^w$, then $Ef_1[f_2] \in A_{\text{ctl}}^w$ \\
\hline
\end{tabular}
\caption{The operator scheme $\Sigma_{\text{ctl}}$ in $A_{\text{ctl}}^w$}
\end{table}

$CTL$ can be defined as the $\Sigma_{\text{ctl}}$-language give in the form $L_{\text{ctl}}=\langle A_{\text{ctl}}^m , A_{\text{ctl}}^w \rangle$, $\Sigma_{\text{ctl}}: A_{\text{ctl}}^m \rightarrow A_{\text{ctl}}^w \wedge A_{\text{ctl}}^w$ is the word algebra of the operator scheme $\Sigma_{\text{ctl}}$ generated of the operations from $O_{\text{ctl}}$ and a finite sets of variables, denoting atomic propositions, $AP$. $A_{\text{ctl}}^m$ represents $CTL$ semantics algebra defined across the satisfy sets of $CTL$ formulas for a given models $M$. $L_{\text{ctl}}$ is a mapping which associates satisfiability sets in $A_{\text{ctl}}^w$ from the $CTL$ expressions in $A_{\text{ctl}}^w$ that they satisfy and $\varepsilon_{\text{ctl}}$ is a homeomorphism that evaluates $CTL$ expressions in $A_{\text{ctl}}^w$ to their satisfy sets in $A_{\text{ctl}}^m$. How long the rules for forming $CTL$ formulas are independent of any model, the signification of the resulting formulas are dependent upon a given model. Thus in the algebraic definition of $CTL$, $A_{\text{ctl}}^w$ is independent from any model while $A_{\text{ctl}}^m$ is dependent on the given model $M$.

The word algebra $A_{\text{ctl}}^w$ is unique to homeomorphism in the class of algebras with operator scheme $\Sigma_{\text{ctl}}$. This has as the carrier set $A_{\text{ctl}}^w$, the collection of $CTL$ formulas, called terms or expressions and whose semantics algebra follows: $\Sigma_{\text{ctl}}$ is the carrier set, $\langle S, \emptyset, \cup, \cap, \equiv, \text{Next}_{\text{all}}, \text{Next}_{\text{some}}, LFP_{\text{all}}, LFP_{\text{some}} \rangle$. These actions in $A_{\text{ctl}}^m$ are defined as follows:

- $S$ is the constant set of all states in $M$ and $\emptyset$ is the constant empty set.
- $\cup$ is the unary operator which produces the complement in $S$ of its argument.
- $\cap$ and $\cup$ are the binary set intersection and union operators.
- For $t \in A_{\text{ctl}}^m$, are unary operators $\text{Next}_{\text{all}}$ and $\text{Next}_{\text{some}}$ are defined by the equations $\text{Next}_{\text{all}}(t) = \{ s \in S | \text{suc}s \subseteq t \}$, and $\text{Next}_{\text{some}}(t) = \{ s \in S | \text{suc}s \cap \text{suc}\emptyset \neq \emptyset \}$, where $\text{suc}s$ denotes the successors of the state $s$ in $M$.
- $LFP_{\text{all}}$ and $LFP_{\text{some}}$ are inspired by the fixed point construction operator $Y$ [4]. For sets $LFP_{\text{all}}(t_1, z_2)$, with $t_1, z_2 \in A_{\text{ctl}}^w$, computes the least fixed point of the equation $Z = t_1 \cup \{ s \in t_1 | \text{suc}s \subseteq z \}$, and $LFP_{\text{some}}(t_1, z_2)$, with $t_1, z_2 \in A_{\text{ctl}}^w$, computes the least fixed point of the equation $Z = t_1 \cup \{ s \in t_1 | \text{suc}s \cap z \neq \emptyset \}$.

2.2 Algebraic structure of the model.

Defined as an algebraic compiler, the $CTL$ model checking algorithm maps $CTL$ formulas in the syntax algebra $A_{\text{ctl}}^w$ into set expressions of a set expression language defined by the model $M$, while preserving the satisfiability semantics of the $CTL$ formulas. In order to understand these mapping we structure the model $M = \langle S, E, P; AP \rightarrow 2^S \rangle$ as a $\Sigma$-language whose syntax algebra contains the sets of the expressions and whose semantics algebra contains the sets in $2^2$. The operator scheme for this language $\Sigma_{\text{set}}=\langle S_{\text{set}}, O_{\text{set}}, \sigma_{\text{set}} \rangle$ where $S_{\text{set}} = \{ S, B \}$, again $S$ is the sort for sets and $B$ is the sort for the boolean values, $O_{\text{set}} = \{ \emptyset, \cap, \cup, \equiv, \subseteq, \text{suc}, \neg \}$.
\[ \land, \lor \] \) and \( \sigma_{\text{sets}} \) is presented in Figure 3. The model \( M \) defined as \( \Sigma_{\text{sets}} \) - language \( L_M=\langle A_{\text{sets}}^w, A_{\text{sets}}^e, \epsilon_{\text{sets}}, A_{\text{sets}}^m \rightarrow A_{\text{sets}}^w \rangle \), where \( \epsilon_{\text{sets}}: A_{\text{sets}}^m \rightarrow A_{\text{sets}}^m \). He evaluates set expressions to the sets they represent.

In this language, \( A_{\text{sets}}^m \) is the semantic algebra with the carrier sets \( A_S^m=2^S \) and \( A_B^m=\{\text{true}, \text{false}\} \). The operators in algebra and their signatures as defined by \( \sigma_{\text{sets}} \) are shown in Figure 3. Semantic algebra \( A_{\text{sets}}^w \) and \( A_{\text{sets}}^m \) are the carrier sets in the relation \( A^w \subseteq A^m \). This permits for the identity show in the map all the elements of the carrier sets of \( A_{\text{sets}}^m \) through their occurrences in the carrier sets of \( A_{\text{sets}}^w \).

### Figure 3: Operator scheme \( \Sigma_{\text{sets}} \) in \( A_{\text{sets}}^m \)

<table>
<thead>
<tr>
<th>Operator ( \quad )</th>
<th>Description in ( A_{\text{sets}}^m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \emptyset \rightarrow S )</td>
<td>( \emptyset \in A_S^m )</td>
</tr>
<tr>
<td>( \land: S \times S \rightarrow S )</td>
<td>if ( S_1, S_2 \in A_S^m ) then ( S_1 \land S_2 \in A_S^m )</td>
</tr>
<tr>
<td>( \lor: S \times S \rightarrow S )</td>
<td>if ( S_1, S_2 \in A_S^m ) then ( S_1 \lor S_2 \in A_S^m )</td>
</tr>
<tr>
<td>( \neg: S \rightarrow S )</td>
<td>if ( S \in A_S^m ) then ( \neg S \in A_S^m )</td>
</tr>
<tr>
<td>( \equiv: S \times S \rightarrow B )</td>
<td>if ( S_1, S_2 \in A_S^m ) then ( S_1 \equiv S_2 \in A_B^m )</td>
</tr>
<tr>
<td>( \subseteq S \times S \rightarrow B )</td>
<td>if ( S_1, S_2 \in A_S^m ) then ( S_1 \subseteq S_2 \in A_B^m )</td>
</tr>
<tr>
<td>( \text{succ: } S \rightarrow S )</td>
<td>if ( S \in A_S^m ) then ( {s \in S</td>
</tr>
<tr>
<td>( \rightarrow B \times B \rightarrow B )</td>
<td>if ( b_1, b_2 \in A_B^m ) then ( b_1 \rightarrow b_2 \in A_B^m )</td>
</tr>
<tr>
<td>( \land: B \times B \rightarrow B )</td>
<td>if ( b_1, b_2 \in A_B^m ) then ( b_1 \land b_2 \in A_B^m )</td>
</tr>
<tr>
<td>( \lor: B \times B \rightarrow B )</td>
<td>if ( b_1, b_2 \in A_B^m ) then ( b_1 \lor b_2 \in A_B^m )</td>
</tr>
</tbody>
</table>

2.3 Algebraic describe of CTL model checker

We define a CTL model checker as an algebraic compiler \( MC:L_{\text{ctl}} \rightarrow L_{\text{ctl}} \) by the pair of embedding morphisms \( (T_{MC}, H_{MC}) \). \( T_{MC}: A_{\text{ctl}}^w \rightarrow A_{\text{ctl}}^w \) maps CTL formulas from word algebra \( A_{\text{ctl}}^w \) to set expressions in \( A_{\text{ctl}}^w \), which evaluate to the satisfiability sets of the CTL formulas, \( H_{MC}: A_{\text{ctl}}^m \rightarrow A_{\text{ctl}}^m \), maps sets in \( A_{\text{ctl}}^m \) by the identity morphism to sets in \( A_{\text{ctl}}^m \). The morphism \( T_{MC} \) is constructed by the following approach:

1. The associate each operation \( o_{\text{ctl}} \) from the algebra \( A_{\text{ctl}}^m \) with a set expression \( d(o_{\text{ctl}}) \) from the algebra \( A_{\text{ctl}}^w \) with the property \( H_{MC}(o_{\text{ctl}}(S_1, ..., S_d)) = \epsilon_{\text{sets}}(d(o_{\text{ctl}})(d_{\text{ctl}}(S_1), ..., d_{\text{ctl}}(S_d))) \). The variables are treated as nullary operators each \( ap \in AP \) is associated with the set expression \( P(ap) \), \( d(ap)=P(ap) \). This construction is presented in the Figure 4.

The sets expressions exists in an extension of \( A_{\text{ctl}}^w \) and contains constants sets (\( \emptyset, S, P(ap)|ap \in AP \)), set variables \( (Z, Z') \), assignment statements, and a while loop constructs. The \( d_{\text{ctl}}(t), d_{\text{ctl}}(t_1), d_{\text{ctl}}(t_2) \) are the set expressions associated with the arguments of the operators \( o \in A_{\text{ctl}}^m \).

<table>
<thead>
<tr>
<th>( o \in A_{\text{ctl}}^m )</th>
<th>( d_{\text{ctl}}(o) \in A_{\text{ctl}}^w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \epsilon_{\text{ctl}}(ap) )</td>
<td>( P(ap) )</td>
</tr>
<tr>
<td>( S )</td>
<td>( S )</td>
</tr>
<tr>
<td>( \emptyset )</td>
<td>( \emptyset )</td>
</tr>
<tr>
<td>( t )</td>
<td>( S \setminus d_{\text{ctl}}(t) )</td>
</tr>
<tr>
<td>( t_1 \land t_2 )</td>
<td>( d_{\text{ctl}}(t_1) \cap d_{\text{ctl}}(t_2) )</td>
</tr>
<tr>
<td>( t_1 \lor t_2 )</td>
<td>( d_{\text{ctl}}(t_1) \cup d_{\text{ctl}}(t_2) )</td>
</tr>
<tr>
<td>( \text{Next}_d(t) )</td>
<td>( {s \in S</td>
</tr>
<tr>
<td>( \text{Next}_\text{sw}(t) )</td>
<td>( {s \in S</td>
</tr>
<tr>
<td>( \text{LFP}_d(t_1, t_2) )</td>
<td>( Z=d_{\text{ctl}}(t_1); Z'=\emptyset; )</td>
</tr>
<tr>
<td>( \text{LFP}_\text{sw}(t_1, t_2) )</td>
<td>( Z'=d_{\text{ctl}}(t_2); Z'=\emptyset; )</td>
</tr>
<tr>
<td>( \text{LFP}_\text{sw}(t_1, t_2) )</td>
<td>( Z={s \in S</td>
</tr>
<tr>
<td>( \text{LFP}_d(t_1, t_2) )</td>
<td>( Z'={s \in S</td>
</tr>
</tbody>
</table>

2. The set expressions the second column of Figure 4 define the operations of an algebra \( A_{\text{sets}}^w \), which is similar to the operations from
$A^w_{ct}$ and from $A^w_{w}$. This algebra is generated by the operators $d(o_{ct})$ using a free generators the constants sets $d(true)$, $d(false)$, $d(ap)$, $ap \in AP$ and is isomorphic with the algebra $A^w_{ct}$. This results from the fact that they are similar and their generators are one to one through the operators in $O_{ct}$. Using the unique extension lemma [8] this one to one mapping on the generators extends to a unique isomorphism $T'_{MC}: A^w_{ct} \rightarrow A^w_{sets}$.

3. $A^w_{sets}$ is a subalgebra of $A^w_{sets}$, the embedding $T_{MC}: A^w_{ct} \rightarrow A^w_{sets}$ is constructed by the composition of $T'_{MC}$ and the injection function $I: A^w_{sets} \rightarrow A^w_{sets}$, with $T_{MC}= T'_{MC} \circ I$.

The morphisms $T_{MC}$ and $H_{MC}$ thus constructed are presented in the diagram from Figure 5. The diagram is commutative. Commutatively assures the fact as $T_{MC}$ keeps the meaning of the formulas from $A^w_{ct}$ when mapping them to set expressions in $A^w_{sets}$. The diagonal mapping $D_{ct}: A^w_{sets} \rightarrow A^w_{sets}$ in Figure 5 is generated by $d_{ct}$ defined in Figure 4 and show the translation process performed by $T_{MC}$ using derived operations [3, 8].

2.4 Algebraic implementation of CTL model checker

Construction of $T_{MC}$ in the Section 2.3 can be entered into an algorithm which implements the CTL model checker. This algorithm is universal in the sense that given operator scheme $\Sigma_{ct}$ and a model $M$ the model checker of $L_{ct}$ is automatically generated from the specifications of $(\Sigma_{ct}, D_{ct})$. This specification is obtained by associating each operation $o \in O_{ct}$ with an operation a derivative $d_{ct}(o) \in D_{ct}$. To define the derived operations that implement the operations of $\Sigma_{ct}$ from the algebra $A^w_{sets}$ using meta-variables that take as values set expressions of the carrier sets of $A^w_{sets}$. For each operation $o \in O_{ct}$ such that $\sigma_{ct}(o)=s_1 \times ... \times s_n \rightarrow s$, $d_{ct}(o)$ takes it as the formal parameters the meta-variables denoted by $@_i$, $1 \leq i \leq 2$, where $@_0$ denotes the set expression associated with $i$ - th argument of $d_{ct}(o)$; the meta-variable $@_1$ is used to denote the resulting set expression, as example $@[o]=d_{ct}(o)(@_i,..., @_o)$.

Since it is not always possible expressed the semantics of all of the operations from $\Sigma_{ct}$ using of the compositions of the operations from $A^w_{sets}$ are written the derivative operations using a semantics expression language which includes the set operations in $A^w_{sets}$ and extends it with local set variables, set assignment statements and looping constructs. The derivative operations written in this extended semantics expression language are called macro-operations.

Formally $T_{MC}$ is implemented by associating with each the operation $o \in O_{ct}$ from $A^w_{ct}$ a parameterized macro-operation denoted by $d(o)$, that is constructed by the following rules:

1. Define the macro-operations for the generators of the $A^w_{ct}$ by assigning $d(true)=S$ and $d(false)=\emptyset$, and for each $ap \in AP$, $d(ap)=P(ap)$.

2. Embed the generators of $A^w_{ct}$ in $A^w_{sets}$ by the function:

$T_{MC}(true)=d(true)$, $T_{MC}(false)=d(false)$, and $\forall ap \in AP$, $T_{MC}(ap)=d(ap)$.

3. Extend the function defined in (2) above to the entire algebra $A^w_{w}$ by the equality:

$\forall f \in A^w_{ct}$ with $f=o(f_1,...,f_k)$,

$T_{MC}(f)(o)(T_{MC}(f_1),..., T_{MC}(f_k))$. Where through $T_{MC}$ is the unique extension of the second function to a homeomorphism, $T_{MC}$ is well-defined. This homeomorphism maps CTL formulas into set expressions which evaluate to the satisfiability sets of the formulas in the model $M$ defining the target language $L_{ct}$. The general aim of the algorithm for computing of the image of a formula $f$ in $A^w_{ct}$ under the embedding $T_{MC}$:

$A^w_{ct} \rightarrow A^w_{sets}$ assumes the existence of the following:

1. The function $S: A^w_{ct} \rightarrow S_{ct} \cup \{\text{none}\}$ recognizes the generators of $A^w_{ct}$ defined by:

$S(f)=\begin{cases} 
    s, & \text{if } f \text{ is a generator in carrier set } A^w_{s} \\
    \text{none}, & \text{otherwise}
\end{cases}$

Fig. 5: $\Sigma$ - language $L_{ct}$ and $L_M$ and the model checker $MC: L_{ct} \rightarrow L_{sets}$.
2. The function
\[ \mathcal{P} : A^w_{\text{ctl}} \setminus AP \rightarrow \bigcup_{n=0}^{\infty} O_{\text{ctl}} \times A^w_{s_1} \times \ldots \times A^w_{s_n} \]
recognizes CTL formulas \( f \in A^w_{\text{ctl}} \) in terms of their constructions operator and their component sub formulas, that is if \( f = \circ(f_1, \ldots, f_n) \) then \( \mathcal{P}(f) = (\circ, (f_1, \ldots, f_n)) \).

\( \mathcal{S} \) is a scanning algorithm that is automatically generated from regular expressions of conditions [9] specifying the generators of \( A^w_{\text{ctl}} \) and \( \mathcal{P} \) is a parser that determines which operation and which sub formulas where used to create the formula \( f \). Since \( \mathcal{P} \) is compositional we use a pattern-matching parser [7] for the suggested aim. For each \( f \in A^w_{\text{ctl}} \), \( s \in S_{\text{ctl}} \), the behavior of the algorithm that implements \( T_{\text{MC}} \) is described by the following recursive functions:

\[
T_{\text{MC}}(f) = \begin{cases} 
S(f) = s, s \in S_{\text{ctl}} & \text{then } d(f) \\
\text{else if } \mathcal{P}(f) = (\circ, (f_1, \ldots, f_n)) & \text{then } d(\circ)(T_{\text{MC}}(f_1), \ldots, T_{\text{MC}}(f_n)) 
\end{cases}
\]

We improve the efficiency of a model checker by replacing the operations of expressions set construction by functions which evaluate the incoming set expressions and thus generates sets that are the values of the set expressions.

3. Conclusion.
The behavior of the model checker algorithm demonstrated in the section 2.4 consists of identifying the sets of states of a model \( M \) whence satisfy each of a sub formula of a given CTL formula \( f \) and constructing the set of states, from these sets, that satisfy the formula \( f \). This is certainly the behavior of the algorithm for the homeomorphism computation performed by an algebraic compilers; Thus is evaluated an expression by repeatedly identifying its generating sub expressions and replacing them with their images in the target algebra. In the case of the model checking algorithm, sub expressions are CTL sub formulas and their images are the sets of states in the model satisfies the sub formulas.

References: