

# The Performances of Different Rotor Flux Observers

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*Abstract:* - In this paper is presented a comparison of performances between three rotor flux observers. Starting from the two-phase induction motor model it is studied the observers' stability and parameters adjustment, as well as the sensitivity to the variation of motor parameters, respectively to rotor resistance variation. At the end of the paper there are presented the simulation results obtained using MATLAB/SIMULINK simulation environment.

*Key-Words:* - Two-Phase Induction Motor, Field-Orientation-Flux-Observer, Gopinath Flux-Observer, MATLAB/SIMULINK

## 1 Introduction

Mechanical sensorless vectorial adjusting control systems have become unanimous accepted for a wide range of applications in electric drive field, starting from the low cost ones and finishing with high performance systems. A great effort has been put into this field research and has materialized especially towards eliminating mechanical sensors and to improve price-performance ratio for computational systems

In vectorial adjusting control systems a special role is played by instantaneous position and rotor flux absolute value. In this way, rotor flux estimation through solid adaptive methods offers notable results using a reasonable computation effort. Even though we don't identify directly the motor parameters, the computation effort represents the main advantage of adaptive flux observers.

Rotor flux estimation can also be accomplished using Kalman filters. These filters allow both rotor flux estimation and motor parameters detection using the stator current and voltage. But usage of these filters encounter great difficulties in implementation of computation algorithms on digital systems and in real time solving of the Riccati equations.

Taking into account the above observations, this paper follows through to resolve the instantaneous position of the rotor flux using adaptive observers, [2]. Starting from the induction motor, alternative methods of rotor flux estimation adaptive observers are presented, a comparison study being made related to their stability, parameters adjustment and sensitivity to the variation of motor parameters, respectively to rotor resistance variation.

## 2 Two-Phase Induction Motor Problem Statement

For determining the structure of adaptive flux observers it was started from the linear model of the induction motor, represented in a d-q system of axes, located on the stator, in which rotative speed of the rotor is considered constant, [1]. The complex way of describing the model is preferred, due to the fact that the stability study of the observers and on-line determination of gate-matrix parameters require fixing of the pole position into the complex plane.

To formulate the two-phase induction motor (TPIM) model using complex dimensions we start from defining the state variables, input and output, according to the relations,

$$\begin{aligned} i_s &= i_{sd} + j i_{sq} \\ \Psi_s &= \Psi_{sd} + j \Psi_{sq} \end{aligned} \quad (1)$$

$$\begin{aligned} i_r &= i_{rd} + j i_{rq} \\ \Psi_r &= \Psi_{rd} + j \Psi_{rq} \end{aligned} \quad (2)$$

$$u_s = u_{sd} + j u_{sq} \quad (3)$$

Stator and rotor equations of the motor become, [3],

$$[\dot{x}(t)] = [F(\omega_r)] \cdot [x(t)] + [G] \cdot [u(t)] \quad (4)$$

in which,

$$[\dot{x}(t)] = \begin{bmatrix} \dot{i}_s \\ \dot{\Psi}_s \end{bmatrix} \quad [x(t)] = \begin{bmatrix} i_s \\ \Psi_s \end{bmatrix} \quad [u(t)] = \begin{bmatrix} u_{sd} \\ u_{sq} \end{bmatrix} \quad (5)$$

$$[F(\omega_r)] = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad [G] = \begin{bmatrix} b_1 \\ 0 \end{bmatrix} \quad (6)$$

where,

$$a_{11} = -\frac{R_s}{\sigma L_s} - \frac{R_r(1-\sigma)}{\sigma L_r} \quad a_{12} = \frac{L_m}{\sigma L_s L_r} \left( \frac{R_r}{L_r} - j\omega_r \right)$$

$$a_{21} = \frac{L_m R_r}{L_r} \quad a_{22} = -\frac{R_r}{L_r} + j\omega_r \quad (7)$$

$$b_1 = \frac{1}{\sigma L_s}$$

$$\sigma = 1 - \left( \frac{L_m^2}{L_s L_r} \right) \quad (8)$$

Mathematical model of the two-phase induction motor was formulated using a matrix state equation because this one lends itself to implementations and simulations using software packages like MATLAB/SIMULINK. For solving the matrix equations different methods can be used (backward substitution, forward substitution, LU...) to get best performances for real-time simulation, but choosing the best fitted method and other fine tuning adjustments might be the subject of a future paper. In Fig.1 is presented the model implemented using SIMULINK environment, where the model described by equation (4) is included in the S-function block named MAB.

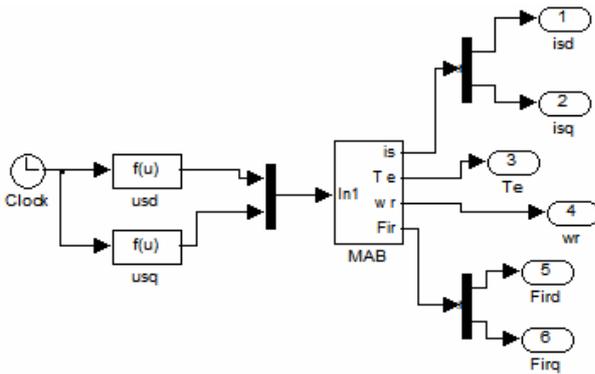


Fig.1

The torque expression from Fig.1 can be expressed in terms,

$$T_e = \left( \frac{P}{2} \right) \left( \frac{L_m}{L_r} \right) (\Psi_{dr} i_{qs} - \Psi_{qr} i_{ds}) \quad (9)$$

The expression for the rotor speed is expressed in terms of torque as,

$$p\omega_r = \frac{P}{2J} (T_e - T_L) \quad (10)$$

where  $T_e, T_L$  are the electromagnetic and the load torque.

### 3 Conventional Flux Simulators

Adaptive observers' structure is based on the combination between a simulator and an estimation error corrector. This corrector is actually a closed-loop control which amplifies the error between the estimate and the real size. This difference is used to make the two sizes, the estimated and the real one, to converge with a controlled speed. For the induction motor there are two possible simulators: one which uses the current model and one which uses the voltage model. For each of these simulators we can use two types of correctors: stator equations and voltage model of the motor. Each corrector can also have three types of closed-loop signals: stator voltage, stator current and differential stator current. From the multiple possible observer combinations using the elements enumerated above, in this paper only three of them were studied in parallel, [3],

OF1. *simulator*: current model; *corrector*: stator equations; *closed-loop signal*: differential stator current. This estimator is also known as field orientation Gopinath observer and its structure is,

$$\hat{\Psi}_r = a_{21} i_s + a_{22} \hat{\Psi} + g \left[ \dot{i}_s - (a_{11} i_s + a_{12} \hat{\Psi} + b_1 u_s) \right] \quad (11)$$

OF2. *simulator*: current model; *corrector*: stator equations; *closed-loop signal*: stator voltage, with the following structure,

$$\hat{\Psi}_r = a_{21} i_s + a_{22} \hat{\Psi} + g \left[ u_s - \frac{1}{b_1} (\dot{i}_s - a_{11} i_s - a_{12} \hat{\Psi}_r) \right] \quad (12)$$

OF3. *simulator*: current model; *corrector*: stator equations; *closed-loop signal*: stator current, and its structure is,

$$\hat{\Psi}_r = a_{21} i_s + a_{22} \hat{\Psi} + g \left[ i_s - \frac{1}{a_{11}} (\dot{i}_s - a_{12} \hat{\Psi} - b_1 u_s) \right] \quad (13)$$

The main element which determine the stability of the flux observers as well as the sensitivity to the motor parameters variation is the  $g$  gate, a complex number with the shape,

$$g = g_a + jg_b \quad (14)$$

where  $a$  and  $b$  represent the imposed coordinates for the two poles of the observer.

If the parameters of TPIM are constant, estimation error is defined by difference between real rotor flux,  $\Psi_r$  and the estimated one,  $\hat{\Psi}_r$ , [3],

$$\varepsilon = \hat{\Psi}_r - \Psi_r \quad (15)$$

Estimation error dynamic is given by,

$$\dot{\varepsilon} = (a_{22} - ga_{12}) \cdot \varepsilon = -h\varepsilon \quad (16)$$

Relation (16) gives the position of the two poles conjugate complex of the flux estimator. Their coordinates are  $\text{Re}(-h)$  and  $\pm j\text{Im}(-h)$ . Because of the coefficients  $a_{22}$  and  $a_{12}$ , the position of the poles depends on the rotor speed. For each value of the rotor speed we need to determine the estimator's gate coefficients ( $g_a$  si  $g_b$ ), and the two poles need to keep system's stability condition. Such a system is stable if and only if the poles are into the negative complex semiplane. By replacing coefficients (7) into relation (16) and by making equal the real and imaginary part, we get a system of two equations from where we can calculate the gate,

$$g_a = \frac{\left( \frac{R_r}{L_r} \alpha + \omega_r \beta \right)}{\left( \left( \frac{R_r}{L_r} \right)^2 + \omega_r^2 \right)} - 1 \frac{\sigma L_s L_r}{L_m} \quad (17)$$

$$g_b = \frac{\omega_r \alpha - \frac{R_r}{L_r} \beta}{\left( \frac{R_r}{L_r} \right)^2 + \omega_r^2} \frac{\sigma L_s L_r}{L_m}$$

where  $a$  and  $b$  represent the coordinates imposed for the two poles of the observer and,

$$\beta = 0 \text{ and } \alpha = K \sqrt{\left( \frac{R_r}{L_r} \right)^2 + \omega_r^2} \quad (18)$$

determine the optimum position of the poles on the negative real axis, minimizing this way the influence of TPIM parameters variation over flux observer stability.

According to relations (17) the gate coefficients depend on the rotor speed which requires a real-time

adjustment of them. For sensorless drive systems this aspect imposes a supplementary closed-loop, from speed estimator to flux estimator.

Replacing coefficients from (7) into relation (11) and taking into account relations (1)-(3), we get the final expression of Gopinath flux observer model,

$$\begin{aligned} & \left( \hat{\Psi}_{rd} + j\hat{\Psi}_{rq} \right) = \\ & = \left( \Psi_{rd} + j\Psi_{rq} \right) \left[ -\frac{R_r}{L_r} + j\omega_r - (g_a + jg_b) \frac{L_m}{\sigma L_s L_r} \left( \frac{R_r}{L_r} - j\omega_r \right) \right] + \\ & + (i_{sd} + ji_{sq}) \left[ \frac{L_m R_r}{L_r} + (g_a + jg_b) \left( \frac{R_s}{\sigma L_s} + \frac{R_r(1-\sigma)}{\sigma L_r} \right) \right] - \\ & - (g_a + jg_b) \frac{1}{\sigma L_s} (u_{sd} + ju_{sq}) + (g_a + jg_b) (i_{sd} + ji_{sq}) \end{aligned} \quad (19)$$

The implemented model using SIMULINK environment is presented in Fig.2. Relation (19), which describes the final model, is encapsulated into the S-function block named Flux\_Obs.

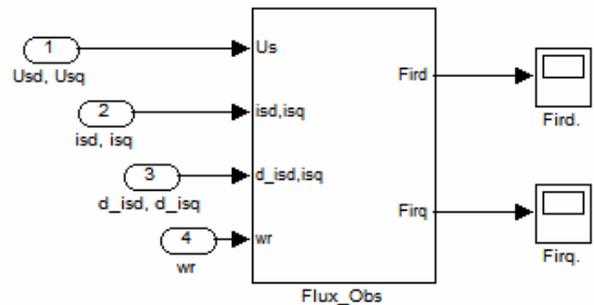


Fig.2

## 4 Computation Results

The behavior of the three adaptive observers has been studied comparatively, using real-time MATLAB/SIMULINK simulations. In order for the simulation to be as accurate as possible, the integration step and time sampling for discrete blocks must be equal with the time sampling of the real system. This is the time necessary to compute a control loop, (200µs).

Table 1

|                                      |                    |
|--------------------------------------|--------------------|
| $P=2$                                | $L_s=1.841 \Omega$ |
| $J=3.3 \times 10^{-5} \text{ kgm}^2$ | $L_r=1.538 \Omega$ |
| $R_s=415 \Omega$                     | $L_m=1.161 \Omega$ |
| $R_r=252.33 \Omega$                  |                    |

A higher order method for solving differential equations must be chosen to meet the requirements and that's why 4<sup>th</sup> and 5<sup>th</sup> order Runge-Kutta formulas were used.

The simulations were realized on a TPIM using the input parameters from Table 1. In Fig.3 is presented the schematic used for simulating the three flux observers.

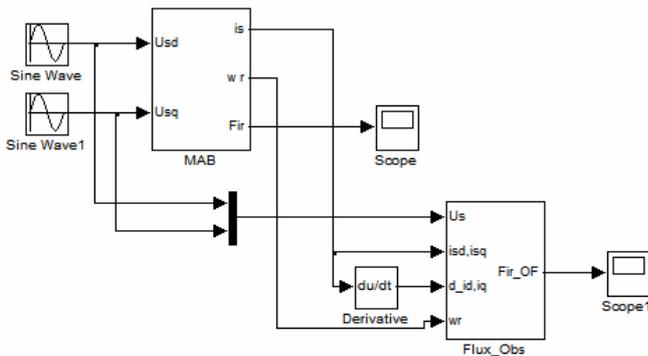


Fig.3

Simulation is the only way to compare the estimated rotor flux with the motor one. This comparison is possible because the TPIM model supplies the rotor and stator currents in a two-phase system of axes, d-q. Rotor flux components are calculated in TPIM computation block (MAB, Fig.1) using the relations,

$$\Psi_{rd} = L_r i_{rd} + L_m i_{sd} \quad (20)$$

$$\Psi_{rq} = L_r i_{rq} + L_m i_{sq} \quad (21)$$

Figures Fig.4-Fig.7 show the absolute value of the estimated rotor flux, (----) and the real one, (\_\_\_), for different values of coefficient  $K$  and a variation of rotor resistance by  $\pm 50\%$ . For  $K=0.5$  (small), the weight of the compensator, based on the stator equations of TPIM is small, instead the observer becomes relative sensitive to the rotor resistance variation. For  $K=2$  (big), the weight of the error compensator due to rotor resistance variation is bigger and implicitly the uncontrollable error of the model based on the stator equations will be also bigger. Because of that, the flux observer with the mathematical model described in equation (12) and with uses the stator voltage as closed-loop, generates a high instability and the gate matrix could not be determined. After repeated tests, the conclusion is that this type of flux observer is stable for values of coefficient  $K$  smaller than 1.2, [4].

The results of the three simulations prove out the fact that the initial estimated error reduces rapidly and the estimated value and the real one converge even for important variations of the rotor resistance.

Each single simulation was evaluated by comparison with the results from the determinations of the rotor flux components using a flux simulator based on the current model of TPIM. Thereby the performances of Gopinath field orientation flux observer are remarkable compared with the observers (12) and (13), for rotor resistance variation and changes of coefficient  $K$ . By changing the coefficient  $K$  we can control the weight of the correction generated by the adjustable model, applied to the reference model.

### 5 Conclusion

In this paper there were analyzed by comparison three flux observers. After this study several conclusion can be deduced:

- The success of designing flux observers is determined by pole assigning. Thereby the coordinates  $a$  and  $b$  determine the gate coefficients which adjust the weight of error compensator due to rotor resistance.
- The stability is given by controllable pole position related to the rotor speed and the observers' sensitivity can be adjusted using coefficient  $K$ .
- The flux observer Gopinath has a remarkable stable behavior to motor parameters variation which makes it an important alternative to Kalman filters in sensorless vectorial adjusting control systems.

$K=2; R_r=378$  (OF1,OF3)

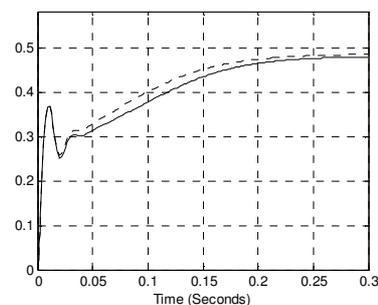
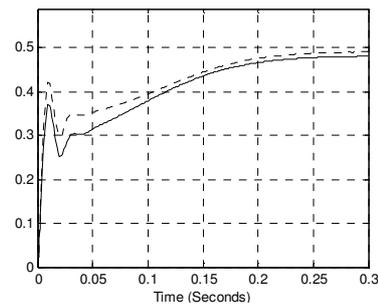
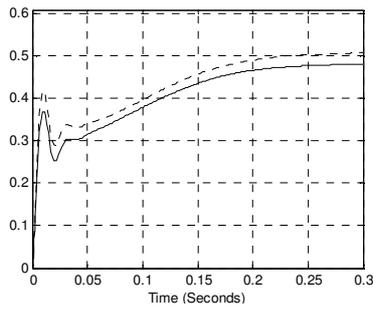


Fig.4

$K=2; R_r=126$  (OF1,OF3)



$K=0,5; R_r=126$  (OF1,OF2,OF3)

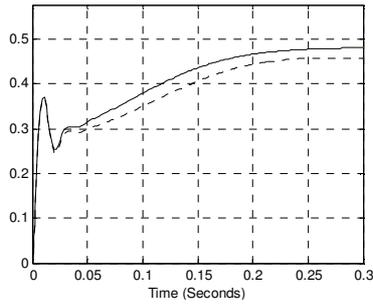
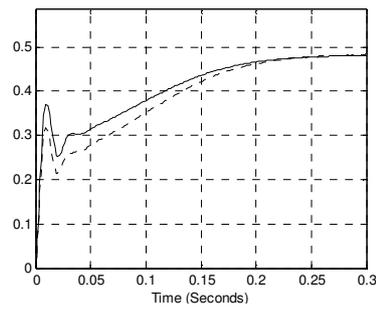
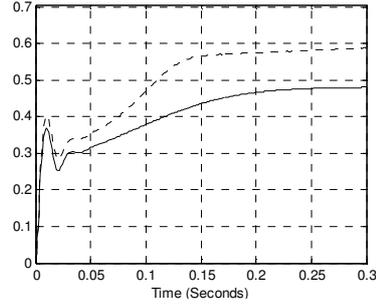


Fig.5



$K=0,5; R_r=378$  (OF1,OF2,OF3)

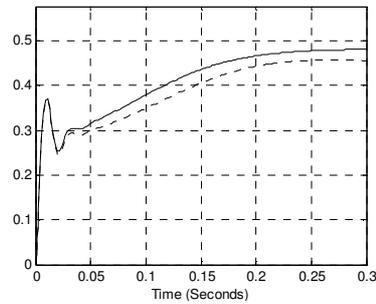
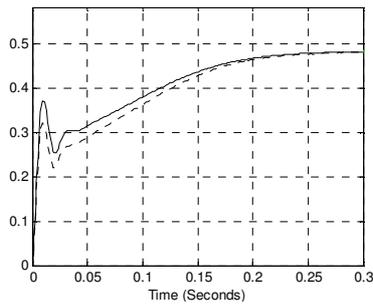


Fig.7

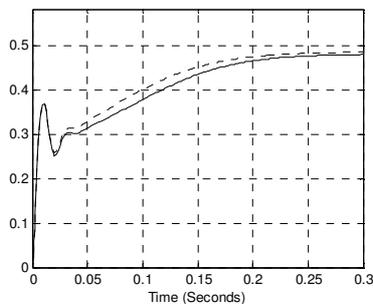
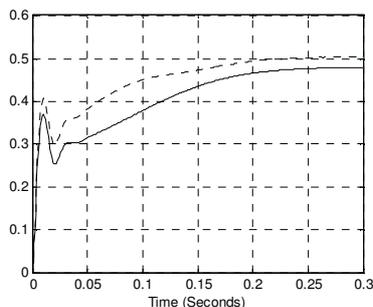


Fig.6

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