Antimonotonicity and Bubbles in a 4th Order Non Driven Circuit

I. N. STOUBOULOS, I. M. KYPRIANIDIS, and M. S. PAPADOPOULOU
Physics Department
Aristotle University of Thessaloniki
Thessaloniki, 54124
GREECE

Abstract: - We have studied an autonomous 4th order circuit, which exhibits sensitive dependence on initial conditions. The circuit contains a non linear resistor R_N of N-type. We have observed the dynamics of the non linear circuit for various values of its parameters. A diversity of phenomena has been detected, such as antimonotonicity, period doubling route to chaos and bubbles. We have seen crisis events, which are seem to lead to the catastrophe of the above phenomena.

Key-Words: - Antimonotonicity, Bubbles, Route to chaos, Reverse period-doubling, Crisis.

1 Introduction

Dawson et al [1] coined the term antimonotonicity to characterize creation and annihilation of periodic orbits. Namely, in contrast to the monotone bifurcations of the logistic map, in many common nonlinear dynamical systems, periodic orbits can be both created and destroyed, via reverse bifurcation sequences, as a control parameter of the system is increased.

The intermittent behavior of a 4th order autonomous nonlinear electric circuit has been studied. The electric circuit contains two active elements, one linear negative conductance and one nonlinear resistor exhibiting a symmetrical piecewise linear v-i characteristic. The two capacitances C_1 and C_2 serve as the control parameters of the system. The antimonotonicity, the formation of "bubbles" in the bifurcation region as well as the chaotic behavior of this circuit were reported in two recent publications [2, 3]. Additionally, a crisis-induced intermittency was detected, occurring when the corresponding spiral attractor suddenly widens to a double-scroll one [2].

Due to the advantages which electric circuits offer to experimental chaos studies, such as robustness and convenient implementation, most chaotic and bifurcation effects cited in the literature, have been observed in such circuits. Moreover, their potential applications in chaotic ciphering are supporting their use in secure communications. Such circuits exhibit among others the period-doubling route to chaos [4, 5], the intermittency route to chaos [6, 7], the quasiperiodicity route to chaos [8, 9] and of course the crisis [10, 11].

2 The Non Driven 4th Order Circuit

The circuit, we have studied, is shown in Fig.1, while in Figs.2 and 3 we can see the v-i characteristic of the nonlinear resistor and negative conductance respectively.

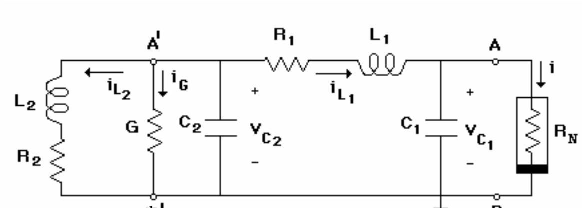

Fig.1. The 4th order non driven electric circuit.

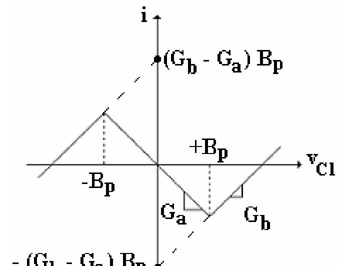

Fig.2. The v-i characteristic of nonlinear resistor R_N.

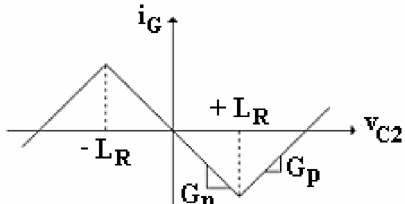

Fig.3. The v-i characteristic of the negative conductance.

The state equations of the circuit are:

$$\frac{dv_{C1}}{dt} = \frac{1}{C_1}\left(i_{L1} - i\right) \quad (1)$$

$$\frac{dv_{C2}}{dt} = -\frac{1}{C_2}\left(G \cdot v_{C2} + i_{L1} + i_{L2}\right) \quad (2)$$

$$\frac{di_{L1}}{dt} = \frac{1}{L_1}\left(v_{C2} - v_{C1} - R_1 i_{L1}\right) \quad (3)$$

$$\frac{di_{L2}}{dt} = \frac{1}{L_2}\left(v_{C2} - R_2 i_{L2}\right) \quad (4)$$

where

$$i=g(v_{C1})=G_b v_{C1}+0.5(G_a-G_b)(|v_{C1}+B_p|-|v_{C1}-B_p|) \quad (5)$$

The circuit's parameters are: L_1=33mH, L_2=100mH, C_2=15nF, R_2=90Ω, G_n=-0.45mS, G_p=0.45mS, L_R=7.5V, G_a=-0.105mS, G_b=7mS and B_p = 0.68V, while R_1 varies from 900Ω to 300Ω and C_1 from 21nF to 1nF.

3 Dynamics of the 4th Order Circuit

The bifurcation diagrams of circuit's dynamics are shown in Figs.4-10. Giving constant values to resistor R_1, we have plotted the bifurcations diagrams i_{L2} vs. C_1. The comparative study of the bifurcation diagrams gives the qualitative changes of system's dynamics, as C_1 takes different discrete values.

The bifurcation diagram, i_{L2} vs. C_1, for R_1=930Ω is shown in Fig.4. As C_1 is decreased, the system always remains in periodic state with period-1. The bifurcation diagram for R_1=890Ω is shown in Fig.5. The system remains in a periodic state, following the scheme: period-1 (for C_1 > 17.63nF) → period-2 (for 15.21nF < C_1 < 17.63nF) → period-1 (for C_1 < 15.21nF). Bier and Bountis [12] named this scheme "primary bubble". The bifurcation diagram for R_1=870Ω is shown in Fig.6. The system remains in a periodic state, following the scheme: period-1 (for C_1 > 17.41nF) → period-2 (for 16.65nF < C_1 < 17.41nF) → period-4 (for 15.60nF < C_1 < 16.65nF) → period-2 (for 14.57nF < C_1 < 15.60nF) → period-1 (for C_1 < 14.57nF). The bifurcation diagram for R_1=865Ω is shown in Fig.7. The system remains again in a periodic state, following the scheme: period-1 (for C_1 > 17.36nF) → period-2 (for 16.77nF < C_1 < 17.34nF) → period-4 (for 16.26nF < C_1 < 16.77nF) → period-8 (for 15.77nF < C_1 < 16.26nF) → period-4 (for 15.34nF < C_1 < 15.77nF) → period-2 (for 14.42nF < C_1 < 15.34nF) → period-1 (for C_1 < 14.42nF).

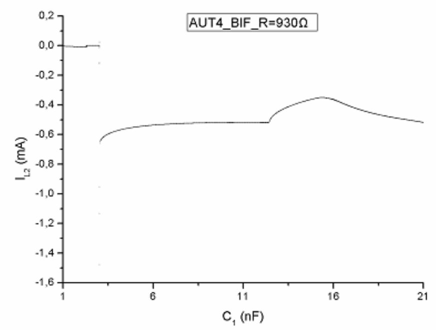

Fig.4. Bifurcation diagram i_{L2} vs. C_1 for R_1=930Ω.

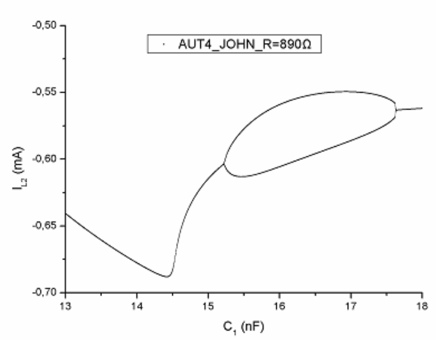

Fig.5. Bifurcation diagram i_{L2} vs. C_1 for R_1=890Ω.

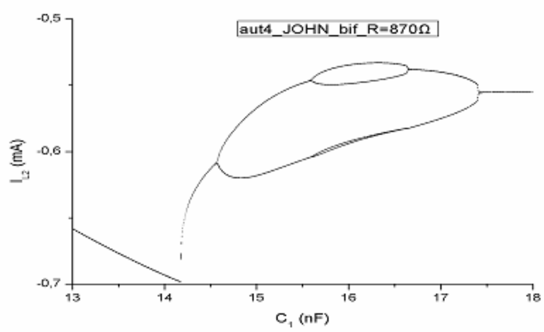

Fig.6. Bifurcation diagram i_{L2} vs. C_1 for R_1=870Ω.

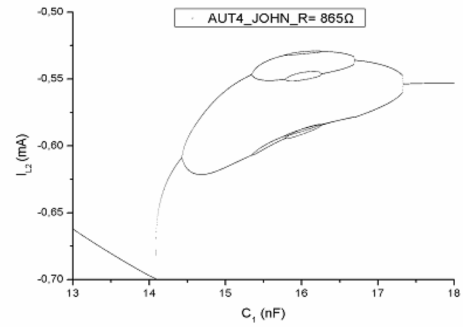

Fig.7. Bifurcation diagram i_{L2} vs. C_1 for R_1=865Ω.

As R_1 is decreased, chaotic states appear, as we can observe in Fig.8, where the bifurcation diagram, i_{L2} vs. C_1, for R_1=862 is shown. The bubbles are now chaotic. The bifurcation diagram for R_1=855Ω is shown in Fig.9, where crisis begins. Finally, the bifurcation diagram for R_1=820Ω is shown in Fig.10, where crisis becomes enlarged in the region 15.00nF < C_1 < 15.80nF.

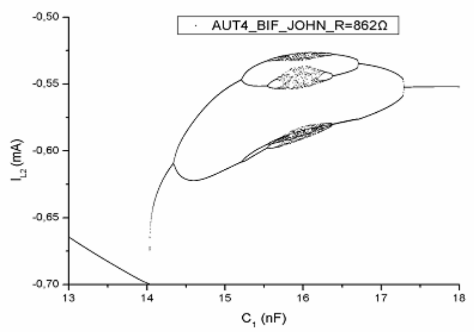

Fig.8. Bifurcation diagram i_{L2} vs. C_1 for R_1=862Ω.

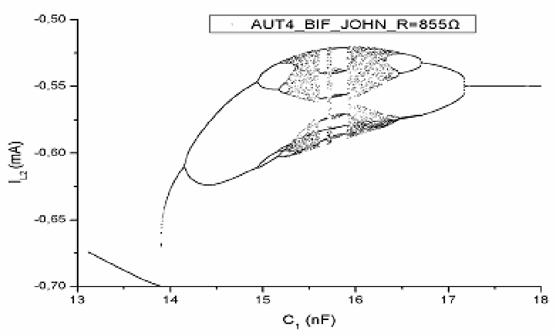

Fig.9. Bifurcation diagram i_{L2} vs. C_1 for R_1=855Ω.

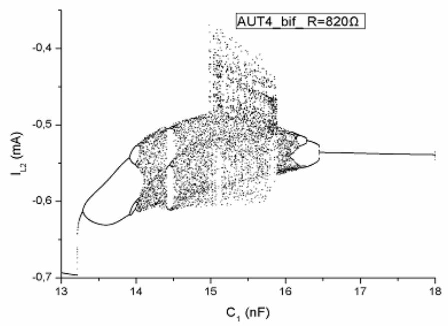

Fig.10. Bifurcation diagram i_{L2} vs. C_1 for R_1=820Ω.

4 Route to Chaos

Using the bifurcation diagram of Fig.9 (R_1=855Ω), we have chosen the values of C_1 to demonstrate the route to chaos through period doubling and reverse period doubling. In the next two paragraphs we present both simulation and experimental results.

4.1 Simulation Results for R_1 = 855Ω

In Figs.11-18 we can see the theoretical phase portraits v_{C2} vs. v_{C1} for R_1=855Ω and various values of capacitance C_1.

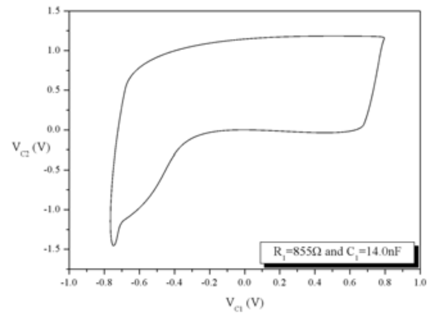

Fig.11. v_{C2} vs. v_{C1} for C_1=14.0nF and R_1=855Ω (period-1).

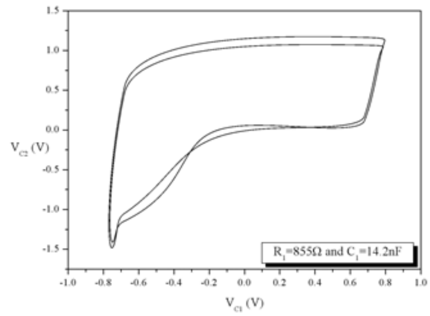

Fig.12. v_{C2} vs. v_{C1} for C_1=14.2nF and R_1=855Ω (period-2).

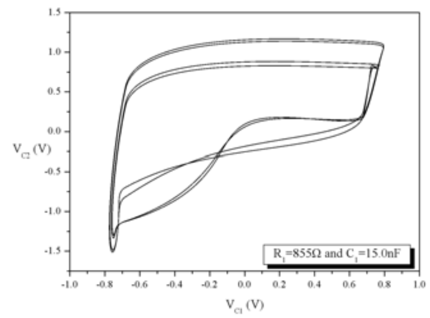

Fig.13. v_{C2} vs. v_{C1} for C_1=15.0nF and R_1=855Ω (period-4).

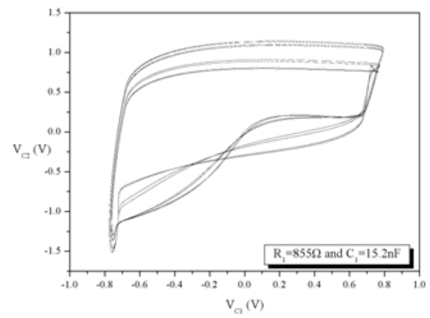

Fig.14. v_{C2} vs. v_{C1} for C_1=15.2nF and R_1=855Ω (period-8).

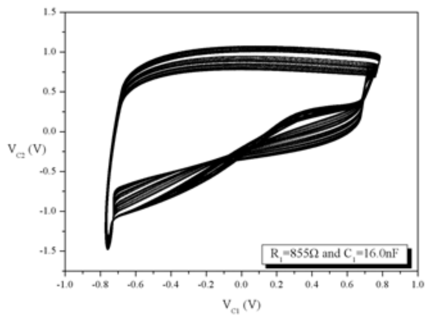

Fig.15. v_{C2} vs. v_{C1} for C_1=16.0nF and R_1=855Ω (chaotic attractor).

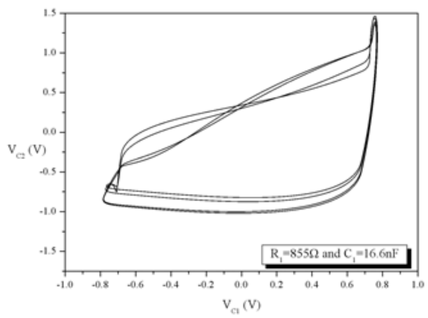

Fig.16. v_{C2} vs. v_{C1} for C_1=16.6nF and R_1=855Ω (period-4).

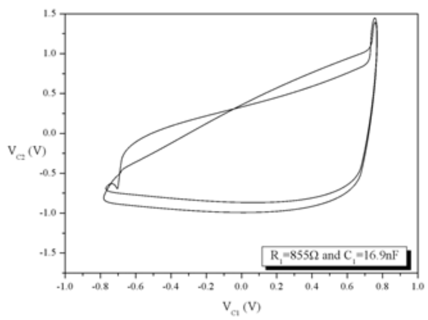

Fig.17. v_{C2} vs. v_{C1} for C_1=16.9nF and R_1=855Ω (period-2).

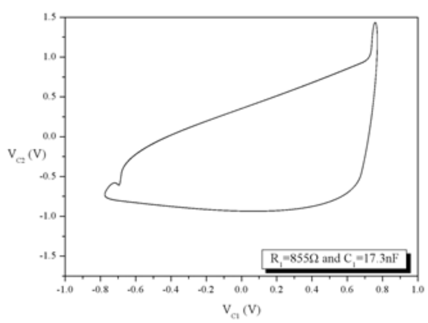

Fig.18. v_{C2} vs. v_{C1} for C_1=17.3nF and R_1=855Ω (period-1).

4.2 Experimental Results for R_1 = 855Ω

In Figs.19-26 we observe the experimental phase portraits v_{C2} vs. v_{C1} for corresponding values of capacitance C_1 as in Figs.11-18, respectively.

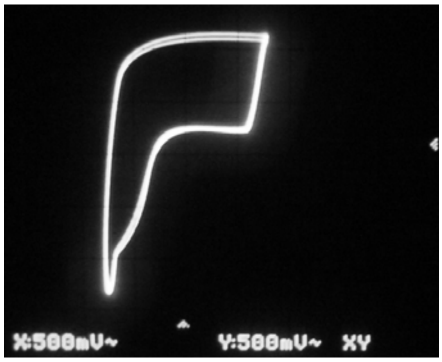

Fig.19. Experimental diagram v_{C2} vs. v_{C1} (period-1).

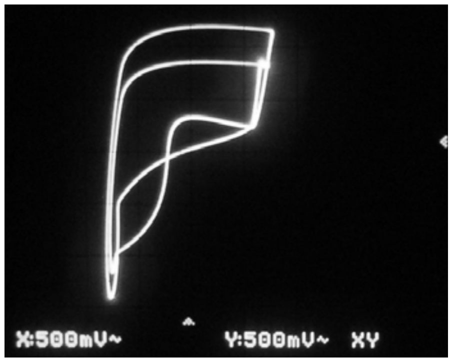

Fig.20. Experimental diagram v_{C2} vs. v_{C1} (period-2).

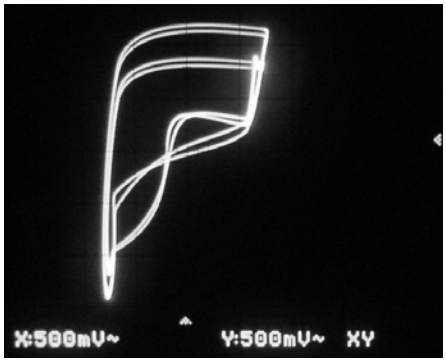

Fig.21. Experimental diagram v_{C2} vs. v_{C1} (period-4).

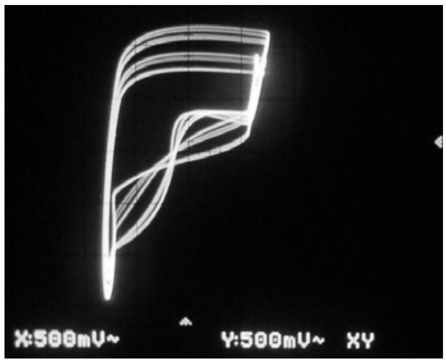

Fig.22. Experimental diagram v_{C2} vs. v_{C1} (period-8).

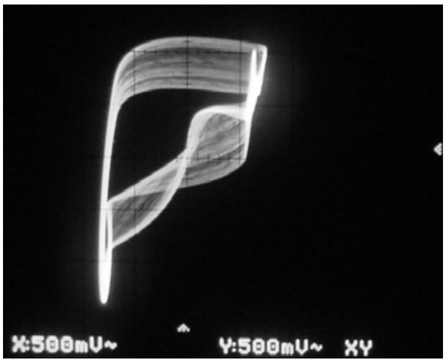

Fig.23. Experimental diagram v_{C2} vs. v_{C1} (chaotic attractor).

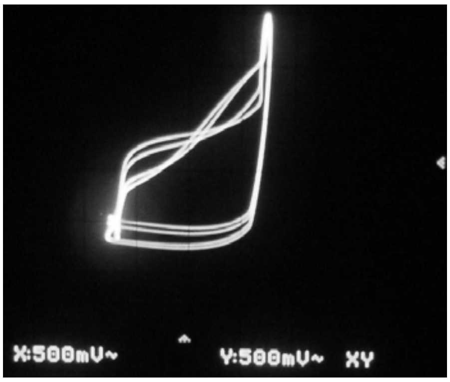

Fig.24. Experimental diagram v_{C2} vs. v_{C1} (period-4).

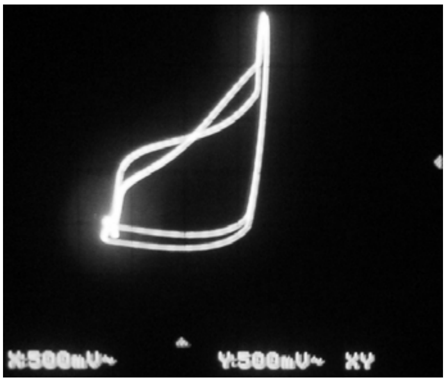

Fig.25. Experimental diagram v_{C2} vs. v_{C1} (period-2).

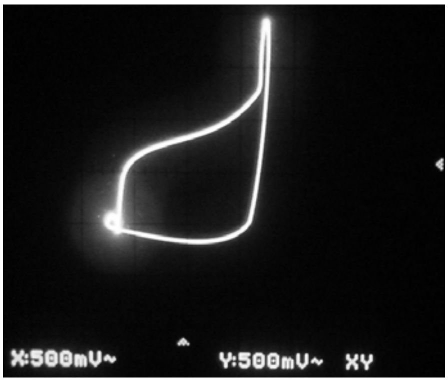

Fig.26. Experimental diagram v_{C2} vs. v_{C1} (period-1).

5 Conclusion

In this paper we have studied a non driven 4th order circuit, which contains a non linear resistor R_N of N-type v-i characteristic. We have observed the dynamics of the circuit for various values of C_1, C_2 and R_1. We have seen a diversity of phenomena, such as antimonotonicity ($800\Omega < R_1 < 870\Omega$), period-doubling route to chaos ($R_1 < 920\Omega$) and bubbles (p-1). We have located crisis phenomena of the system inside the bubbles ($840\Omega < R_1 < 855\Omega$) and crisis phenomena outside the bubbles ($800\Omega < R_1 < 840\Omega$) which appear function like a mechanism of catastrophe of the above phenomena.

Acknowledgements:
This work has been supported by the research program "EPEAEK II, PYTHAGORAS II", code number 80831, of the Greek Ministry of Education and E.U.

References:

[1] Dawson, S.P., Grebogi, C., Kan, I., Kocak, H., and Yorke, J.A., Antimonotonicity: Inevitable Reversals of Period-doubling Cascades, *Physics Letters A*, Vol.162, 1992, pp. 249-254.

[2] Kyprianidis, I.M., Stouboulos, I.N., Haralabidis, P. and Bountis T., Antimonotonicity and chaotic dynamics in a fourth-order autonomous nonlinear electric circuit, *International Journal of Bifurcation and Chaos*, Vol.10, No.8, 2000, pp. 1903-1915.

[3] Koliopanos, Ch.L., Kyprianidis, I.M., Stouboulos, I.N., Anagnostopoulos, A.N., and Magafas, L., Chaotic behaviour of a fourth-order autonomous electric circuit, *Chaos Solitons and Fractals*, Vol.16, 2003, pp. 173-182.

[4] Linsay, P.S., Period doubling and chaotic behavior in a driven anharmonic oscillator, *Physical Review Letters*, Vol.47, No.19, 1981, pp. 1349-1352.

[5] Buskirk, R.V., and Jeffries, C., Observation of chaotic dynamics of coupled nonlinear oscillators, *Physical Review A*, Vol.31, No.5, 1985, pp. 3332-3357.

[6] Jeffries, C., and Perez, J., Observation of a Pomeau-Manneville intermittent route to chaos in a nonlinear oscillator, *Physical Review A*, Vol.26, No.4, 1982, pp. 2117-2122.

[7] Huang, J.-Y., and Kim, J.-J., Type-II intermittency in a coupled nonlinear oscillator: Experimental observation, *Physical Review A*, Vol.36, No.3, 1987, pp. 1495-1497.

[8] Matsumoto, T., Chua, L.O., and Tokunaga, R., Chaos via. Torus Breakdown, *IEEE Transactions Circuits Systems*, Vol.34, 1987, pp. 240-253.

[9] Chua, L.O., Yao, Y., Yang, Q., Devil's staircase route to chaos in a nonlinear circuit, *International Journal of Circuit Theory and Applications*, Vol.14, 1986, pp. 315-329.

[10] Jeffries, C., and Perez, J., Direct observation of crises of the chaotic attractor in a nonlinear oscillator, *Physical Review A*, Vol.27, No.1, 1983, pp. 601-603.

[11] Ikezi, H., deGrassie, J.S., and Jensen, T.H., Observation of multiple-valued attractors and crises in a driven nonlinear circuit, *Physical Review A*, Vol.28, No.2, 1983, pp. 1207-1209.

[12] Bier, M., and Bountis, T. C., Remerging Feigenbaum trees in dynamical systems, *Physics Letters A,* Vol.104, No.5, 1984, pp. 239-244.