Wavelet Image Compression Based on Mathematical Expectation and Standard Deviation

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Abstract

The aim of this paper is to propose an efficient method for filtrating wavelet coefficients in image compression. The method filtrates coefficients based on wavelet transform, mathematical expectation and standard deviation. Image quality is measured objectively, using peak signal-to-noise ratio, and subjectively, using mean opinion score. Experimental results indicate that this method based on mathematical expectation and standard deviation has good performances in image compression.

Keywords: Wavelet transform; MRA; Mathematical expectation; Standard deviation; MOS

1. Introduction

Wavelet transform is a widely-used technique in image compression. In the process of image compression, the filtration of wavelet coefficients directly affects the quality of synthetical images. However, how to filtrate wavelet coefficients is still a problem. For this reason, precisely grasping the characteristics of statistic on wavelet coefficients to explore better filtrating method is the key issue of improving coding performance in image compression. Many researchers have done a lot of fruitful work in this aspect, e.g. coding based on entropy[1], coding based on energy[2,3,4] and so on.

With different sub-images, the separation and importance of coefficients are different. Generally speaking, the bigger of the coefficient amplitude, the more important it is. We research the expectations and standard deviations of each coefficient matrix and propose the method of wavelet image compression based on mathematical expectation and standard deviation.

The content of this paper is organized as follows: mathematical foundations is presented in section 2; wavelet transform is presented in section 3;process wavelet coefficients is presented in section 4; image quality evaluation is given in section 5; Experimental results and conclusions are presented in section 6;The relative references are listed in section 7.

2. Mathematical foundations

2.1. Multi-resolution analysis

Multi-resolution analysis（MRA）, formulated by Mallat in 1980s, provides a convenient framework for studying image compression.It is the basis of Mallat pyramid algorithm, which decomposes and synthesizes the image information in the different sub-bands. There are two basic ingredients for a multi-resolution analysis:

1) An infinite chain of nested linear function spaces V^m , $m = 0,1,2\cdots$. It can be described as $V_0 \subset V_1 \subset V_2 \subset \cdots$ in mathematics.

2) An inner product of two functions $f, g \in V^m$. It is defined as

$$\langle f, g\rangle = \int_{-\infty}^{\infty} f(x)\overline{g(x)}dx$$

Supposed $\phi(x)$ is a scaling function

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of $L^2(R)$, p is the shift-invariance, m is the scale-invariance, the set of functions $\{\phi_{mn}\}$, $p \in R$ with

$$\phi_{mp}(x) = 2^{\frac{-m}{2}} \phi(2^{-m}x - p)$$

constitutes a basis for subspace V^m.

Detail information of subspace V^m can be captured in an additional subspace W^{m-1} defined by

$$W^{m-1} \equiv \left\{ f \in V^{m-1} \,|\, \langle f, g \rangle = 0, \forall g \in V^{m-1} \right\}$$

Such that: $V^m = V^{m-1} \oplus W^{m-1}$. The space V^m can be fully decomposed as

$$V^m = V^0 \oplus W^0 \oplus \cdots \oplus W^{m-1}.$$

The subspace $\{W_m\}_{m \in R}$ are spanned by wavelet functions ψ_{mp} that are constructed from a mother wavelet ψ by binary dilation and dyadic translation:

$$\Psi_{mp}(x) = 2^{\frac{-m}{2}} \psi(x^{-m}x - p), m, p \in R.$$

Thus, a hierarchical representation of function f_{m_0} at level m_n can now be written as the linear combination of its coarse shape at level M and the detail information of all intermediate levels $m_n < m \le M$

$$f(x) = \sum_{p \in R} C_{Mp}\phi_{Mp}(x) + \sum_{m=m_0}^{M} \sum_{p \in R} d_{mp}\psi(x).$$

The coefficients c_{mp} and d_{mp} are determined by the inner products of f with ϕ_{mp} and ψ_{mp}, respectively:

$$c_{mp} = \langle f, \phi_{mp} \rangle \qquad d_{mp} = \langle f, \psi_{mp} \rangle$$

2.2.mathematical expectation

Mathematical expectation describes an important position in random variable X (a coefficient matrix). Its effect equals to the static moment in mechanics. In other words, it is the kernel of the mass distribution. Mathematical expectation is defined as:

$$E(X) = \sum_i \sum_j f(x_i, y_j) \bullet p_{i,j}$$

Where $f(x_i, y_j)$ is the value of $X(i, j)$, $p_{i,j}$ is the probability,.

2.3.standard deviation

Standard deviation is an instrument for measuring the deviation between random variable X and its expectation $E(X)$. It is defined as follows:

$$\sigma(\mathrm{X}) = (\mathrm{E(X\text{-}EX)^2})^{1/2}$$

If the value of each $X(i, j)$ approach to $E(X)$,the $\sigma(\mathrm{X})$ will be small, and the matrix X is dense. Otherwise, $\sigma(\mathrm{X})$ will be big, and X is sparse.

3. Wavelet transform

Wavelet transform has excellent space-freque-ncy localization characterizations. Discrete wavelet transform (DWT) corresponds to two sets of analysis/synthesis digital filters, $g / \tilde{g}$ and $h / \tilde{h}$, where h is a low pass filter and g is a high pass filter. In two dimensions, filters are usually applied to both horizontal and vertical directions.

According to Mallat's arithmetic, images can be decomposed into a low-frequency image and three high-frequency images, marked as LL1, HL1, LH1, HH1 respectively.Then LL1 also can be decomposed into one low-frequency and 3 high-frequency images, marked as LL2,HL2, LH2, HH2, respectively.This process can be described in fig.1. The decomposition splits the frequency space into multiple scales and orientateons[5] in Fig.2.

<table>
<tr><td>LL2</td><td>HL2</td><td rowspan="2">HL1</td></tr>
<tr><td>LH2</td><td>HH2</td></tr>
<tr><td colspan="2">LH1</td><td>HH1</td></tr>
</table>

Fig.1.two-level wavelet decomposition

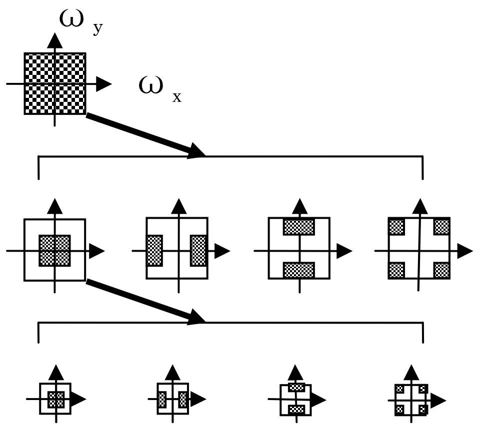

Fig.2.An idealized multi-scale and orientation decomposition of frequency space. Shown, from top to bottom, are levels 0,1,and2,and from left to right, are the low-pass, vertical, horizontal, and diagonal sub-bands.

4. Process wavelet coefficients

After processing an image using DWT, a great deal of energy focused on the low-frequency coefficients in LL2,which reflect the basic feature and guarantee the basic quality of the original image. But the large number of high-frequency coefficients have a little of energy. Analyzing the high-frequency coefficients (HL1,LH1,HH1,HL2, LH2,HH2),we get such conclusions:firstly,the expectations of each high-frequency matrix approach to zero.So we can set coefficients to zero, it has small effect on image quality.Secondly, for the standard deviations are much bigger than zero, there are some big coefficients which take an important role in enhancing image quality,so we can not set all the coefficients to be zero.

Computing mathematical expectation and standard deviation of each high-frequency matrix, we found the fact that the expectations are not always positive or negative. So the distribution of coefficients and the thresholds of negative and positive coefficients are unsymmetri-
cal. According to this fact,we discuss the thresholds in different conditions:

1).In the case of $E(X) > 0$,

If the coefficients are positive, $\mathrm{T{=}EX} + \sigma\left(\mathrm{X}\right)$.

If the coefficients are negative, $\mathrm{T}=\left|\mathrm{EX}\text{-}\sigma\left(\mathrm{X}\right)\right|$.

2).In the case of $E(X) < 0$,

If the coefficients are positive, $\mathrm{T}=\left|\mathrm{EX}\text{-}\sigma\left(\mathrm{X}\right)\right|$.

If the coefficients are negative, $\mathrm{T{=}EX} + \sigma\left(\mathrm{X}\right)$

A wavelet coefficient that has an absolute value above or equal to the threshold is called a significant coefficient; otherwise it is an insignificant coefficient. The insignificant wavelet coefficient is coded with one bit “0” while the significant wavelet coefficient is coded with normal coding (e.g. Huffman coding). A sub-band that has no significant wavelet coefficient is called an insignificant sub-band. The insignificant sub-band is entirely coded with one bit. This means that a large number of insignificant coefficients are coded with one symbol (one bit “0”).By coding a large number of insignificant wavelet coefficients with one bit realizes the image compression ,and it is the main purpose of the new coding algorithm.

5. Image quality evaluation

The image quality can be evaluated objectively and subjectively[6].A standard objective measure of coded image quality is peak signal-to-noise (PSNR)[7] which is defined as the ratio between signal variance and reconstruction error variance [mean-square error(MSE)] usually expressed in decibels(dB)

$$MSE = \left\{ \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} \left[f(i,j) - \hat{f}(i,j) \right]^2 \right\}^{1/2}$$

Where $f(i,j)$ is the value of original image, $\hat{f}(i,j)$ is the value of synthetical image, M and N is the size of the image.When the input signal is an R-bit discrete variable, the variance or energy can be replaced by maximum input symbol energy$(2^R\text{-}1)^2$. For the common case of 8 bits per picture element of input image, the PSNR can be defined as

$$PSNR(dB) = 10\lg(\frac{255^2}{MSE})$$

In fact, in image compression systems, the truly definitive measure of image quality is perceptual quality. So in addition to the commonly used PSNR,we chose to use a perception based subjective evaluation, quantified by MOS[8].The method uses the five-grade impairment scale[9] with proper description for each grade:5- imperceptible;4-perceptible,but not annoying;3-slightly annoying;2-annoying;and 1-very annoying. For the set of distorted images,the MOS values were obtained from an experiment involving 9 viewers.

The original source image without compression was used as the reference condition.The assessor is asked to vote on the second, keeping in mind the first. At the end of the series of sessions, MOS for each test condition and test image are calculated

$$MOS = \sum_{i=1}^{5} i \bullet p(i)$$

where i is grade and $p(i)$ is grade probability.

At last,the compression ratio(CR)[4] is defined as

$$CR = \frac{N_1}{N_2}$$

Where N_1 is the number of the wavelet coefficients in original image; N_2 is the number of retained wavelet coefficients after the filtration.

6. Experimental results and conclusions

In order to verify the validity of image compression based on mathematical expectation and standard deviation, this paper realized the algorithm using Matlab6.5 and bior3.7 wavelet. Good effect has received when experiments is done accoding to many kinds typical image sequence. Here explaining it through listing two groups of experiments: the classical images such as figure, woman and babara with 8 bit (256*256) as testing images, the original images are shown as Fig.3 and testing images are shown as Fig.4. The quality of synthetical images was scaled using CR,PSNR and MOS,shown as table 1.

(a).figure image

(b).woman image

(c).babara image

Fig.3. Original images for experiment

(a). figure image

(b).woman image

(c).babara image

Fig.4. Testing images for experiment

Table1.Comparession performance

image	CR	PSNR	MOS
figure	1.973	21.551	4.89
woman	2.7973	24.429	4.22
babara	3.1408	27.466	4.11

As is shown in table 1,the method based on mathematical expectation and standard deviation has good performances in image compression.But many things should be researched in further,such as how to choose the best wavelet, how to decided how many levels is the best to decompose,what image does this method match better and so on.

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