# A BIVARIATE SHRINKAGE FUNCTION FOR COMPLEX DUAL TREE DWT BASED IMAGE DENOISING

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*Abstract:* -

For many natural signals, the wavelet transform is a more effective tool than the Fourier transform. The wavelet transform provides a multi resolution representation using a set of analyzing functions that are dilations and translations of a few functions. The wavelet transform lacks the shift-invariance property, and in multiple dimensions it does a poor job of distinguishing orientations, which is important in image processing. For these reasons, to obtain some applications improvements, the Separable DWT is replaced by Complex dual tree DWT. In this paper, we propose a new simple non Gaussian bivariate probability distribution function to model statistics of wavelet coefficients of images. The model captures the dependence between a wavelet coefficient and its parent. Using Bayesian estimation theory we derive from this model a simple non-linear shrinkage function for wavelet denoising, which generalizes the soft thresholding approach. The new shrinkage function, which depends on both the coefficient and its parent, yields improved results for complex wavelet-based image denoising.

*Key-Words:* - Complex DWT, Denoising, Bivariate Shrinkage Function

## 1 Introduction

Many scientific datasets are contaminated with noise, either because of the data acquisition process, or because of naturally occurring phenomena. A first pre-processing step in analyzing such datasets is denoising, that is, estimating the unknown signal of interest from the available noisy data. There are several different approaches to denoise images.

Spatial filters have long been used as the traditional means of removing noise from images and signals. These filters usually smooth the data to reduce the noise, but, in the process, also blur the data. In the last decade, several new techniques have been developed that improve on spatial filters by removing the noise more effectively while preserving the edges in the data. Some of these techniques borrow ideas from partial differential equations and computational fluid dynamics. Other techniques combine impulse removal filters with local adaptive filtering in the transform domain to remove not only white & mixed noise, but also their mixtures. A different class of methods exploits the decomposition of the data.

This paper presents the concept of complex dual tree Discrete Wavelet Transform to denoise the digital images by using soft thresholding algorithm with new shrinkage function. This transform is nearly shift invariant and is oriented in 2D. The 2D dual tree wavelet transform produces six sub bands at each scale, each of which is strongly oriented at distinct angles. Section 2 involves

Separable DWT. The complex Dual Tree DWT is discussed in Section 3 & Section 4 deals with bivariate shrinkage functions. Section 5 gives the general method involved in image denoising. Section 6 deals with results & discussions & the Conclusion are given in Section 7.

## 2 Separable DWT

### 2.1 1-D Discrete Wavelet Transform

The analysis filter bank decomposes the input signal x(n) into two sub band signals, c(n) and d(n). The signal c(n) represents the low frequency part of x(n), while the signal d(n) represents the high frequency part of x(n). We denote the low pass filter by af1 (analysis filter 1) and the high pass filter by af2 (analysis filter 2). As shown in the figure, the output of each filter is then down sampled by 2 to obtain the two sub band signals c(n) & d(n).

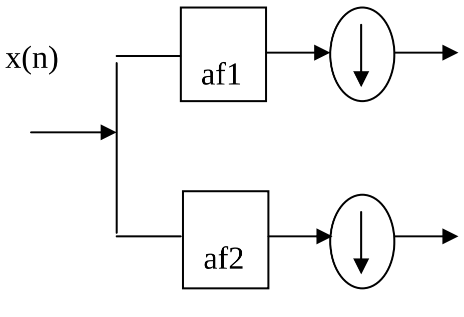



Fig. 1. Separable DWT (Analysis)

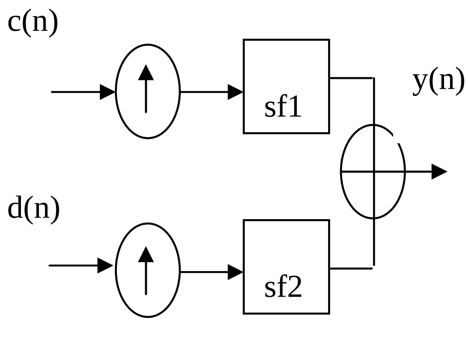



Fig. 2. Separable DWT (Synthesis)

The Synthesis filter bank combines the two sub band signals c(n) & d(n) to obtain a single signal y(n). The synthesis filters bank up-samples each of the two sub band signals. The signals are then filtered using a low pass and a high pass filter. We denote the low pass filter by sf1 (synthesis filter1) and the high pass filter by sf2 (synthesis filter 2). The signals are then added together to obtain the signal y(n). If the four filters are designed so as to guarantee that the output signal y(n) equals the input signal x(n), then the filters are said to satisfy the perfect reconstruction condition.

### 2.2 2-D Discrete Wavelet Transform

To use the wavelet transform for image processing we must implement a 2D version of the analysis and synthesis filter banks. In the 2D case, the 1D analysis filter bank is first applied to the columns of the image and then applied to the rows. If the image has N1 rows and N2 columns, then after applying the 1D analysis filter bank to each column we have two sub band images, each having N1/2 rows and N2 columns; after applying the 1D analysis filter bank to each row of both of the two sub band images, four sub band images are obtained, each having N1/2 rows & N2/2 columns. This is illustrated in the diagram below. The 2D synthesis filter bank combines the four sub band images to obtain the original image of size N1 by N2.

## 3 Dual Tree CDWT

It turns out that, for some applications of the discrete wavelet transform, improvements can be obtained by using an expansive wavelet transform in place of a critically sampled one. There are several kinds of expansive DWT's; here the dual tree complex discrete wavelet transform is described. The dual tree complex DWT of a signal x is implemented using two critically sampled DWT's in parallel on the same data, as shown in the figure.

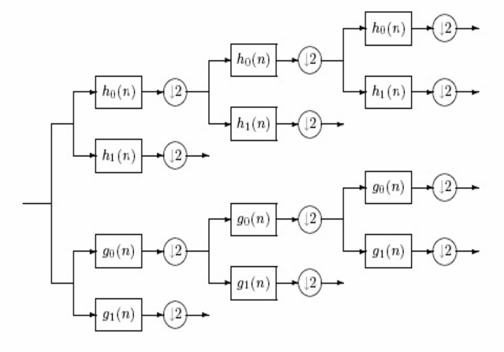



Fig. 3. Dual Tree Complex DWT

The transform is 2-times expansive because for an N-point signal it gives 2N DWT coefficients. If the filters in the upper and lower DWTs are the same, then no advantage is gained. However, if the filters are designed is a specific way, then the sub band signals of the upper DWT can be interpreted as the real part of a complex wavelet transform, and sub band signals of the lower DWT can be interpreted as the imaginary part. Equivalently, for specially designed sets of filters, the wavelet associated with the upper DWT can be approximate Hilbert transform of the wavelet associated with the lower DWT.

When designed in this way, the dual tree complex DWT is nearly shift invariant, in contrast with the critically sampled DWT. Moreover, the dual tree complex DWT can be used to implement 2D wavelet transforms where each wavelet is oriented, which is especially useful for image processing. The dual tree complex DWT outperforms the separable DWT for applications like image denoising and enhancement.

There are two versions of the 2D dual tree wavelet transform: the real 2-D dual tree DWT is 2 times expansive, while the complex 2-D dual tree DWT is 4-times expansive. Both types have wavelets oriented in six distinct directions.

## 4 Bivariate Shrinkage Function for Image Denoising

To model the statistics of wavelet coefficients of images, a new simple non-Gaussian bivariate probability distribution function is proposed in this paper. The model captures the dependence between a wavelet coefficient and its parent. Using Bayesian estimation Theory, this model is derived, which generalizes the soft thresholding approach. The new shrinkage function, which depends on both the coefficient and its parent, yields improved results for wavelet based image denoising.

Let w2 represent the parent of w1. Then, y = w+n. Where w = (w1, w2), y = (y1, y2) and n = (n1, n2). The noise values n1,n2 are zero mean Gaussian. Based on the empirical histograms, the non-Gaussian bivariate PDF is given by,

$$p_{\mathbf{w}}(\mathbf{w}) = \frac{3}{2\pi\sigma^2} \cdot \exp\left(-\frac{\sqrt{3}}{\sigma}\sqrt{w_1^2 + w_2^2}\right)$$

With this PDF, w1 and w2 are uncorrelated, but not independent. The MAP estimator of w1 yields the following bivariate shrinkage function

$$\hat{w}_1 = \frac{\left(\sqrt{y_1^2 + y_2^2} - \frac{\sqrt{3}\sigma_n^2}{\sigma}\right)_+}{\sqrt{y_1^2 + y_2^2}} \cdot y_1 .$$

For this bivariate shrinkage function, the smaller the parent value, the greater the shrinkage. This is consistent with other models, but here is derived using a Bayesian estimation approach beginning with the new bivariate non-Gaussian model.

### 4.1 Local Adaptive Image Denoising

Using the bivariate function above, an effective and low complexity locally adaptive image denoising algorithm is developed. The algorithm is summarized as follows.

1. Calculate the noise variance.
2. For each Wavelet Co-efficient,

    a. Calculate signal variance.
    b.Estimate each coefficient using the bivariate shrinkage function.

## 5 General Method for Image Denoising

The classical method which employs soft thresholding image denoising algorithm and the new local adaptive denoising algorithm via bishrinkage function are as follows.

### 5.1 Soft Thresholding

A denoising method called soft thresholding is applied to wavelet coefficients through all scales and subbands. The coefficients with values less than the threshold (T) to 0, then subtracts T from the non-zero coefficients.

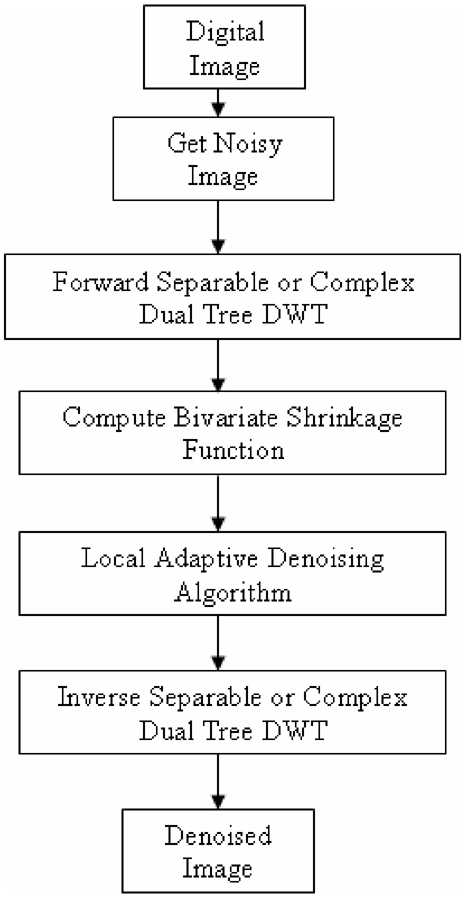
### 5.2 PSNR

$$PSNR = 20 \log_{10} \left( \frac{255}{RMSE} \right)$$

Where,
RMSE – Root Mean Square Error

### 5.3 New Method: Bivariate Shrinkage Function + Local Adaptive denoising algorithm

The Method which uses both Bivariate Shrinkage Function together with Local adaptive denoising algorithm is described here.



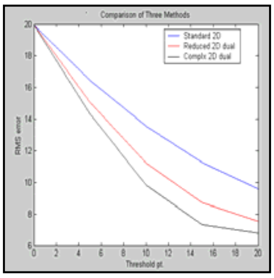

## 6 Results & Discussions

From the resulting images, it is clear that the Complex 2-D dual tree method removes more noise signal than separable 2-D method does. Actually, 2-D dual tree method outperforms separable method. This can be proved by the rms Vs threshold points.

### 6.1 Results via Soft Thresholding

Original Noisy

Separable DWT Complex Dual Tree DWT




Rms Vs Threshold Plot

### 6.2 Results via Bivariate Shrinkage Function

Separable DWT

Complex Dual Tree DWT

The denoised image obtained using soft thresholding has a PSNR of 31.45 dB. The denoised image obtained using bivariate shrinkage function has a PSNR of 34.58 dB. Thus the local adaptive thresholding algorithm via bivariate function gives better performance over the classical method. The values are tabulated as follows.

| Methods | Soft Threshloding in dB | Bivariate Shrinkage in dB |
|---|---|---|
| Separable Method DWT | 28.46 | 31.25 |
| Complex Dual Tree DWT | 31.45 | 34.58 |

## 7 Conclusion & Future Scope

In this paper, first a new bivariate PDF is proposed for wavelet coefficients and second a new bivariate shrinkage function is derived from it. The proposed PDF is a Laplacian bivariate PDF. This new rule maintains the simplicity, efficiency and intuition of classical soft thresholding approach. In future, this work can be extended by using different types of wavelets also.

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