

# The Transfer Mechanism of Operational (Dynamic) Complexity in Supply Chains

AHMAD MAKUI – ALIREZA MADADI  
 Industrial Engineering Dept.  
 Iran University of Science and Technology  
 Tehran-Narmak-16844  
 IRAN

*Abstract:* - In a dynamic environment such as a supply chain, even basic supplier-customer systems with structurally simple information and material flow formations have a tendency to exhibit operational complexity. The operational complexity of supplier-customer systems is associated with the uncertainty of information and material flows within and across organizations. Operational complexity or dynamic complexity can be variable with the ad hoc order, unreliable deliveries, demand fluctuations, alterations to specifications, effectiveness of the management and other unpredicted variations in information and material variation in supply chain. A few works have been accomplished to measure and define operational complexity of supply chain. But there is not any convincing proof about the transfer mechanism of operational complexity from one tier/echelon to another tier/echelon. The focus of this research and the purpose of it, is to indicate how operational complexity transfer across supply chain by demand amplification or Bullwhip effect.

*Key-Words:* - Supply Chain, Operational Complexity, Lyapunov Exponent, Chaos.

## 1 Introduction

Many natural phenomena may be called complex systems, and complexity science is highly interdisciplinary. Examples of complex systems include ant-hills, ants themselves, human economies, nervous systems, cells and so forth, as well as human beings and their psychology, emotions, bodies and interactions. The effect and the role of complexity can be seen widely in a dynamic environment such as supply chain. The supply chain represents the flow of materials, information, and finances as they move in a process from supplier to manufacturer to wholesaler to retailer to consumer. Today's competitive global environment means accelerated new product introductions, production relocation, and increased outsourcing. At the same time, new market segments, customers, channels, and logistics partners are being added at a rapid rate. The result? Increasingly complex supply chains. But top-performing companies are able to control the escalation of complexity and manage the resulting supply chain more effectively. One of the key issues known to impact on the effectiveness of a supply chain that of uncertainty. Uncertainties in supply and demand are recognized to have a major impact on the performance of the manufacturing

function. This uncertainty is because of the dynamic environment of supply chain.

Three interacting yet independent effects would seem to cause the dynamic behavior experienced within supply chain [7]. These are deterministic chaos, parallel interactions and demand amplification. The combination of these effects can significantly increase the degree of uncertainty within a supply chain system. As the uncertainty of the supply chain increases, the system becomes operationally more complex to monitor and manage. The Operational Complexity of the supply chain is determined as the amount of information required to monitor the uncertainty associated with information and material flows [2], [4], [6], and [12]. As the Operational Complexity increases, there is an associated increase in the amount of information required to describe that system [3]. Shannon information theory proposes entropy as a measure for the expected amount of information required to describe the state of the system. In this research, we try to show how Operational Complexity transfers from one tier/echelon to another tier/echelon, by increasing variability of demand further upstream in the supply chain. This phenomenon is called Bullwhip effect [10].

## 2 Supply Chain and Supply Chain Management

In the previous section it is pointed that a Supply Chain refers to the distribution channel of a product, from its sourcing, to its delivery to the end consumer (also known as the value chain)(Figure 1). The supply chain is typically comprised of multiple companies who are increasingly coordinating activities via an extranet. Supply chain management is way to goes into improving the way your company finds the raw components it needs to make a product or service, manufactures that product or service and delivers it to customers. The following are five basic components for supply chain management.

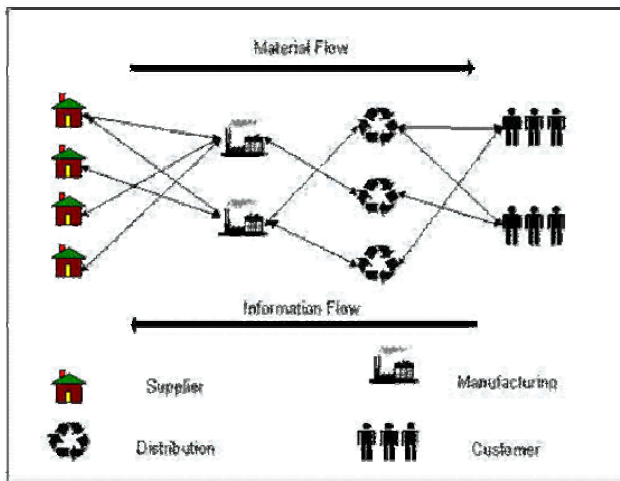


Fig. 1, a simple supply chain model

## 3 Bullwhip Effect

The term "Bullwhip Effect" was first popularized by three professors at Stanford University in 1995. The basic premise is that relatively small downstream changes in consumer demand result in progressively broader order fluctuations back up through the supply chain, which resembles the shape of a bullwhip. This results in blow up inventories at every level of the supply chain. (Figure 2)

Increasing variability of demand further upstream in the supply chain is named bullwhip effect [9], [10]. In other word, the variability of the orders received by the supplier is greater than the demand variability observed by the buyer [4]. Sheu showed

a multi-layer demand-responsive logistics control strategy for alleviating, effectively and efficiently, the bullwhip effect of a supply chain [16].

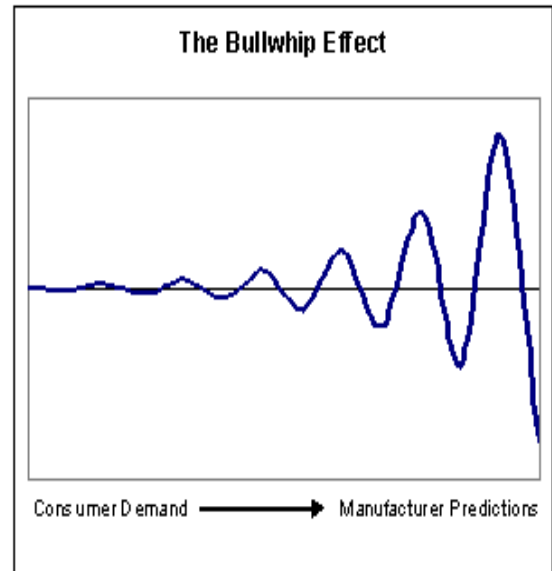


Fig 2, Bullwhip effect

## 4 Complex System

The term complex system formally refers to a system of many parts which are coupled in a nonlinear fashion [1]. Such a system may be discrete (such as a cellular automata system or set of difference equations), or it may be continuous as in a system of differential equations. Nonlinearity causes the complex systems to seem more than the sum of their parts because a linear system is subject to the principle of superposition, and hence is literally the sum of its parts, while a nonlinear system is not. Put another way: a linear relationship is simply one whose graph is a straight line, so a linear connection between two things is one in which change on one side of the connection induces proportional change in the other. A nonlinear connection means that a change in one side is not proportional to the amount of the change on the other side. When there are many non-linear ties in a system (many components),

behavior can be as unpredictable as it is interesting.

## 5 Chaos and Complexity

Complexity and chaos theory have already generated an impressive literature, and a specialized vocabulary to match. This introduction can, at most, sketch in the general area of intellectual activity, and hope to draw the sting of the terminology. The works cited above are possible starting points for those wishing to pursue the subject in more depth.

The more general name for the field is complexity theory (within which 'chaos' is a particular mode of behavior). It is concerned with the behavior over time of certain kinds of complex systems. Over the last 30 years and more, aspects of this behavior became the focus of attention in a number of scientific disciplines. These range as widely as astronomy, chemistry, evolutionary biology, geology and meteorology. Indeed there is no unified field of complexity theory, but rather a number of different fields with intriguing points of resemblance, overlap or complementarily. While some authors refer to the field as 'the science of complexity', others more modestly and appropriately use the phrase in the plural.

The systems of interest to complexity theory, under certain conditions, perform in regular, predictable ways; under other conditions they exhibit behavior in which regularity and predictability is lost. Almost undetectable differences in initial conditions lead to gradually diverging system reactions until eventually the evolution of behavior is quite dissimilar. The most graphic example of this is the oft-quoted assertion that the flapping of a butterfly's wing can in due course decisively affect weather on a global scale.

The systems of interest are dynamic systems – systems capable of changing over time – and the concern is with the predictability of their behavior. Some systems, though they are constantly changing, do so in a completely regular manner. For definiteness, think of the solar system, or a clock pendulum. Other systems lack this stability: for example, the universe (if we are to believe the 'big bang' theory), or a bicyclist on an icy road. Unstable systems move further and further away from their starting conditions until/unless brought up short by some over-riding constraint – in the case of the bicyclist, impact with the road surface.

## 6 Chaos in Supply Chain

Since the late 1950s it has been recognized that the systems used internally within supply chains can lead to oscillations in demand and inventory as orders pass through the system. The uncertainty generated by these oscillations can result in late deliveries, order cancellations and an increased reliance on inventory to buffer these effects. Despite the best efforts of organizations to stabilize the dynamics generated, industry still experiences a high degree of uncertainty from this source. From Wilding [7] point of view the supply chain complexity triangle can be describes as the interaction of deterministic chaos, parallel interactions and demand amplification. Supply chains according to chaos theory can be viewed as system that is highly sensitive to initial conditions. According to Wilding [7], "an infinitesimal change to a system variable's initial condition may result in a completely different response." In supply chain networks, chaos can be generated by both management decision-making and the computer control algorithms. The ground moves in a Supply Chain network such that the collective dynamics of the system at conditions far from equilibrium can be shifted in many sectors by what one or two links in the chain do. In sum, complex response patterns result from what a supply chain manager, a computer program protocol, or suppliers do in a seemingly benign and isolated part of the network.

## 7 Theoretical Development

In this paper two methods are defined to evaluate the complexity of supply chain. One is information-theoretic point of view [13], [5], and the other one is based on chaos theory and uses the Lyapunov exponent to estimate the rate at which a system loses information. [7], [14], [15].

Operational complexity  $H_D$  is defined as the expected amount of information necessary to describe the state of the system's deviation from the schedule. The calculation involves measurement of the difference between system's actual performance and the expected performance predicted in the schedule. It can be calculated according to the equation below.

$$H_D = -P \log_2 P - (1-P) \log_2 (1-P) - (1-P) \sum_{i=1}^m \sum_{j=1}^{n_s} p_{ij} \log_2 p_{ij}$$

(1)

Where:

$P$  is the probability of the system being “in a tolerated state”, e.g. the states that comprise the schedule would correspond to the tolerated states,

$(1-P)$  is the probability if the system being in a “non-tolerated state”, i.e. a state deviating from the scheduled state or the tolerated range as scheduled,

$m$  is the number of resources,

$n_s$  is the number of non-tolerated states,

$p_{ij}$  is the probability of resource  $i$  being in non-tolerated state  $j$ ,

$i, j$  are indices of resource and non-tolerated state respectively.

Operational complexity formula hence reflects uncertainty both in quantity and variety coming from the customer, the process and the supplier, goodness of the planning and scheduling adaptable to the uncertainty, and the level of flexibility or agility of the process which restricts the planning or scheduling in dealing with the uncertainty.

Chaos theory investigates the irregular dynamic behavior of non-linear systems. Also Chaos theory investigates the properties of system that can be defined according to equations of the form:

$$X_{n+1} = f(X_n) \quad (2)$$

Such systems may show extreme sensitivity to the initial value of  $x$ , i.e.,  $x_0$ . However, time-plots of the state of the system may show its behavior as being approximately cyclical (orbiting around and attractors). Deshmukh [15] uses chaos – theoretic measure for Operational Complexity, i.e. the Lyapunov exponent.

The Lyapunov exponent [11] is used to measure the extent to which ever-smaller “infinitesimal” errors is amplified. On the other hand, the Lyapunov exponent measures how sensitive a system is to errors.

Lyapunov exponent i.e.  $\lambda$  is defined as bellow [3]:

$$|x_{n+1} - x_n| = |x_1 - x_0| \exp(\lambda n) \quad (3)$$

Rearranging:

$$\lambda = \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=0}^{n-1} \ln |f'(x_k)| \quad (4)$$

If this number is positive ( $\lambda > 0$ ), then this means that small errors will tend to magnify, and the system is showing sensitivity. If this number is negative ( $\lambda < 0$ ), then the small errors tend to be reduced and the system is showing stability. So, the Lyapunov exponent gives us a way to measure the

stability of our system. In order to show how Operational Complexity is transferred between supply chain, Bullwhip effect is investigated. The Bullwhip effect, which characterizes the dynamic of supply chain operation, reflects an increase in demand variability while moving upwards the supply Chain [8]. The bullwhip can be measured by the following equation:

$$\frac{\text{var}(\mathbf{X}^g)}{\text{var}(\mathbf{D})} \quad (5)$$

That  $X_g$  is the amount ordered in echelon “ $g$ ” of the supply chain and “ $D$ ” is the customer demand.

Referring to the definition of Lyapunov exponent, it can be seen that in presence of bullwhip effect,  $\lambda$  will be greater than zero. Suppose again equation (3).

$$|x_{n+1} - x_n| = |x_1 - x_0| \exp(\lambda n)$$

$(x_{n+1} - x_n)$  can be interpreted as the order variance and  $(x_1 - x_0)$  as the demand variance.

So we will have:

$$(1/n) \ln(|x_{n+1} - x_n| / |x_1 - x_0|) = \lambda \quad (6)$$

And as in the presence of bullwhip,  $(|x_{n+1} - x_n| / |x_1 - x_0|)$  is greater than one, equation (6) and  $\lambda$  will be greater than zero. This means that the bullwhip effect can cause the chaos in system.  $\lambda$  is the expected amount of the information that are being lost to the system, and the dynamic complexity is the expected amount of information necessary to describe the state of the system’s deviation from the schedule. To keep a supply chain on schedule, these two amounts of information must be equal. So by transferring the bullwhip through echelons in the supply chain, a magnified  $\lambda$  is transferred and with it, operational complexity goes by echelons.

## 8 Case Study

The case study involved a simple two-stage in a supply chain consisting a single retailer and a single manufacture. It can be shown when there is an increase or amplification in demand variability (Bullwhip effect). Then Lyapunov exponent ( $\lambda$ ) can be equal positive quantities. Also the relationship between Operational Complexity, chaos theory and Bullwhip effect can be obtained. On the other hand, it can be proved when Bullwhip effect increases, then chaos measure and Operational Complexity will increase too.

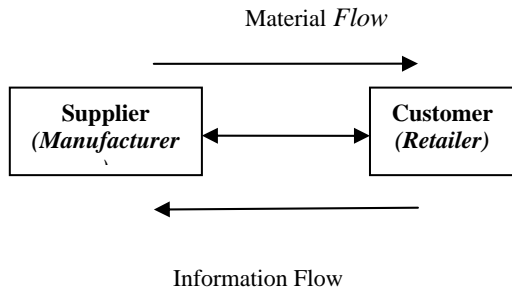


Fig.3, Supposed model

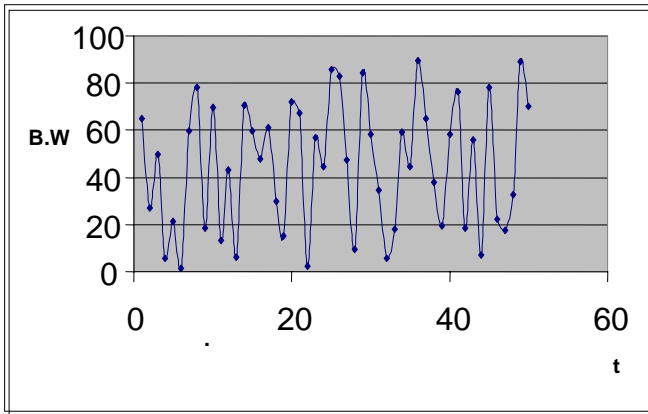


Fig.4, Demand amplification between retailer and manufacturer

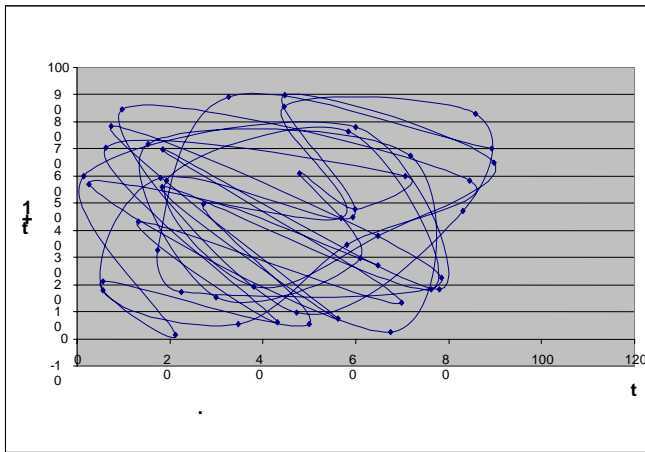


Fig.5, Return map of data from figure 4 that shows a cyclical behavior

## 9 Conclusion

The contribution of this research is three-fold. First, when Bullwhip effect increases, then the system is showing sensitivity, so that the errors

tend to be magnified across the supply chain. Second, there is a direct relationship between Bullwhip effect and operational complexity. Third, it is defined that one of the effects would seem to cause the dynamic behavior within supply chain is demand amplification and it can increase the degree of uncertainty within a supply chain system. Also, the operational complexity is referred to uncertainty of supply chain. At the end, because of the moving property of the Bullwhip effect within tiers of supply chain, operational complexity can be transferred.

### References:

- [1] G. Frizelle, *The Management of Complexity in Manufacturing*, Business Intelligence Ltd. 1998.
- [2] S. Sivadasan, *Operational Complexity of Supplier-Customer Systems*, Ph.D. thesis, Oxford University, 2001.
- [3] J. Efstathiou, S. Kariuki, L. Huaccho Huatuco, S. Sivadasan and Calinescu, *The Relationship between Information-Theoretic and Chaos-Theoretic Measures of the Complexity of Manufacturing Systems*, Technical Report, Department of Engineering Science, Oxford University, 2001.
- [4] S. Sivadasan, J. Efstathiou, An Information-Theoretic Methodology for Measuring the Operational Complexity of Supplier-Customer Systems, *International Journal of Operations & Production Management*, Vol.22, No.1, 2002.
- [5] A.V. Deshmukh, J.J. Talavage, M.M. Barash, Complexity in Manufacturing Systems, Part 1: Analysis of Static Complexity, *IIE Transactions*, Vol 30, 1998, pp.645-655.
- [6] S. Sivadasan, J. Efstathiou, A. Calinescu, and L. Huatuco Huatuco, Interpreting Information Theoretic Measures of Complexity at the Supplier-Customer Interface, *Proceedings of the 11th Institute of Operations Research and the Management Sciences Applied Probability Society Conference (INFORMS APS 2001)*, 25 th-27 th July 2001.

- [7] R. Wilding, The Supply Chain Complexity Triangle, *International Journal of physical Distribution & Logistics Management*, Vol.28 No.8, 1998.
- [8] F. Chen, Z. Drezner, J.K. Ryan, D. Simchi-Levi, Quantifying the Bullwhip Effect in a Simple Supply Chain: The Impact of Forecasting, Lead Times, and Information, *Management Science*, Vol.46, No.3, March 2000.
- [9] H.L. Lee, V. Padmanabhan, and S. Whang , The Bullwhip Effect in Supply Chains, *Sloan Management Review*, Vol.38, Issue 3, Spring 1997.
- [10] H.L. Lee, V. Padmanabhan, and S. Whang, Information Distortion in a Supply Chain: The Bullwhip Effect, *Management Science*, Vol. 43, No.4, 1997, pp.546-548.
- [11]A. Wolf, J.B. Swift, H.L.Swinney , Determining Lyapunov exponents from a time Series, *Physica D*, pp. 285—317, Vol. 16, 1985.
- [12] S. Sivadasan, J. Efstathiou, A. Calinescu, L. Huaccho Huatuco, Advances on Measuring the Operational Complexity of Supplier-Customer Systems, *European Journal of Operational Research*, Vol. 171, Issue. 1, 2006, pp.208-226.
- [13] G. Frizelle, E.Woodcock, Measuring Complexity as an Aid to Developing Operational Strategy, *International Journal of Operations & Production Management*, Vol. 15, No.5, 1995.
- [14] R. Wilding, *An investigation into sources of uncertainty within industrial supply chains; Amplification, deterministic chaos and parallel interactions*, Ph.D. thesis, University of Warwick, 1997.
- [15] A.V. Deshmukh, *Complexity and chaos in Manufacturing Systems*, Ph.D. thesis, Purdue University, 1993.
- [16] Sheu Jih-Biing, A multi-layer demand-responsive logistics control methodology for alleviating the bullwhip effect of supply chains, *European Journal of Operational Research*, Vol. **161**, No.3, **2005**, pp. **797-811**.