Comparison of MCDM Methods for Assessment of Proposed Investment Alternatives in an Environmental System

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Abstract: This paper presents the possibility of using a Multiple Criteria Decision Making (MCDM) in a real environmental system. It is concerned with an assessment of proposed investment alternatives not only under the economical viewpoint, but also under all the other viewpoints required, i.e. environmental and social. The MCDM methods with criteria weighting have been compared for evaluation of proposed environmental investments alternatives for the Central Sewage Plants. Three options of final liquidation of sewage sludge have been proposed. Algorithms of methods were processed using the program system MATLAB.

Key-Words: Environmental System, Assessment of Investment Alternatives, Sewage Sludge, MCDM

1 Introduction

During the last few years, we have seen a significant advance in public administration environment, as a result of adoption of computational methods of management science [4]. In this field, the application of methods that uses a criteria weighting in the decision making process [3,5,7,10,15] can be used. As a good example, the application of MCDM methods in the environmental planning and management can be mentioned [6].

Generally speaking, a decision problem [4,12] involves a set of objects (alternatives, actions, courses of action, states, competitors, etc.) described or evaluated by a set of attributes (criteria, features, issues, etc.). Independently of further interpretation, a decision situation may be represented by a table of rows which corresponds to objects and columns to attributes; for each pair (object-attribute) there is a corresponding descriptor. We can also say that the table represents facts know about the decision situation. Typically, one or several decision makers (experts, agents, nature, etc.) are also involved in a decision making process. A decision maker is a person who has a number of alternatives and who must choose the best one under a reduced level of uncertainty. The objective of the decision-maker is to choose the best alternative. Each alternative will have one of several possible consequences, and the probability of the occurrence for each consequence is known.

As soon as the decision is made, the events may occur, that are out of the decision-maker’s control. Each combination of alternatives, followed by an event to occur, leads to a measurable value outcome. Managers make decisions in complex situations. They must reduce the complex problems to their simplest possible form. The attributes used to describe the objects are built on some elementary features of the objects. They may be nominal (also called categorical or quantitative, e.g. male or female) or cardinal (also called non-nominal or quantitative, e.g. finance ratios or temperature). Three most common decision problems can be distinguished: choice, classification and ranking.

The selection can be realized on the basis of numerous methods. The methods are applied with the knowledge of decision makers. Knowledge can be expressed by evaluation of the criteria, i.e. by the definition of point importance or weights for criteria and/or by an elimination of the subordinate criteria. The most general question is the probability involved in the explanation of the decision situation. The explanation means discovering the important facts and dependencies in the table describing the decision situation. The information about the decision situation is usually vague because of uncertainty and imprecision resulting from many sources. The degree of uncertainty may be described by fuzzy logic [1,9,10,11].

2 Problem Formulation

Environmental system can be defined as large and complex system which works with a great number of dependent factors [6,8]. These factors can be expressed by a quick change, incompleteness and disproportion. The creation of a general decision making process for the environmental system is difficult.
As an example of environmental system can be used a problem environmental investments assessment [8]. Its main problem is to define the environmental benefits of a project. There exist options of monetary and non-monetary evaluation in the case of choosing among a lot of alternatives (variants) of end of pipe technologies. The importance of environmental investment will continue to grow both in the form of end of pipe technologies and which is even better, as an integrated investment of preventive approach. If the effectiveness of an environmental investment is assessed, the problem would be assessed from two viewpoints. The first one is the degree of a goal achievement, e.g. minimization of harmful substances emitted, and the second one is the increase in the value of the achieved effect per unit of the invested costs. In a purely environmental investment, the summary in one economic criterion is often rather problematic for environmental effects.

There are different stakeholders and their information needs in the environmental area: suppliers and customers (householders, citizens), financiers, investors, employees, communities, authorities, environmental organizations. The term “efficiency of environmental investments” should be extended, in particular to company view, with the LCA (life cycle assessment) viewpoint. Eco-efficiency expresses the efficiency with which ecological resources are used to meet the economic goals already established [2,14]. The efficiency is the achievement of specific output goals with minimum level if inputs.

The means spent by the society in an ineffective manner present a loss no matter whether these product or non-product investments. As for investments ranking in the second category, it should be taken into account that their assessment is much more problematic and demanding, and also risky as far as the result is concerned. The minimum costs, the maximum effects or a combination of both, are the criteria for absolute assessment (to implement or to dismiss) or for a selection from a certain number of variants. At present, there is a great number of methods used to assess the effectiveness and efficiency of investments. Each of them is a certain guideline for further analysis and decision-making, however none of the recommended methods can serve as a generally applicable one.

The contribution focuses on the eco-efficiency as an integration of economy and ecology criteria. In formal way, eco-efficiency is a ratio. It can be monitored through two generic indicators: eco-intensity and its inverse, the eco-productivity. Some costs and benefits are intangible, as far as environment is concerned. The valuation of environmental benefits (stability of the ecosystem, visibility, value of the nature, etc.) considering the environmental investment (like sewage, dust in cyclones, etc.) presents problem itself.

The reason is the impossibility of comparable assessment of the individual consequences. It is possibility to be solved on the basis of MCDM methods. The use of traditional MCDM process P, is determined by the set of decision alternatives $A = \{a_1, a_2, \ldots, a_n\}$ and by the set of criteria $C = \{c_1, c_2, \ldots, c_m\}$, according to which the desirability of an alternative is to be evaluated. It can be expressed in a decision matrix $R(n,m)$. Process $P$ can be expressed by the following way:

$$P = \{ A, C, R(n,m) \}. \quad (1)$$

where a matrix element $r_{ij}$ indicates the performance rating of the $i$-th alternative $a_i$ with respect to the $j$-th criterion $c_j$.

3 MCDM Methods

The problem in selection of the most feasible alternative $A$ is solved by classic method [4,12], i.e. the method based on point evaluation (PO) $e_i$ of criterion $c_j$ the weight method (WE) of evaluation criteria and the method of analytic hierarchy process (AHP). It is necessary to determine the maximum and minimum criteria and transform value $r_{ij}$ to value $\ast r_{ij}$, where $\ast r_{ij} \in [0,1]$.

The PO method can use a scale from 1 to 10 (or 9), in which 10 represents the important criterion. On the basis of $e_i$ it is possible to compute a vector of general weights $WE = \{we_1, we_2, \ldots, we_m\}$ for criteria $j$ in WE method. Elements $we_m$ can be defined the following way:

$$we_j = e_j / \sum_{j=1}^{m} e_j. \quad (2)$$

A large system is appropriately broken up into subsystems with their respective collection and interrelations. Then, a multi-level is created hierarchical structure. An elementary example of a hierarchy structure has three levels (the first level-goal of the decision, the second level-criteria and the third level-alternatives). If the criteria $c_j$ have a different importance in MCDM process, a criterion importance, related to some other criterion, is given by a weight $w_j$. One of the methods for calculation of weights $w_j$ is given in the AHP method [5,6,13].

The AHP method points out to an effective decision under difficult situations. It is the method of analysis of difficult unstructured situation which separates the problem hierarchically into several different groups (also called levels, clusters, stratum) with easy elements so called hierarchy structure. Hierarchy is a particular type of system, which is based on the assumption that the entities, which we have identified, can be grouped into disjointed sets, with the entities of one group influencing the entities of only one other group and being influenced by the entities of only one other group [4]. The AHP [13] is possible applied to the easiest type of hierarchy
structure of MCDM. We wish to find their weights of influence: the vector \( w = \{w_1, w_2, ..., w_m\} \) and the matrix \( V = \{v_{ij}, v_{1j}, v_{2j}, ..., v_{mj}\} \) on some \( A = \{a_1, a_2, ..., a_n\} \) and \( C = \{c_1, c_2, ..., c_m\} \). Basic problems of the method are [4,5,13]:

- Subjective evaluation of the pair wise comparison of the criteria, which assigns to individual components a numerical value. It expresses relative importance individual criteria by weights \( w_j \) and \( v_{ij} \), where \( i = 1, 2, ..., n \) and \( j = 1, 2, ..., m \);
- Determine the judgment of the selection of alternative \( a_i \) with the highest priority on the basis of the multiplication i-row of the matrix \( V \) and the vector \( w = \{w_1, w_2, ..., w_m\} \);

There are situations when criteria \( c_j \) or their attainment by alternatives \( a_i \) cannot be defined or evaluated crisply. In such cases, we can use the method of fuzzy analytic hierarchy process (FAHP) [3,5].

The MCDM including subordinated criteria has been solved by independent assumption among criteria or by elimination of the criteria with subordination relation [10]. Therefore, it is suggested that the method of subordination relation has obtained the weighs of criteria eliminated subordination.

### 3.1 Fuzzy AHP Method

In this section, the MCDM problems are represented using fuzzy logic (fuzzy sets), as they were introduced in [1]. The method of FAHP makes use of the theory of fuzzy logic. The optimization of the selection of alternatives \( A \) in accordance with the compromise criteria \( C \) is realized by the application of the FAHP [3,5]. Basic problems of the method are:

- Create “fuzzy standard”, that means, to establish a membership function of criteria \( C \);
- Use method of FAHP to calculate aggregate weights for selection alternatives \( A \);

The algorithm of FAHP consists of the following steps.

The 1st step: Express MCDM process with a set of alternatives, a set of criteria and a decision matrix on the basis formula (1).

The 2nd step: On the basis of knowledge of experts (decision-makers), it is necessary to assign a fuzzy score \( \mu_{ai} \) for criteria \( C \) and alternatives \( A \) and give different weights \( \sim a_{ij} \) to each of the \( m \) criteria according to their attitude about the importance of the criteria. We can used PO method on the basis of the scale from 1 to 9 (it represents the important criterion). We come out from Saaty’s scale of relative importance [13] and the sets of fuzzy score \( \mu_{ai} \) and weights \( \sim a_{ij} \) are triangular fuzzy numbers [3] which are defined by the following way:

\[
\{(1,1,2), \sim a_{1j}, \sim a_{2j}, ..., \sim a_{ij} \}\]  

The 3rd step: Compute [5,6] the total fuzzy score \( T(a_{ij}) \) for \( a_i \) as taken as the fuzzy number \( \sim A \) (3). If \( S = a_1, a_2, ..., a_n \) then \( T(a_{ij}) \) can be calculated by the following expression:

\[
\sim T(S) = \sim a_{ij} \sim a_{m}\]  

The 4th step: Calculate the degree \( N_S \) (for \( S = a_i \)) by which the alternative selection matches the criteria by the following way:

\[
N_S = p_i / (p_1 + p_2 + ... + p_i), S = a_i, N_S = p_n / (p_1 + p_2 + ... + p_n), S = a_n,
\]

where \( p_i = (f_j + f_k) / 2 \); \( p_n = (G_k + H_k) / 2 \); \( G \) and \( H \) are triangular fuzzy numbers; \( f_k \) and \( h_k \) are left width of triangular fuzzy number; \( f_j \) and \( h_j \) are right width of triangular fuzzy number. The 5th step: Select the great value \( N_{OUT} \) of the best alternative \( a \) on the formula:

\[
a = N_{OUT} = \max\{N_S\}, S = a_1, a_2, ..., a_n.
\]

### 3.2 Fuzzy Subordination Relations Method

At first the subordination is investigated by interviewing the decision maker(s) and then the pure common weights are separately recomputed for the criteria using subordination relations. Secondly, the similar way of weight computation is applied to the modification of the evaluation score. With the revised weights and the evaluation score a more reasonable decision can be made [10]. It is important to establish the appropriate criteria and to determine the weight that represents the relative importance of selected criteria. Fuzzy subordination relations (FSR) in Cartesian product \( K \times K \) a membership function [1,11] of two variables

\[
d : K \times K \rightarrow [0,1],
\]

whose the different values \( d_{ij} \) denote the grade of the relation ship between any two evaluations criteria \( i \) and \( j \) in \( K \). Thus, \( d_{ij} \) means the grade in which criterion \( i \) is subordinated to \( j \).

The algorithm of FSR consists of the following steps:

The 1st step: Consider that the weights of criteria are \( w_{ei} \). Construct fuzzy subordination matrix \( D(m \times m) \) with elements \( d_{ij} \). The matrix represents a fuzzy subordination relation among criteria and is made through interview with decision makers. The FSR is reflexive

\[
d_{ii} = 1, \forall i, j \in K \text{ and } d_{ij} = \left\{ \begin{array}{ll}
1 & \text{if } d_{ij} = d_{ji} \\
0 & \text{if } d_{ij} \neq d_{ji}
\end{array} \right.,
\]

\[
d_{ii} = \{0,1\}, d_{ij} = d_{ik} \cdot w_{ei} / w_{ek},
\]

\[
d_{ik} = 0, d_{ki} = 0.
\]

(8)
where (1) criterion \( i \) is completely subordinate to criterion \( k \); (2) criterion \( i \) is partially subordinate to criterion \( k \); (3) criterion \( i \) and criterion \( k \) are mutually disjointed.

The 2nd step: Compute the weight matrix \( WW \) (pure matrix) consists of pure weights exclusive of the subordinate part among criteria \( WW = \{w_{w11}, w_{w12}, ..., w_{w1m}; w_{w21}, w_{w22}, ..., w_{w2m}; ..., w_{wm1}, w_{wm2}, ..., w_{wmn}\}. \)

In order to simplify the computation it is supposed that the subordinate part among three or more criteria is disregarded and only subordinate part between two criteria exists. It is considered that the subordinate part between two criteria is halved and distributed to each criterion. The elements \( w_{wj} \) of \( WW \) are weights of subordinate part between criteria \( i \) and \( j \) which are defined by the following way:

\[
w_{wj} = w_{wi} - \sum_{i \neq j} w_{wi} / 2 . \tag{9}\]

In the same way, it is denoted a score matrix \( HO^k \) for alternative \( k \) \( HO^k = \{ho_{11}^k, ho_{12}^k, ..., ho_{1m}^k; ho_{21}^k, ho_{22}^k, ..., ho_{2m}^k; ..., ho_{mn}^k\} \) for \( k = 1, 2, ..., n \). These elements \( ho_{ij}^k \) are represented by the evaluation score of subordinate part between criteria \( i \) and \( j \), which are defined by the following way:

\[
ho_{ij}^k = ho_{ji}^k = (R^k_i \cdot d_{ij} + R^k_j \cdot d_{ji}) / 2, i \neq j, \tag{10}\]

where \( R^k_i \) is defined on the basis of (1).

The 3rd step: Compute the total evaluation score \( CO^k \) \((k = 1, 2, ..., n)\) for each alternative by summing the value of multiple between the pure weight and the upper elements of matrix \( WW \) and the correspondent value of the matrix \( HO^k \) by the following expression:

\[
CO^k = \sum_{i=1}^{m} \sum_{j=i}^{n} w_{wj} \cdot ho_{ij}^k . \tag{11}\]

The 4th step: Choose the optimal alternative \( *a \) (it means \( CO^k \)) of by means of the total evaluation scores of alternatives on the formula:

\[
a = \max \{ a_k \} = \max \{ CO^k \} . \tag{12}\]

4 Applications of MCDM Methods in Environmental Systems

An example was presented for the judgment of optimal selection of the proposed investment alternatives. Methods of assessment were verified in an example of investment alternatives to dispose of sludge from sewage plant in the Central Sewage Plant in Prague (Czech Republic) in an economically effective manner.

The sludge drying is a basis for consideration because the use of other methods leading to complete stabilisation of sewage sludge in the environment may only be taken into account for highly drained sludge or dried sludge. The suitable alternative must respect the valid regulations for waste management.

Three alternatives of sewage sludge the liquidation are offered (see details in [8]):

- Alternative A: agricultural use of (dried) sludge in a controlled manner;
- Alternative B: incineration of dried sludge in the waste incineration plant in Malešice;
- Alternative C: incineration of dried sludge in the thermal power station in Mělník.

For further assessment of individual alternatives, an analysis of transport has been made in accordance with further criteria: the price, the level of risk for the environment, the transport restrictions during transportation, etc. It should be noted that the optimum solution to the problem already encountered has been looked for at this stage. Prevention is the strategy of the years to come and therefore the alternative of applying the strategy of a cleaner production which could minimize the waste created at the source have been taken into account. The best alternative has been selected on the basis of the total investment cost and total annual cost\(^1\) in million CZK [6]: for alternative A are 386.5 and 103.0; for alternative B is 406.0 and 123.9; for alternative C are 418.0 and 139.5. Verbal assessment of the impact of the investment on the environment:

- Alternative A: impact on soil-IMPORTANT, if the conditions of sludge handling are complied with, it is within legal standards; if the impact on the air-VERY LOW, it is within legal standards in the course of drying;
- Alternative B: impact on soil-ashes is expected to be deposited in a landfill secured in a respective manner, re-allocation of land for non-agriculture is necessary; impact on the air-LOW, use of the respective degree of flue gas cleaning;
- Alternative C: impact on soil-ashes is supposed to be disposed on a respective secured landfill, re-allocation of land for non-agriculture use is necessary; impact on the air-VERY LOW during the incinerating process the waste gases are considered within legal standards.

\(^1\) Total annual costs for individual alternatives of sludge liquidation including impact of inflation are show for the amount in sludge production; in year 2000: 219 metric tonnes a days with 30 percents of the dry substance in sludge, maximum 234 metric tonnes of drained sludge a day or 76 metric tonnes of dried sludge a day.
The selected methods of liquidation have undergone a procedure of gradual selection of alternatives—the methods of the set level of criteria. The total number of 14 criteria $C_i$ has been defined and has been divided in 4 groups according to the viewpoints: technical viewpoint $\{C_1, C_2\}$, economic viewpoint $\{C_3 \text{ to } C_5\}$, environmental viewpoint $\{C_6 \text{ to } C_9\}$ and social viewpoint $\{C_{10} \text{ to } C_{14}\}$, where: $C_i$ is the risk of technical feasibility [RU²], $C_2$ is the period of realisation [number of years], $C_3$ is the investment cost [mil. CZK], $C_4$ is the annual operating cost [mil. CZK], $C_5$ is the fee charged for liquidation [mil. CZK], $C_6$ is the noise load [RU], $C_7$ is the negative impact on soil [RU], $C_8$ is the bad smell load [RU], $C_9$ is the negative impact on air [RU], $C_{10}$ is the impact of harmful substances on the population during transport [RU], $C_{11}$ is the risk of contamination in the case of an accident [RU], $C_{12}$ is the transport distance to the place of liquidation [km], $C_{13}$ is the number of transits through the city [number of journey in a day] and $C_{14}$ is the transport demand [tonne kilometres per a year]. For the example of MCDM process $P$ was defined by the decision matrix on the basis of the Table 1. An expert (it means stakeholder) order of criteria is in Table 2, where 1 is the best important criterion.

### Table 1. Definition of decision problem

<table>
<thead>
<tr>
<th>Alternative</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>3</td>
<td>386.5</td>
<td>103.0</td>
<td>15.6</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>4</td>
<td>406.0</td>
<td>123.9</td>
<td>43.7</td>
</tr>
<tr>
<td>C</td>
<td>4</td>
<td>3</td>
<td>418.0</td>
<td>139.6</td>
<td>48.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
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<td>3</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{13}$</th>
<th>$C_{14}$ [10³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>3</td>
<td>45</td>
<td>3.9</td>
<td>912.6</td>
</tr>
<tr>
<td>B</td>
<td>2</td>
<td>15</td>
<td>3.9</td>
<td>304.2</td>
</tr>
<tr>
<td>C</td>
<td>2</td>
<td>55</td>
<td>3.9</td>
<td>1115.4</td>
</tr>
</tbody>
</table>

### Table 2. Expert order evaluation of criteria

<table>
<thead>
<tr>
<th>Evaluation</th>
<th>$C_1$</th>
<th>$C_2$</th>
<th>$C_3$</th>
<th>$C_4$</th>
<th>$C_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>order</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>4</td>
<td>5</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_6$</th>
<th>$C_7$</th>
<th>$C_8$</th>
<th>$C_9$</th>
<th>$C_{10}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>order</td>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>10</td>
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<table>
<thead>
<tr>
<th>Criteria</th>
<th>$C_{11}$</th>
<th>$C_{12}$</th>
<th>$C_{13}$</th>
<th>$C_{14}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>order</td>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>

2 RU is relatives units for individual verbal numerical scale of the respective criteria: 1 is zero effect, none, 2 is very little impact, 3 is little impact, 4 is considerable impact.

5 Conclusion

The AHP, FAHP and FSR are modern, progressive and effective methods for decision making process - being appropriate for the solution of problems of uncertainty related to environmental management. These methods also enable the synthesis of quantitative and qualitative information from multiple stakeholders and the integration of ecological indicators by ranking ecosystems in terms of environmental conditions and suggesting cumulative impacts across a large region. The application of these methods using MATLAB program provides the best alternative of a decision making process through computational models. It is interesting to note that application of each method has pointed to a different solution as the best alternative.

The authors have assumed that the choice based on the FSR method is better because it obtains the weights of criteria by eliminating the subordination of criteria in the process of decision system definition. Realization programs in MATLAB were designed for three alternatives and for arbitrary calculation of some criteria, with accent its graphic processing. This provides the user a compendious and easy work, besides the import of attributes with the help of scroll windows and the numerical computing and consecutive graphic depictions consequential attributes. This program can find its application in the assessment of

3 The consistency index as an indicator of „closeness to consistency“ for AHP method has been 0.0498.
new alternatives, especially in point decision making about selection of optimal variants.

References:

Appendix: The algorithm of AHP
The 1st step: Define of the hierarchy structure $H S$ of system by the following way: $H S = \{L_k\}, k = 1,2,3,$ where: $L_1 = \{g\}$ is the global goal of decision; $L_2 = \{c_1, c_2, ... , c_m\}$ are criteria and $L_3 = \{a_1, a_2, ... , a_n\}$ are alternatives (low level of the hierarchy structure).

The 2nd step: Define the Saaty’s matrix $S(m \times m)$ of the pair wise comparison of the criteria. This matrix $S$ is positive and reciprocal. Any set $S$ is a binary relation, which satisfies the reflexive, anti-symmetric and transitive law. The matrix has elements $s_{ij}$, where:

$$s_{ij} = w_i / w_j, s_{jj} = 1 / s_{ij}, s_{jj} = s_{ji} = 1.$$ (13)

Saaty’s scale of relative importance was used for assigning the values of matrix elements $s_{ij}$ [13].

The 3rd step: Calculate the largest eigenvalue $\lambda_{max}$ and eigenvector $\sigma = \{\sigma_1, \sigma_2, ... , \sigma_m\}$, $\sigma_i \geq 0$ of the matrix $S$ and vector of weights $w = \{w_1, w_2, ... , w_m\}$ on the basis of formula:

$$S \cdot \sigma = \lambda_{max} \cdot \sigma \text{ and } w_i = \sigma_i / \sum_{j=1}^{m} \sigma_j.$$ (14)

We take the consistency index (CI) as our indicator of “closeness to consistency” by the following way: $CI = (\lambda_{max} - m) / (m - 1)$. Generally, if this number is less than 0.1, we may be satisfied with our judgment.

The 4th step: Create new values $r_{ij}$ for maximum or minimum criteria $C$ by the following way:

$$r_{ij} = r_{ij} / \sum_{i=1}^{n} r_{ij} \text{ or } r_{ij} = (1 / r_{ij}) / \sum_{i=1}^{n} (1 / r_{ij}).$$ (15)

The 5th step: On the basis of principle of hierarchy composition compute $v_{ij}$ as element of the matrix $V$. We have to define of the Saaty’s matrix $S(n \times n)$, where $i = 1, 2, ... , m$ for the pair wise comparison of the values of “normalization” criteria and alternatives $A$. Calculate the largest eigenvalue $\lambda_{max}$ and eigenvector $\sigma = \{\sigma_1, \sigma_2, ... , \sigma_m\}$, $\sigma_i \geq 0$ of the matrix $S$ and vector of weights $v_{ij}$ on the basis of formula (14) and $CI$. 

The 6th step: Compose weights for alternatives $a_i$ in the hierarchy $H$ and $CI_{II}$ of the hierarchy by the following way:

$$a = \max \{a_i\} = \max \left\{ \sum_{j=1}^{m} v_{ij} \cdot w_j \right\}, i = 1,2, ..., n$$

and $CI_{II} = \max \{CI, \sum_{j=1}^{m} w_j \cdot CI_j\}$. (16)