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Model Reference Adaptive Control and Fuzzy Model Reference Learning Control for the Inverted Pendulum. Comparative Analysis

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Abstract: - The Inverted Pendulum is one of the most important classical problems of Control Engineering. Broom Balancing (Inverted Pendulum, pole on a cart) is a well known example of nonlinear, unstable control problem. The basic control objective of the inverted pendulum problem is to maintain the unstable equilibrium position, by controlling the force applied to the mobile cart in the horizontal direction.

In this paper we provide a comparative analysis of two model reference adaptive control (MRAC) methods, the first one based on the stability theory of Lyapunov and the other one based on fuzzy logic (FMRLC – Fuzzy Model Reference Learning Control). The performances of the proposed control algorithms are evaluated and shown by means of digital simulation.

Key-Words: - Lyapunov, Model Reference Adaptive Control, Fuzzy Model Reference Learning Control

1 Introduction

The control of the inverted pendulum system is a widely studied problem in the field of Control Engineering. Due to its characteristics (nonlinear, unstable), this system is used to demonstrate and test various control algorithms.

The system consists of a rigid pole connected by a hinge at its base, to a cart, which is constrained to move along a linear horizontal direction. A force is applied to this cart: if appropriate forces are applied the pole can be kept in various positions from falling over.

This paper uses this simple case study to examine the implementation of two model reference adaptive control algorithms and to compare their control performances. In MRAC, the technical demands and the desired input-output behavior of the closed-loop system is given via the corresponding dynamic of the reference model. Therefore, the basic task is to design such a control, which will ensure the minimal error between the reference model and the plant outputs despite the uncertainties or variations in the plant parameters and working conditions.

The first approach is the design of a model reference adaptive controller using the stability theory of Lyapunov. This theory assures that the equilibrium point 0 is asymptotically stable.

The design of controllers, using conventional techniques, for plants with nonlinear dynamics and modeling uncertainties can be often quite difficult. Fuzzy control is a practical alternative for a variety of challenging control applications, since it provides a convenient method for constructing nonlinear controllers via the use of heuristic information. However, some of the problems encountered in practical control problems, such as model uncertainties or the difficulty to choose some of the fuzzy controller's parameters, demand a way to automatically tune the fuzzy controller so that it can adapt to different operating conditions.

Based on a simple fuzzy logic controller we then focus on the design of a Fuzzy Model Reference Learning Controller (FMRLC).

In the end the performances of the two proposed control algorithms are evaluated and shown by means of digital simulation.

2 Dynamical Model of the Plant

The dynamic of the inverted pendulum system shown in Figure 1 is described by the following nonlinear equations.

The coordinates of the centre of gravity point *cg* in terms of pivoting point *p*, angle *Θ*, and length *l*:

$$\begin{aligned} X_{cg} &= X_p + l\sin(\Theta) \\ Y_{cg} &= Y_p + l\cos(\Theta) \end{aligned} \tag{1}$$

The sum of the forces in the horizontal and vertical direction:

$$\begin{aligned} H &= m\ddot{X}_{cg} \\ V &= mg + m\ddot{Y}_{cg} \end{aligned} \tag{2}$$

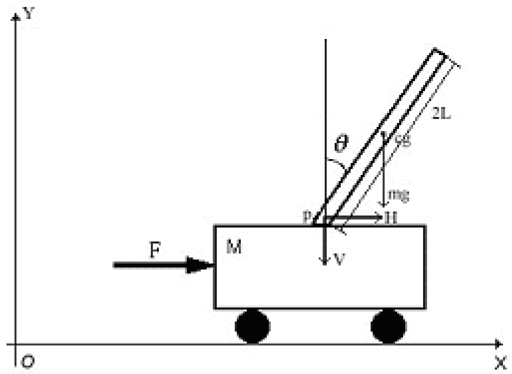

Fig.1: Inverted pendulum on a cart

Summing all the forces in the horizontal direction we get the first equation of motion:

$$F = M\ddot{X}_p + H \qquad (3)$$

The second equation of motion is given by the sum of moments around the centre of gravity *cg*:

$$Vl\sin(\Theta) - Hl\cos(\Theta) = I\ddot{\Theta} \qquad (4)$$

Assuming the pendulum is a uniform rod, its moment of inertia $I=ml^2/3$.

Combining these equations we get the final form of the nonlinear equations which describe the motion of the inverted pendulum system:

$$\begin{cases} (M+m)\ddot{X}_p + ml\ddot{\Theta}\cos(\Theta) - ml\dot{\Theta}^2\sin(\Theta) = F \\ mgl\sin(\Theta) - ml^2\ddot{\Theta} - m\ddot{X}_p l\cos(\Theta) = I\ddot{\Theta} \end{cases} \qquad (5)$$

The plant parameters are given in Table 1.

m	0.5 kg	Mass of pendulum
M	1 kg	Mass of cart
l	0.5 m	Distance between the pivot point *p* and the centre of gravity *cg* of the pendulum
G	9.81 m/s^2	Gravitational constant
I	0.0833 kg·m^2	Inertia of pendulum

Table 1: Plant parameters

A linear model for the system was developed to help designing the MRAC. Equations (5) were linearized about the equilibrium point $\Theta = 0$. For small angles we assumed the following approximations:

$$\sin(\Theta) = 0$$
$$\cos(\Theta) = 1 \qquad (6)$$
$$\frac{d^2\Theta}{dt^2} = 0$$

The following linear ISO (input state output) model was determined for the inverted pendulum:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \dfrac{g}{\frac{4}{3}l - \frac{ml}{m+M}}x_1 - \dfrac{1}{(m+M)(\frac{4}{3}l - \frac{ml}{m+M}} \\ y = x_1 \end{cases} \qquad (7)$$

Where $x_1 = \Theta$ is the angle of the pendulum, x_2 is the rotational speed of the rod, $u = F$ is the input to the system and $y = x_1$ is the system's output.

3 Model Reference Adaptive Control

This section discusses the design of the conventional Model Reference Adaptive Control (MRAC) as applied to the inverted pendulum system.

When the plant parameters and the disturbance are varying slowly, or slower than the dynamic behavior of the plant, then a MRAC control scheme can be used. This adaptive structure offers a superior performance and robustness in time than a classical PID controller [1].

Figure 2 shows the structure of the MRAC scheme designed for the inverted pendulum system.

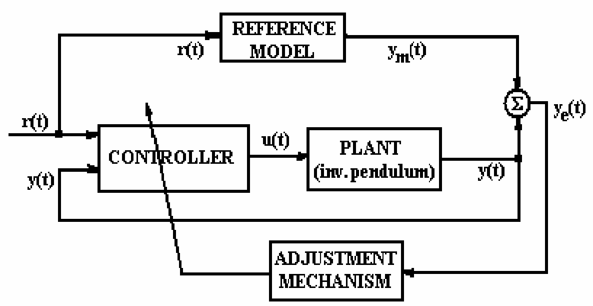

Fig. 2: Model Reference Adaptive Control

The MRAC structure consists of four main parts: the plant, the controller, the reference model and the adjustment mechanism.

The *reference model* is chosen to generate the desired trajectory, y_m, for the plant output y to follow. A standard second order differential equation was chosen as the reference model and the second order linear model of the plant was determined (8).

$$\begin{aligned} \ddot{y}_m &= -2\xi\omega_n\dot{y}_m - \omega_n^2 y_m + \omega_n^2 r \\ 0.5\ddot{y} &= 9.8y - 0.66u \end{aligned} \qquad (8)$$

Where: $\omega_n = 3$ *rad/sec* and $\xi = 1$.

The tracking (adaptation) error $y_e=y_m-y$ represents the deviation of the plant output from the desired trajectory. The *adjustment mechanism* uses this adaptation error to adjust the controller's parameters.

Depending on the method used to determine the adjustment mechanism, MRAC assures the stability and convergence of the adaptation error.

Due to the specifics of the plant and the reference model (second order model), the feedforward feedback controller for the inverted pendulum has three parameters k_1, k_2, k_3 and the control law takes the following form:

$$u = k_1 r - k_2 y - k_3 \dot{y} \tag{9}$$

For $k_1 = k_2$ the controller becomes a proportional derivative (PD) controller with the derivative component on the feedback loop.

The ISO dynamic equation that describes the closed-loop system, which contains both the controller and the plant, is given next:

$$\dot{x} = Ax + Br \tag{10}$$

where $x = \begin{bmatrix} y & \dot{y} \end{bmatrix}^T$ and

$$A = \begin{bmatrix} 0 & 1 \\ \frac{0.66k_2 + 9.8}{0.5} & \frac{0.66}{0.5}k_3 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ -\frac{0.66}{0.5} \cdot k_1 \end{bmatrix}.$$

The reference model is given by the following ISO system:

$$\dot{x}_m = A_m x_m + B_m r \tag{11}$$

where $x_m = \begin{bmatrix} y_m & \dot{y}_m \end{bmatrix}^T$ and

$$A_m = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\xi\omega_n \end{bmatrix},\ B_m = \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix}^T.$$

The differential equation that describes the adaptation error may be expressed by:

$$\dot{x}_e = A_m x_e + (A_m - A)x + (B_m - B)r \tag{12}$$

where $x_e = \begin{bmatrix} y_e & \dot{y}_e \end{bmatrix}^T$.

The adaptation laws for the controller's parameters are determined using Lyapunov's theory of stability.

The first major problem is to choose the proper positive definite function $V(t,x_e)$.

$$V = x_e^T P x_e + tr\{(A_m - A)^T \gamma_A (A_m - A)\} + \\ + tr\{(B_m - B)^T \gamma_B (B_m - B)\} \tag{13}$$

The equilibrium point $x_e=0$ is asymptotically stable if [1]:

- V is positive definite (P is positive definite);
- $V(0)=0$;
- dV/dt is negative definite.

The second problem is to obtain the derivative of the function V.

Matrix P ($2x2$) is the symmetric and positive definite solution of the Lyapunov equation (14). We assume that Q is the identity matrix.

$$A_m^T P + PA_m = -Q < 0 \tag{14}$$

The derivative of the Lyapunov function V is negative definite if we choose the following adaptation laws for the variant closed-loop system matrix:

$$\begin{aligned} \dot{A}(t) &= \gamma_A \cdot P \cdot x_e \cdot x^T \\ \dot{B}(t) &= \gamma_B \cdot P \cdot x_e \cdot r^T \end{aligned} \tag{15}$$

Solving equations (15), the adaptation laws for the controller's parameters are:

$$\begin{aligned} \dot{k}_1 &= -\gamma_1 \left(p_{12} \cdot y_e + p_{22} \cdot \dot{y}_e \right) \cdot r \\ \dot{k}_2 &= \gamma_2 \left(p_{12} \cdot y_e + p_{22} \cdot \dot{y}_e \right) \cdot y \\ \dot{k}_3 &= \gamma_3 \left(p_{12} \cdot y_e + p_{22} \cdot \dot{y}_e \right) \cdot \dot{y} \end{aligned} \tag{16}$$

In the adaptation laws (16) some terms were absorbed into the adaptation gains γ_1, γ_2, γ_3.

4 Fuzzy Model Reference Learning Control

This section discusses the development of the Fuzzy Model Reference Learning Controller (FMRLC) as applied to the inverted pendulum system. The FMRLC algorithm was first introduced in [2].

In order to develop the FMRLC a direct fuzzy controller was first designed.

4.1 Direct Fuzzy Control

The design of a direct fuzzy controller can be resumed to choosing and processing the inputs and outputs of the controller and designing its four component elements (the rule base, the inference engine, the fuzzification and the defuzzification interfaces) [4].

We consider the inputs to the fuzzy system: the error:

$$e(kT) = \Theta_r - \Theta \tag{17}$$

and change in error:

$$c(kT) = (e(kT) - e(kT-T)) / T \tag{18}$$

and the output variable the force applied to the cart

$$u = F \tag{19}$$

The universe of discourse of the variables (that is, their domain) was normalized to cover a range of [-1, 1] and scaling gains (g_e, g_c, g_u) were used to normalize. A standard choice for the membership functions was used with five membership functions for the three fuzzy variables (meaning $25 = 5^2$ rules in the rule base) and symmetric, 50% overlapping triangular shaped membership functions (Figure 3), meaning that only 4 ($=2^2$) rules at most can be active at any given time.

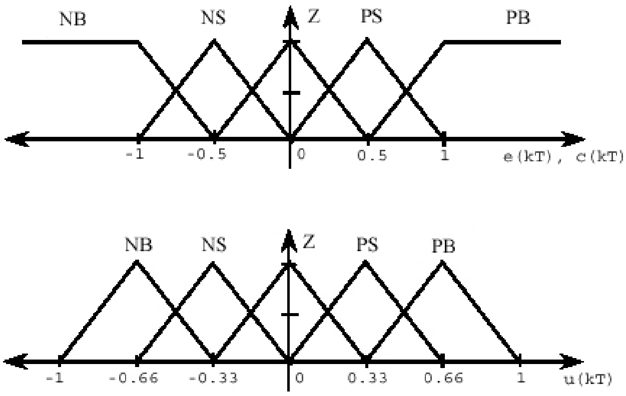

Fig. 3: Membership functions for the fuzzy controller

The fuzzy controller implements a rule base made of a set of IF-THEN type of rules. These rules were determined heuristically based on the knowledge of the plant. An example of IF-THEN rule is the following:

IF e is negative big (NB) and c is negative big (NB) THEN u is positive big (PB)

This rule quantifies the situation where the pendulum is far to the right of the vertical and it is moving clockwise, hence a large force (to the right) is needed to counteract the movement of the pendulum so that it moves toward zero.

The resulting rule table is shown in the Table 2.

"force" u		"change in error" c				
		NB	NS	Z	PS	PB
"error" e	NB	PB	PB	PB	PS	Z
	NS	PB	PB	PS	Z	NS
	Z	PB	PS	Z	NS	NB
	PS	PS	Z	NS	NB	NB
	PB	Z	NS	NB	NB	NB

Table 2: Rule base for the fuzzy controller

The *min-max inference engine* was chosen, which for the premises, uses maximum for the OR operator and minimum for the AND operator. The conclusion of each rule, introduced by THEN, is also done by minimum. The final conclusion for the active rules is obtained by the maximum of the considered fuzzy sets.

To obtain the crisp output, the *centre of gravity (COG)* defuzzification method is used. This crisp value is the resulting controller output.

4.2 Adaptive Fuzzy Control

In this section we design and implement a Fuzzy Model Reference Learning Controller (FMRLC), which will adaptively tune on-line the centers of the output membership functions of the fuzzy controller determined earlier.

Figure 3 shows the FMRLC as applied to the inverted pendulum system.

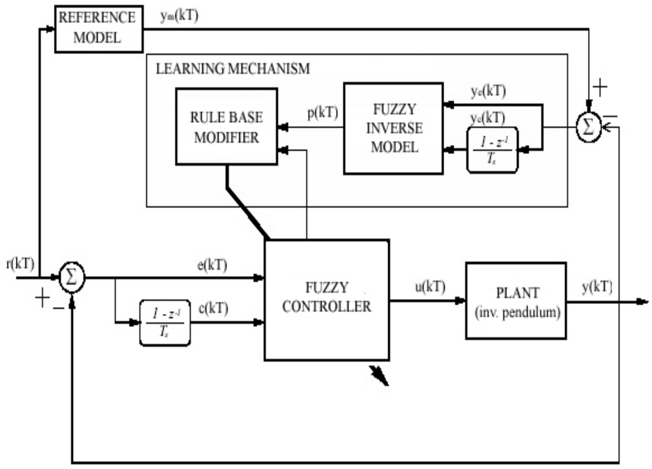

Fig. 4: Fuzzy Model Reference Learning Control

The FMRLC uses a *learning mechanism* that a) observes data from a fuzzy control system (i.e. *r(kT)* and *y(kT)*) b) characterizes its current performance, and c) automatically synthesizes and/or adjusts the fuzzy controller so that some pre-specified performance objectives are met [2].

In general, the *reference model*, which characterizes the desired performance of the system, can take any form (linear or nonlinear equations, transfer functions, numerical values etc.). In the case of the inverted pendulum FMRLC, no explicit dynamical reference model is used. The reference model takes the same form as the reference trajectory $y_m(kT) = r(kT)$ ($= 0$ for balance).

An additional fuzzy system is developed called *"fuzzy inverse model"* which adjusts the centers of the output membership functions of the fuzzy controller, which still controls the process, developed earlier [2, 3]. This fuzzy system acts like a second controller, which updates the rule base of the fuzzy controller by acting upon the output variable (its membership functions centers). The output of the inverse fuzzy model is an adaptation factor *p(kT)* which is used by the rule base modifier to adjust the centers of the output membership functions of the fuzzy controller. The adaptation is stopped when $p(kT)$ gets very small and the changes made to the rule base are no longer significant.

The *fuzzy controller* used by the FMRLC structure is the same as the one developed in the previous section.

The *fuzzy inverse model* has a similar structure to that of the controller (the same rule base, membership functions, inference engine, fuzzification and defuzzification interfaces. See section 4.1), and considering that the reference model is identical to the reference trajectory, the only notable difference between the two fuzzy systems is the normalizing gains values (g_{ye}, g_{yc}, g_p).

The inputs of the fuzzy inverse model are:

$$y_e(kT) = y_m(kT) - y(kT) = r(kT) - y(kT) \tag{20}$$

$$y_c(kT) = (y_e(kT) - y(kT)) / T \tag{21}$$

and the output variable is the adaptation factor $p(kT)$

An example of IF-THEN rule for the fuzzy inverse model is the following:

IF y_e is negative big (NB) and y_c is negative big (NB) THEN p is positive big (PB)

The *rule base modifier* adjusts the centers of the output membership functions in two stages:

1. the active set of rules for the fuzzy controller at time (k-1)T is determined

$$\mu(e(kT-T), c(kT-T)) > 0 \tag{22}$$

2. the centers of the output membership functions, which were found in the active set of rules determined earlier, are adjusted. The centers of these membership functions (b_j) at time kT will have the following value:

$$b_j(kT) = b_j(kT-T) + p(kT) \tag{23}$$

The centers of the output membership functions, which are not found in the active set of rules, will not be updated. This ensures that only those rules that actually contributed to the current output $y(kT)$ were modified. We can easily notice that only local changes are made to the controller's rule base. This local learning plays an important part since it will allow the controller to remember the adjustments made in the past, when it will encounter similar working conditions that led previously to those adjustments.

5 Results. Comparative analysis

This section presents the results we obtained using the two control schemes for the inverted pendulum problem. Both control structures were tested on the real plant (non-linear), to balance the pendulum in the vertical position ($\Theta = 0$) and to follow a square trajectory.

For the first control problem, balancing the pendulum in the vertical position, an angle $\Theta = 0.08rad$ was chosen as the initial condition for the simulation.

The second control problem is to test the capability of the closed-loop adaptive system to adapt or learn (in FMRLC case) a desired trajectory. The desired trajectory proposed for the simulation is a square input between [-π/10, π/10].

Figure 5 shows the response of the MRAC system as compared to the reference model response. The simulation shows that the MRAC scheme is practically impossible to use due to the high amplitude of the response (angle).

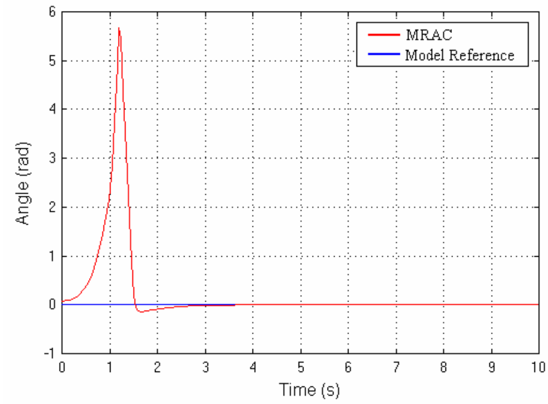

Fig. 5: Pendulum position. Equilibrium

The response of the MRAC system to a square trajectory is shown in Figure 6.

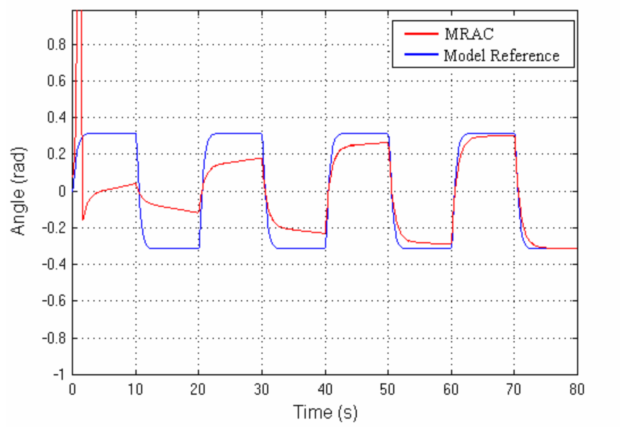

Fig. 6: Pendulum position. Square trajectory

The MRAC adaptation gains used for MRAC simulation were chosen experimentally: $\gamma_1=-90$, $\gamma_2=70$ and $\gamma_3=0.8$.

The FMRLC control algorithm was implemented with a digital structure, using a sampling time $T = 0.001s$. The input and output gains for the fuzzy controller and for the fuzzy inverse model were heuristically tuned to: $g_e=\pi/10$, $g_c=20$, $g_u=10$, $g_{ye}=\pi/10$, $g_{yc}=0.5$ and $g_p=1$

The same angle *Θ = 0.08rad* was chosen as the initial condition for the FMRLC simulation.

Figure 7 shows the response of the FMRLC system as compared to the response of the system controlled by the direct fuzzy controller, which served as the base for the FMRLC design. The improvement is obvious.

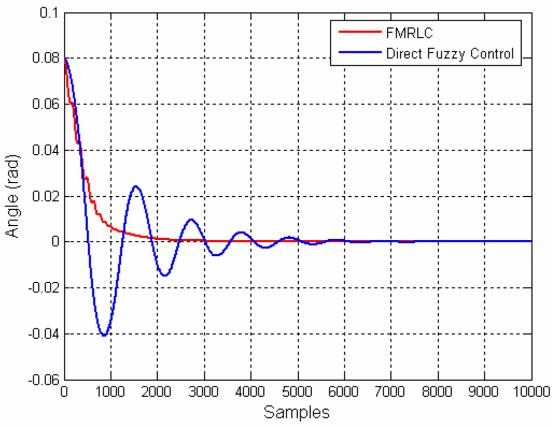

Fig. 7: Pendulum position. Equilibrium

To test the FMRLC's capability to learn (adapt) to different working conditions, the square trajectory between [-π/10, π/10] was used. The following figure shows the response of the system for this case.

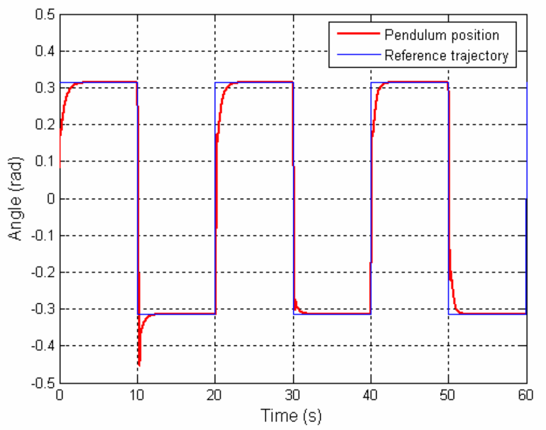

Fig. 8: Pendulum position. Square trajectory

All of the results presented above were obtained using digital simulation

6 Conclusions

This paper studied the implementation of two adaptive control techniques as applied to the balancing of the inverted pendulum system.

The MRAC scheme applies to systems with known dynamic structure, linear (second order system for our linearized plant) or nonlinear, but with unknown constants or slowly varying parameters. The adaptive controller designed for the inverted pendulum is inherently nonlinear. The MRAC system can handle large variations of the plant parameters with slow varying dynamic response. Otherwise, the stability of the closed-loop system and the convergence of the adaptation error are assured by the Lyapunov theory of stability.

The direct fuzzy controller allowed the use of heuristics (which model the way a human would control the process) via the use of the rule table. Since we generally know the way to balance the pendulum, the heuristics we chose in the design of the fuzzy controller proved very useful. The FMRLC took the controller design a step further by supplying an inductive update method, which produced an adaptive fuzzy controller.

Having analyzed the response of both control schemes it is obvious the FMRLC seems to perform better. Having a superior response time and a faster capacity to learn / adapt, the FMRLC proved itself capable to adapt to a wide range of working conditions (not only square trajectory) without the need to modify its parameters. On the other hand the MRAC wasn't able to provide a suitable solution to balance the pendulum, due to the very high overshoot. This shows that for this case study the adaptive fuzzy method we investigated has an advantage with respect to the more conventional MRAC method, since it allows more design flexibility, especially in the use of the reference model. However, there are certain tradeoffs required to achieve this success. These include a greater computational complexity for the intelligent controller and a greater design time required.

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