Limitations of one and two dimensional simulation of natural river flow

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Abstract: This paper describes the fundamental equations for two and one dimensional flow. It shows the assumptions which are necessary for their application for the simulation of natural river flow. Different resistance laws with the consideration of vegetation drag are presented. Furthermore, problems of the practical computation as well as of the theoretical analysis are pointed out. Common errors are reviewed.

Key-Words: flow in natural rivers, numerical simulation, two dimensional flow, one dimensional flow, DARCY WEISBACH resistance law, vegetation resistance, computational errors

1 Introduction

The simulation of flood flows through river systems has long been of interest and many models have been proposed and developed. The main interest is the description of the deformation of the hydrograph when a flood flow propagates along a water shed like in Figure 1.

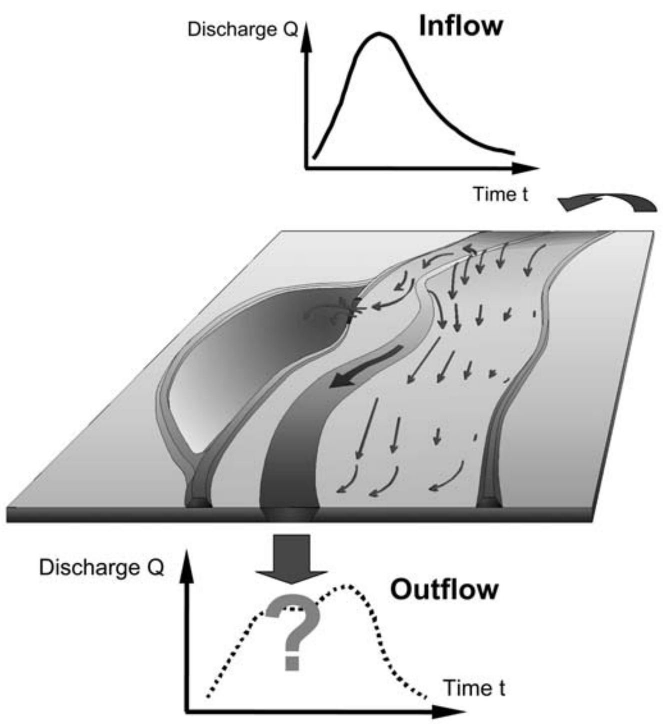

Fig. 1: Sketch of the deformation of a hydrograph in natural river flow with retention effects

Flood flow is a changing discharge with time, where at the beginning the discharge is increasing. It depends on the properties of the watershed how the hydrograph is deformed.

There are some models to describe the deformation with hydrological methods which are called flood rooting methods. But these models are empirical and do not consider the physics of the flow. Physically based are the hydrodynamic models which are derived from the Navier Stokes equations. They will be considered in this paper. It is the object of this article to show the assumptions which are necessary to derive them.

2 Governing Equations

The basic equations for hydrodynamics of river flow are the NAVIER STOKES Equations. They can be derived from the law of conservation of momentum with the assumption of a linear correlation between the gradients of velocities and the stress tensor. Furthermore the continuity equation for incompressible fluids holds in the following form:

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}+\frac{\partial v_z}{\partial z}=0 \qquad (1)$$

In natural rivers the flow is always turbulent. Therefore it is sufficient to calculate with an average velocity.

$$v_i=\overline{v}_i+v_i' \quad \text{with} \quad \overline{v}_i=\frac{1}{\Delta t}\int_{t_0}^{t_0+\Delta t} v_i\,dt \qquad (2)$$

The interval Δt has to be chosen in a way that the average velocity remains constant for steady flow. If equation (2) is applied to the NAVIER STOKES Equations the REYNOLDS Equations (3) are obtained.

$$\rho\left(\frac{\partial v}{\partial t}+\overline{v}_j\frac{\partial \overline{v}_i}{\partial x_j}\right)-=\overline{f}_i-\frac{\partial \overline{p}}{\partial x_i}+\frac{\partial}{\partial x_j}\left(\mu\frac{\partial \overline{v}_i}{\partial x_j}-\rho\overline{v_i' v_j'}\right) \qquad (3)$$

Together with the continuity equation (1) they are the hydrodynamic basis for simulation of river flow. To solve the equations assumptions for the last term of (3) are necessary. BOUSSINESQ 1877 suggested to estimate the so called REYNOLDS stress terms in the following form:

$$-\overline{v_i' v_j'} = \nu_T \left(\frac{\partial \overline{v_i}}{\partial x_j} + \frac{\partial \overline{v_j}}{\partial x_i} \right) + \frac{2}{3} k \delta_{ij} \tag{4}$$

The factor ν_T is the eddy viscosity, which is not a constant. The value depends on the turbulence of the flow. The kinetic energy is symbolized by k

$$k = \frac{1}{2} \left(\overline{v_x' v_x'} + \overline{v_y' v_y'} + \overline{v_z' v_z'} \right) \tag{5}$$

3 Three dimensional flow

Equations (1) and (3) are the basis for three dimensional flow simulation. The eddy viscosity should be calculated with the assumption of equation (4). For the bottom shear stress τ_b the logarithmic velocity profile and the relation that the square of the velocity is proportional to τ_b at the water depth z_0 has to be an input.

$$\vec{\tau}_b = \rho \cdot |\vec{v}(z')| \cdot \vec{v}(z') \left(\frac{1}{\kappa} \ln \frac{z'}{z_0} \right)^{-2} \tag{6}$$

Three dimensional models are seldom used for the simulation of natural rivers. Occasionally the equations are simplified with the assumption of hydrostatic pressure

$$\frac{\partial p}{\partial x_i} = \rho \cdot g \frac{\partial \overline{z_{Sp}}}{\partial x_i} \tag{7}$$

where p = pressure. In this way the pressure is eliminated completely.

4 Two dimensional flow

By averaging equation (1) over the water depth h

$$\frac{\partial h}{\partial t} + \frac{\partial (h \overline{v_x})}{\partial x} + \frac{\partial (h \overline{v_y})}{\partial y} = 0 \tag{8}$$

is obtained. The velocity components in (8) are averaged over time according to (2) and water depth. Averaging of equation (3) over water depth yields the shallow water equations.

$$\frac{\partial \overline{v_x}}{\partial t} + \overline{v_y} \frac{\partial \overline{v_x}}{\partial y} + \overline{v_x} \frac{\partial \overline{v_x}}{\partial x} =$$

$$-g \frac{\partial}{\partial x} (h + z_b) + \frac{1}{h\rho} \left(\frac{\partial (h\tau_{xy})}{\partial y} + \frac{\partial (h\tau_{xx})}{\partial x} \right) - \frac{1}{h} \frac{\tau_{b,x}}{\rho} \tag{9}$$

$$\frac{\partial \overline{v_y}}{\partial t} + \overline{v_x} \frac{\partial \overline{v_y}}{\partial x} + \overline{v_y} \frac{\partial \overline{v_y}}{\partial y} =$$

$$-g \frac{\partial}{\partial y} (h + z_b) + \frac{1}{h\rho} \left(\frac{\partial (h\tau_{xy})}{\partial x} + \frac{\partial (h\tau_{yy})}{\partial y} \right) - \frac{1}{h} \frac{\tau_{b,y}}{\rho}$$

5 One dimensional flow

If the flow is approximately one dimensional, the shallow water equations (9) can be further simplified. The following conditions are necessary for a flow which should be described with one dimensional formulas:

- $v_x \gg v_y, v_z$,
- smooth change of water depth
- smooth change of cross sectional area
- no horizontal and vertical mass transfer
- horizontal water level perpendicular to the main flow direction
- equal energy slope for main channel and flood plains

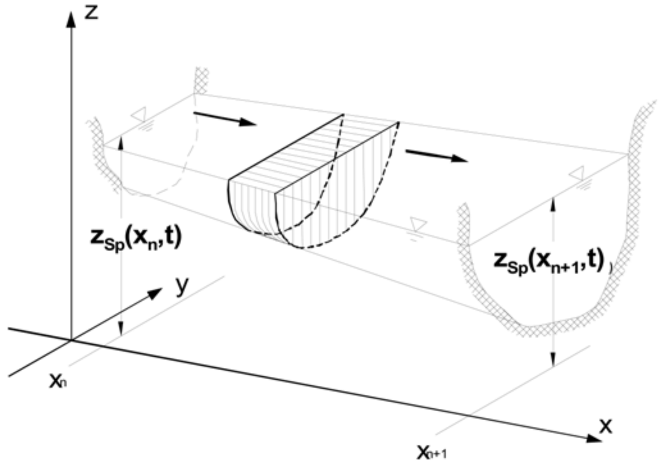

Fig. 2: Sketch of one dimensional flow

Often there is a tributary inflow rate/unit length q_e. Sometimes the costs for two dimensional flow simulation are not justifiable although they would be physically necessary. So one dimensional models are used at the limit of there validation. In this case the results should be interpreted very carefully. If the continuity equation is averaged over the width of the channel

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_e \tag{10}$$

is obtained where A = cross sectional area and Q = channel flow. Averaging the shallow water equations yields

$$\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{\beta Q^2}{A} \right) + gAS_f + gA \frac{\partial z_{Sp}}{\partial x} - v_e \cos(\varphi) q_e = 0 \tag{11}$$

where z_{Sp} = waterlevel above datum, x = space variable, S_f = friction slope, v_e = velocity and φ = angle of tributary inflow. β is often called the BOUSSINESQ coefficient.

$$\beta = \frac{1}{\bar{v}^2 A} \iint_A v^2(x) dA \tag{12}$$

Equations (10) and (11) are the SAINT VENANT Equations for one dimensional channel flow. In the case of steady flow the first term in (11) is zero and equation (11) can be solved approximately

$$z_{Sp}(x_2) - z_{Sp}(x_1) + \int_{x_1}^{x_2} S_f(x)dx + \frac{2}{g(A_1 + A_2)} \cdot \left(\frac{\beta_2 Q_2}{A_2} - \frac{\beta_1 Q_1}{A_1} \right) - \frac{v_e \cos(\varphi)}{g(A_1 + A_2)} \int_{x_1}^{x_2} q_e(x)dx = 0 \tag{13}$$

6 Determination of friction

For the calculation of friction it is assumed that the bottom shear stress is a function of $|v|^2$

$$\tau_b = c_f \cdot \rho \cdot \left(\bar{v}_x^2 + \bar{v}_y^2 \right) \tag{14}$$

The components of the shear stress are

$$\tau_{b,x} = c_f \cdot \rho \cdot \bar{v}_x \sqrt{\bar{v}_x^2 + \bar{v}_y^2}, \; \tau_{b,y} = c_f \cdot \rho \cdot \bar{v}_y \sqrt{\bar{v}_x^2 + \bar{v}_y^2} \tag{15}$$

For one dimensional flow it follows from (14) that

$$\tau_b = c_f \cdot \rho \cdot \left(\frac{Q}{A} \right)^2 \quad \text{with} \quad c_f = \frac{\lambda}{8}. \qquad \text{(16) and (17)}$$

The friction factor is determined by the COLEBROOK/WHITE formula

$$\frac{1}{\sqrt{\lambda_b}} = -2{,}03 \log \left(\frac{2{,}51}{f \cdot Re \sqrt{\lambda_b}} + \frac{k_s / r_{hy}}{14{,}71 \cdot f} \right) \tag{18}$$

where f = form factor; Re = Reynolds number; k_s = equivalent sand roughness; r_{hy} = hydraulic radius. Sometimes the empirical formula from STRICKLER is used

$$c_f = \frac{g}{k_{St}^2 r_{hy}^{1/3}} \tag{19}$$

The value of k_{St} depends on water depth. For one dimensional flow the friction slope S_F can be determined according to DARCY-WEISBACH with the following expression:

$$S_F = \frac{c_f}{g \cdot r_{hy}} \frac{Q \cdot |Q|}{A^2} \tag{20}$$

To consider friction due to vegetation, an assumption is made that the resistance factor for the plants is added to the factor of bottom resistance

$$\lambda_{total} = \lambda_b + \lambda_{plant} \tag{21}$$

6.1 Consideration of vegetation

The resistance of non flexible vegetation can be calculated with the formula of LINDNER [8] and PASCHE [13]:

$$\lambda_{plant} = c_{WR} \frac{4 \cdot h \cdot d_{plant}}{a_x \cdot a_y} \tag{22}$$

where d_m = diameter of plants, a_x and a_y = average distance between the plants. The exact computation of the drag coefficient c_{WR} is very difficult and not completely solved yet. See BWK99 for more details.

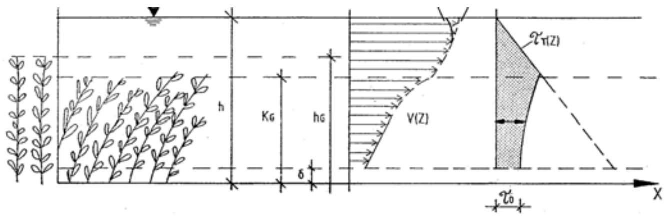

Fig. 3: Flow over flexible vegetation

As an example for the computational difficulties of the resistance of vegetation the flow over flexible vegetation is chosen. Figure 3 shows the flow situation. Investigations of KOUWEN have shown that the following relation holds

$$\frac{1}{\sqrt{\lambda}} = a + b \cdot \log \left(\frac{h_G}{k_G} \right) \tag{23}$$

The height of the vegetation in the flow k_G is estimated with the empirical formula

$$k_G = 0{,}14 \cdot h_G^{0{,}59} \left(\frac{MEI}{\tau_b} \right)^{0{,}4} \tag{24}$$

The values of a and b are found in tables. Both values vary between 0,15-0,29 and 1,85-3,5. The stiffness MEI could be found in experiments after KOUWEN [8]. For green grass it has been found that $319(h_G)^{3{,}3}$ is a good approximation and for dead grass $25{,}4(h_G)^{2{,}26}$. The method of KOUWEN describes the physical process correctly. The main difficulty is the estimation of the stiffness MEI. Although there are some theoretical approaches for the calculation of MEI, a lot of research is necessary in this field.

6.2 Very rough flow

If the bottom roughness is very rough the logarithmic velocity profile is not valid anymore. The flow has two layers. The approach of AGUIERRE-PE/FUENTES [1] shows the estimation of the factor λ in this case.

$$\frac{1}{\sqrt{\lambda}} = 0{,}88 \cdot \frac{\beta_w d_m}{r_{hy}} + 2{,}03 \log \left(\frac{11{,}1 \cdot r_{hy}}{\alpha_t \cdot d_m} \right) \tag{25}$$

The parameter d_m is the diameter of the roughness elements, α_t is a parameter for the form and the geometry of the roughness elements and β_W is a wake parameter. The transition between equation (18) and (25) should be continuous, but it is not. Also the range of dispersion of the parameters α_t and β_W is too wide. For both topics further research is necessary. The resistance laws of one dimensional flow with vegetation are often transferred to two dimensional flow. Even though this is the right approach, more systematical research is necessary at this area.

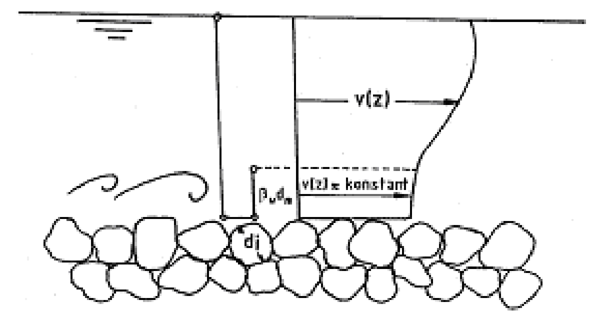

Fig. 4: Velocity profile for very rough flow

7 Limits of computation

7.1 Limits of practical two dimensional flow simulation

For the calculation of two dimensional flow often a constant eddy viscosity is assumed. Calibration of the models are done for steady flow situations. The change of eddy viscosity for different discharges is not considered. A comparison of calibrated and measured eddy viscosity values shows a difference of a factor 10 or even more [10]. Computations with the measured value of eddy viscosity are sometimes unstable. If the resistance factor from STRICKLER in equation (19) is used in two dimensional flow computation, its dependence from water depth is very often neglected, because it is unknown in most of the cases. Another difficulty is, that the value k_{St} is the output of an averaging over the cross sectional area and not over the water depth like it should be for a coefficient in two dimensional flow simulation. The grid generation is a very complex procedure. To get more precise results and a fast computation the optimization of the grid is necessary. The criteria for the optimal grid, especially for the difficult topography of natural rivers are still a topic of research.

7.2 Limits of practical one dimensional flow simulation

When applying the assumptions and conditions according to section 5 the one dimensional equations (10) and (11) are simple but efficient model equations. However, an exact analysis of the computation area is necessary to examine the validity of the assumptions made. If a geometrical separation of different flow areas is apparent, a segmentation in stream tubes is possible. The cost of computation is small in comparison to three or two dimensional simulations. To plot the results however can requires more effort, for example if inundation borders have to be plotted. From the physical point of view the one dimensional equations represent a rough simplification of the natural processes. The friction slope S_f contains all resistance forces resulting from:

- molecular and turbulent viscosity,
- form drags,
- horizontal and vertical secondary flow
- bottom friction, and/or the friction at the banks and the flow around obstacles.

Even if individual forces can be neglected, the majority of these effects has to be considered with an appropriate resistance law and/or by drag coefficients. To sum up all these effects to only one parameter S_f is problematic. For reasons of efficiency the one dimensional equations are sometimes used at the limits of their validity. It becomes evident when parameters have to be set with unrealistic values. This applies in particular to the equivalent sand roughness ks. In these cases special care has to be dedicated to the reliability of the results. The illustration of retention phenomena with the one dimensional equations is still a research topic, TESCHKE [18]. By representation of the entire river by only a single stream tube the complex exchange between the main channel and the floodplains can not be described correctly. Especially the great difference of the flow velocities result in the deformation of the hydrograph. Another major question is whether the flow resistance due to bottom roughness changes for unsteady conditions in relation to steady flow and therefore contributes to the hysteresis effect. So far the same approach is made for both cases. A further theoretical difficulty should be mentioned. The SAINT-VENANT equations can also be derived by applying the energy theorem, see e.g. BWK 2000 [4], SCHROEDER [16] and FIELD et. al. [6]. Then the correction factor for a non uniform velocity profile is replaced with the CORIOLIS coefficient:

$$\alpha = \frac{1}{\bar{v}^3 A} \iint_A v^3(x)\,dA \qquad (26)$$

It is obvious that even when the forces of resistance are computed in the same way, the resulting water levels are not identical for both cases. BWK 2000 [4] examines this problem for steady flow conditions and FIELD et al. [6] for unsteady cases.

8 Errors

The Error is the difference between the computed and the exact solution. The error of the estimated solution has the following reasons

- errors of discretization
- errors at the boundary conditions
- errors of the numerical procedure
- errors of iteration processes
- round off errors
- measurement errors of input data

See OERTEL/LAUREIN [12] for more details.

8.1 Estimation of errors

The exact estimation of the errors is difficult. Two dimensional models should compute the water depth with an accuracy of ±5 cm. One dimensional models compute the water depth with an accuracy of ±10 cm after calibration.
If the models are not calibrated before there is more inaccuracy.

8.2 Natural limitations of computer models

Natural limitations of every computation are:
- The result of a computation can not be better than the knowledge of the person doing the computation.
- The results of a hydrodynamic computation can not be better than the precision of the input data.

9 Conclusion and further steps

It has been shown that for the computation of water levels in natural rivers several assumptions are necessary. With the presented formulas it is possible to achieve good results in simulating natural river flow for most practical problems.
Research is for example necessary in the following areas
- Consideration of retention effects for one dimensional flow simulation
- Measurement of eddy viscosity respectively turbulent stress in natural rivers
- Automatic grid generation for two dimensional flow simulation

Some ideas of this topics are sketched in figures 5, 6 and 7.

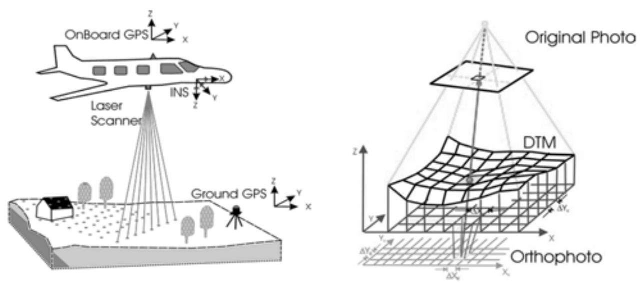

Fig.5: Principle of automatic grid generation with laser scanning [14]

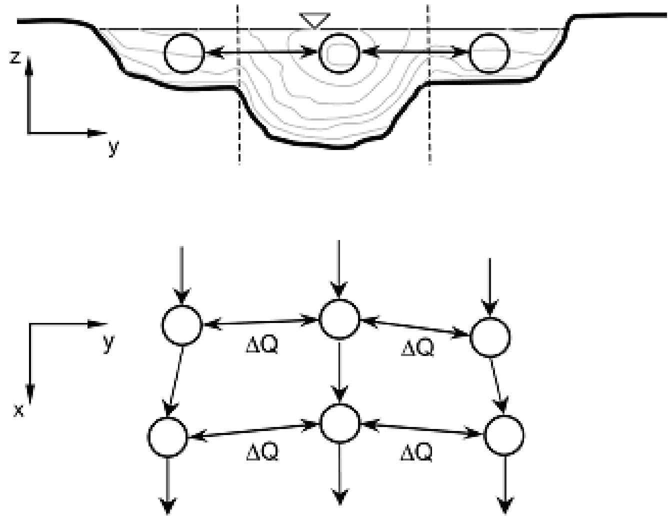

Fig.6: Concept of stream pipes for natural rivers with floodplains. The conservation of continuity, energy and momentum should be satisfied at every node. But a mass transfer is allowed in this model perpendicular to the flow direction [18].

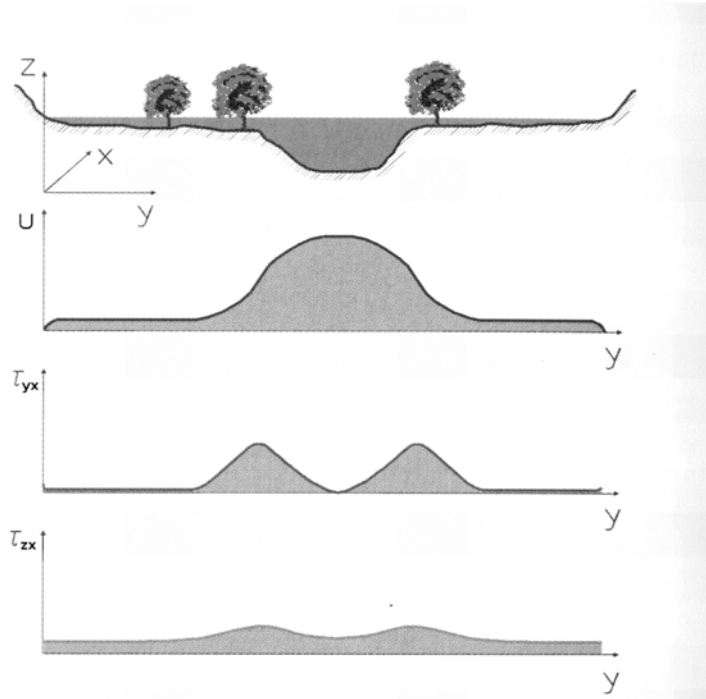

Fig.7: Idealized distribution of the velocity and shear stress in a natural river with floodplains. Measurements have to be done to verify and improve the used theory [10]

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