Comparison of 1-D inversion schemes for TDEM data

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Abstract: - In this study we examine schemes for the 1D inversion of Time Domain Electromagnetic (TDEM) Data. We conducted several model tests using conventional techniques, and we also included in our study a re-weighted least squares dumping technique. The performance of the tested techniques is compared by means of synthetic examples for the case of data-sets with erratic data points which is of particular practical interest. Further, tests with field data are also included. Results indicate that the re-weighting least squares dumping technique gives more satisfactory results when outliers exist in TDEM data sets.

Key-Words: 1-D inversion, Time Domain Electromagnetic, re-weighted least squares

1 Introduction

TDEM soundings are widely employed in Geophysics for mineral resources investigations, geothermal and groundwater studies. Several methods for the 1-D inversion of TDEM data have been proposed in literature [1], [2], [3].

In order to investigate the performance of 1D inversion scheme when TDEM data sets have outliers, we applied two of the most widely used techniques, ridge regression and Occam's inversion. Further, tests were also conducted using a re-weighted least squares dumping technique. After several tests with models, the results indicate that the last tested scheme, provided better results than the two conventional techniques. Provided that the interpreter chooses the right number of layers, the algorithm automatically isolates erratic data by locating them and attributing to them small weights in order to decrease their importance during the inversion procedure.

2 Erratic data sets

In every day's practice fieldwork yields with large data sets. When there is low level of noise, or when noise is normally distributed, all conventional methods give satisfactory results. But when there are data sets with erratic data points, then usually conventional methods fail to reveal a model close to reality, regardless the small misfit errors.

Outliers can occur either due to systematic error in the configuration of the instrument or due to noise, either geological, or, most of the times, man made.

One way for tackling this problem is to carefully examine the data-set and remove outliers manually. This technique however is based on the interpreter's expertise and can become quite time consuming when large data-sets are involved.

Further, automated smoothing - denoising techniques can be applied. Yet, these techniques lack physical insight and may lead into the loss of important information, or even worse it can lead the system to a solution that is mathematically valid but geologically unacceptable.

An alternative technique for eliminating outliers is to use a re-weighted least squares scheme. Techniques based on such schemes can reject indirectly outliers by assigning to them small statistical weights during the inversion procedure. The main advantage is that the technique is embedded within the inversion procedure and the assigned weights are decided by the iterative testing of the goodness of fit of the individual modeled data to the field measurements.

In the next section we will introduce such a scheme, proposed originally from Morelli and LaBreque [4] for the geoelectrical technique and applied to the 1D inversion of TDEM data by [5].

3 Inversion schemes in TDEM

Considering a measured data vector **d** the inverse problem seeks to find the model vector **m** representing the subsurface parameters for which **d**=**f**(**m**), where **f** is the forward modeling operator. By solving the system in least squares sense, we are trying to minimize the misfit error, given by the equation $q = (d - Am)^T (d - Am)$. (1)

In non-linear inversion, the procedure is iterative, and the correction of the model for the k^{th} iteration is given by

$$dm_k = (A_k{}^T A_k)^{-1} A_k{}^T dy_k , \quad (2)$$

where $\mathbf{A}^T\mathbf{A}$ is the generalized Jacobian matrix, and the new model is given by $m_{k+1} = m_k + dm_k$.

3.1 Ridge regression

The inversion of the generalized Jacobian matrix for the TDEM case is ill-conditioned. To tackle these type of problems Marquardt [1] used ridge regression proposed by Levenberg [6]. In ridge regression the purpose is to minimize the misfit q, under the constraint that the solution dm is bounded by a constant quantity, say *ct*, so that

$$dm^T dm < ct , \quad (3)$$

where *ct* can be the data noise level.

Combined minimization leads to the iterative model correction equation

$$dm_k = (A_k{}^T A_k + \lambda_k I)^{-1} A_k{}^T dy_k , \quad (4)$$

where **I** is the unit matrix and λ_k is the lagrangian multiplier. The above technique is routinely used for 1D interpretation of TDEM data.

3.2 Smoothness constrain

Constable et al. [2] proposed a smoothness constrained inversion scheme, where resistivities of layers are allowed to oscillate smoothly between extreme values but layer thicknesses are fixed.

The idea is to find the smoothest model which could fit the data in the sense that the model should depart from the simplest case only as far as necessary to fit the data.

Apart from minimizing q, Constable et al. introduced a scheme which also seeks to minimize the roughness term **R**, which has the form

$$R = \|Cm\|^2 , \quad (5)$$

Matrix **C** describes the dependencies between model parameters (resistivities). **C** will have the form

$$C = \begin{vmatrix} 1 & 0 & \dots & \dots & 0 \\ 0 & 1 & -1 & \dots & \\ 0 & 0 & \dots & \dots & 0 \\ 0 & 0 & \dots & 1 & -1 \\ 0 & \dots & \dots & & 1 \end{vmatrix} . \quad (6)$$

The correction of the model then is given by the equation

$$dm_k = (A_k{}^T A_k + \lambda_k C^T C)^{-1} A_k{}^T dy_k , (7)$$

where λ_k is the lagrangian multiplier.

3.3 Re-weighted Least Squares Technique

Karmis [7], proposed a hybridic technique for the 1-D inversion of TDEM data. It combines the smoothness scheme for inverting for the layer resistivities and the Marquadt scheme for reconstructing the layer thicknesses.

The misfit function q to be minimized is given by

$$q = (Wd - WAm)^T (Wd - WAm) , \quad (8)$$

where **W** is the deviation matrix. Assuming that data noise is uncorrelated and independent, **W** is a diagonal matrix with elements $\mathbf{W}_{ii}=1/\sigma$, σ the mean standard deviation.

After each iteration the model correction is given by the equation

$$dm_k = ((A_k W)^T A_k W + \lambda_k C^T C)^{-1} (W A_k)^T W dy_k \quad (9)$$

where **C** is a matrix that handles the smoothness of the model, similar to the one described in the previous section.

In the case of the re-weighted least squares, for a model with *n*-layers, **C** will be an (*2n-1*)x(*2n-1*) matrix, with the first *n* lines controlling the smoothness of the resistivities of the model (Occam's smoothness scheme), while the rest (*n-1*) lines allow the thicknesses of the layers to vary independently, resembling in this way the ridge regression method. The matrix **C** will have the form of eq. (10).

$$C = \begin{vmatrix} -1 & 1 & 0 & ... & 0 \\ 0 & -1 & 1 & ... & \\ 0 & ... & ... & ... & 0 \\ 0 & ... & 0 & 1 & 0 \\ 0 & ... & 0 & 0 & 1 \end{vmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}a\\a\end{matrix}}\right\} n \\ \\ \left.\vphantom{\begin{matrix}a\\a\end{matrix}}\right\} n-1 \end{matrix} \quad (10)$$

Starting from an initial model, with resistivities and thicknesses of layers, and a value of mean standard deviation σ, synthetic data are produced, by performing inverse Laplace transform using the Gaver-Stehfest method [8]. The Frechet derivatives are being calculated using the perturbation technique, and each element is given by the equation

$$A_{ij} = \frac{\log(f_i(m_j + \delta) - \log(f_i(m_j))}{\delta} \quad (11)$$

where δ is a percentage of the resistivity perturbation, e.g. 3%.

The Lagranian multiplier is found using a line search [2]. Eq. (9) is solved for numerous values of λ_k, calculating each time the average percent data error ε%, defined as

$$\varepsilon\% = (10^{\sqrt{x^2}} - 1) \times 100\% \quad (12)$$

where χ^2 is the reduced chi-square value, defined as

$$\chi^2 = \frac{1}{M-N} \sum_{i=1}^{M} \frac{d_i^2 - m_i^2}{m_i^2} \quad (13).$$

If the value of λ_k that is found after the line search gives a value of ε% above threshold, it is considered as the optimum value for the *k* iteration. If ε% is below threshold, then a new λ_k is sought, that gives value of ε% equal or slightly above the threshold.

In literature, there are several techniques proposed for multi re-weighting least squares inversion [9], [10]. We examined Porsani et al. [11], Labreque and Ward [12] and Morelli and Labreque [4] methods, and after several tests, it is concluded that Morelli and Labreque's proposal best fitted our needs.

According to Morelli and Labreque a trial-weighting matrix is being calculated, using the equation

$$_{trial}W_{i,i} = ({}_{old}W_{i,i}^{1/2}/e_i) \left[\sum_j ({}_{old}W_{j,j}^{1/2} e_j) / \sum_j ({}_{new}W_{j,j}^{1/4} e^{1/2})\right], \quad (14)$$

and every new element of the weighting matrix is obtained by the following relation

$$_{new}W_{i,i} = \begin{cases} {}_{old}W_{i,i} \rightarrow \text{if } {}_{trial}W_{i,i} > {}_{old}W_{i,i} \\ {}_{trial}W_{i,i} \rightarrow \text{if } {}_{trial}W_{i,i} < {}_{old}W_{i,i} \end{cases} \quad (15)$$

Weighting factors are recalculated during every iteration, until the L1-norm defined as

$$L1 = \frac{\sum_j ({}_{old}W_{j,j} e_j)}{\sum_j ({}_{new}W_{j,j} e_j)}. \quad (16)$$

becomes equal to 1.

When the inversion starts, L1-norm has high values. After a few iterations less and less elements of **W** are changing as L1-norm approaches unity. At this time re-weighting must stop; otherwise the solution keeps tracking the noise of the data.

4 Synthetic Examples

To demonstrate the effectiveness of the re-weighted least squares technique, we present examples of 2-layers and 3-layers synthetic examples, in which

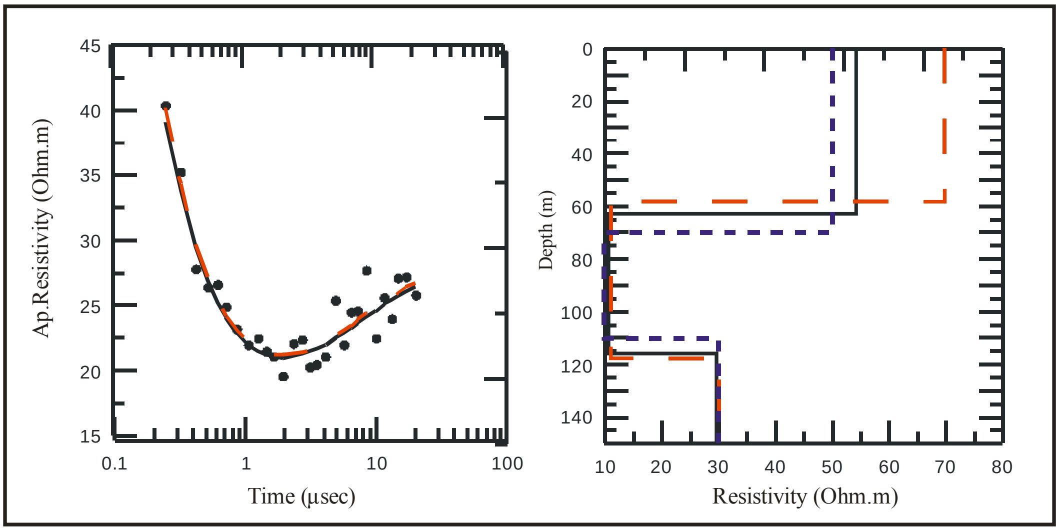

Fig. 1. *Left part:* Data points in black dots. Theoretical response of re-weighted least squares presented in black solid line, and ridge regression in red dashed line. *Right part:* theoretical model in black dashed line, re-weighted least squares in black solid line and ridge regression in red dashed line.

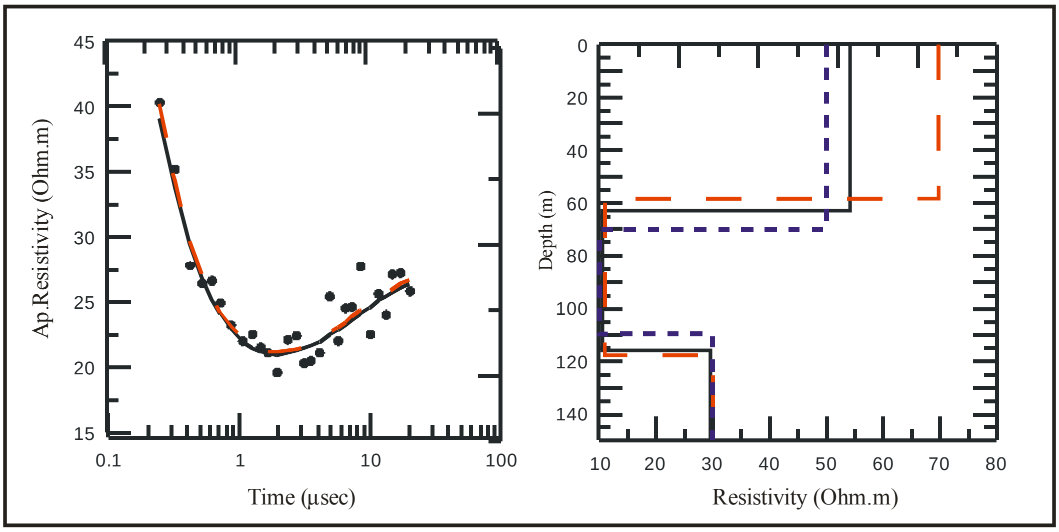

Fig. 2. *Left part:* Data points in black dots. Theoretical response of re-weighted least squares presented in black solid line, and ridge regression in red dashed line. *Right part:* theoretical model in black dashed line, re-weighted least squares in black solid line and ridge regression in red dashed line.

7% normally distributed noise was added, and moreover some of the data points were manually changed to simulate the required outliers. The results are compared with those from ridge regression for the same models.

4.1 2-Layers model

In figure 1 the synthetic model is presented in the right part, with blue dashed line, along with the re-weighted least squares result, presented in black solid line, and ridge regression result presented in dashed red line. The theoretical data points are presented at the left part, with black dots. The theoretical response of the re-weighted algorithm is presented with black solid line and ridge regression response with dashed red line. Although both re-weighted least squares scheme and ridge regression results fit the real data satisfactory, inversion model from the first scheme is closer to the real model.

4.2 3-Layers model

In figure 2 the synthetic model, the results from re-weighted least squares and ridge regression are presented using the same notation as in the previous section. Since data have been modified to a large extent, we didn't expect either of the schemes to fully recover the theoretical model. But it can be seen that re-weighted least squares algorithm managed to reconstruct the initial model in a more exact manner.

5 Real example

We tested the re-weighted least squares dumping algorithm with real data measured at an area of high man-made noise level, near a civil airport radar. The measurement configuration was that of a coincident loop with a 50x50m square loop, and measurement ramp time was 80 μsec. The results of the inversion using ridge regression and re-weighting least squares dumping technique are presented in figure 3, using the same notation as in the previous figures.

As it can be seen from figure 3, both techniques managed to fit the real data in similar manner. The inversion results though, compared with geotechnical data from a drill nearby, showed that re-weighted dumping least-squares technique provided a geoelectrical model closer to reality.

6 Concussions

We examined commonly used schemes for the 1-D inversion of TDEM data sets with erratic data points. Further, we tested an inversion algorithm for TDEM data that automatically isolates erratic data points by giving them small statistical weight so that the system takes them into less account during the inversion procedure.

When data with normal distributed data noise are considered, all methods perform equally well. But when outliers exist into the data sets, simple least squares approaches seek to find a solution that fits data as close as possible, resulting in a model that tracks down the noise that contaminates the data set.

On the other hand, by automatically assigning small weight to erratic data points, re-weighted least squares technique tries to find a stable model that can be closer only to the points with small misfit error. This results into a more realistic model, in the sense that the reconstruction does not suffer by erratic data induced artifacts, though the

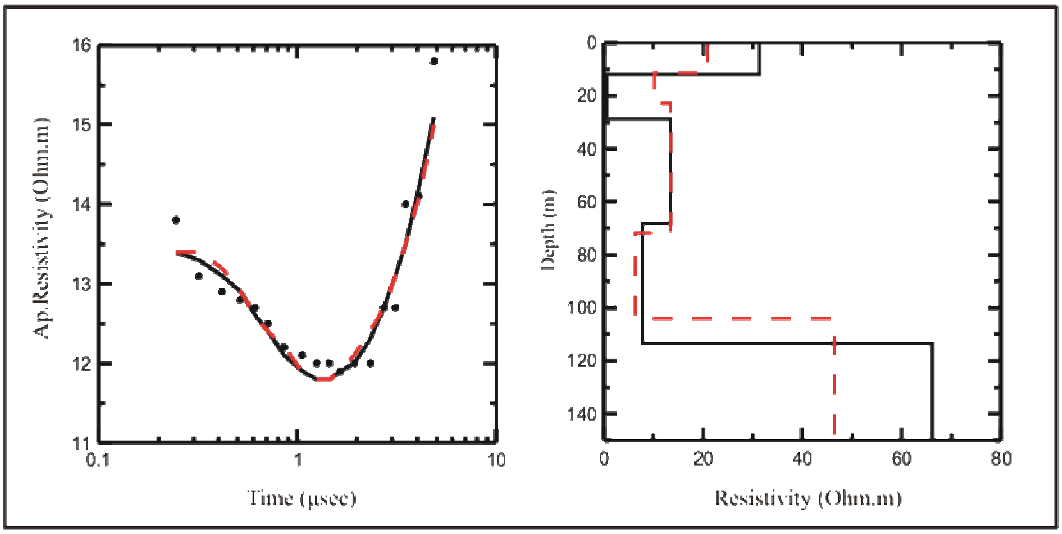

Fig. 3. *Left part:* Data points in black dots. Theoretical response of re-weighted least squares presented in black solid line, and ridge regression in red dashed line. *Right part:* re-weighted least squares in black solid line and ridge regression in red dashed line.

overall misfit can be higher that the other methods.

The main advantage of the approach is that it is fully automated and it treats the problem within the inversion procedure on the basis of the physical parameters.

Using constrain matrix **R** creates a weak dependency between resistivities of layers, leaving the depths to vary independently.

The main disadvantage of the re-weighted least squares technique is that the user must provide the algorithm with the correct initial guess about the number of layers, a disadvantage that is common to all the techniques that invert data sets using discrete number of layers.

Overall, we consider the algorithm as optimum for routine TDEM field data interpretation since it can automatically deal with data sets which suffer from high levels of noise.

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