Abstract - The paper examines problems of an anti-sway system for crane control. The crane is modelled and simulated; anti-sway control results are presented and suggestions for further work are made.

Index Terms - Matlab, crane, sway, anti-sway, control.

I. INTRODUCTION

In crane control swaying of the load is dangerous for workers, operators, cranes and load itself, see fig. 1; additionally sway decreases productivity of container handling and increases working cycle [1].

The basic sway caused by the acceleration of the trolley is:
1. **primary sway**. In this case the container moves in an arc below the trolley, parallel to the trolley travel direction.
2. in the trolley travel direction there is so-called **secondary sway**, where the container rotates about the head block sheaves.
3. **longitudinal sway**, i.e. movement of the container back and forth in the gantry travel direction
4. **rotation** of the container about the vertical axis extending down from the trolley. This rotational sway is caused primarily by **eccentrically loaded containers**. Rotational sway is the most difficult type of sway.
5. Sway coming from wind action on a container or from horizontal impact

Several types of methods have been created to eliminate the sway of the load [2].

- **Hydraulic sway damping** with high reliability dissipate the sway energy of the load but do not prevent the sway from starting.
- Inclined **auxiliary ropes** damp the sway, since the head block is subject to very heavy mechanical shock loads and increase the weight of the loaded load and decrease the free space available for maintenance purposes, see fig. 2.
- **Electronic anti-sway** prevent the sway by manipulating the acceleration and deceleration of the trolley or gantry motion. Mathematical algorithms modify the velocity of the trolley or gantry so that the sway is compensated but the crane driver looses control of the travelling motions; they are not capable of preventing rotational sway, because the systems manipulate the acceleration/ deceleration of the trolley / gantry motions, not each individual hoist rope fall.

The overall system, see fig. 3, transfer function [3]:

\[
\frac{X}{E} = \frac{g}{ml^5 + mlas^4 + mgs^3 + mgas^2}
\]  

(1)

where \(m\) is the trolley mass, \(l\) is the length of the cable between the trolley and the payload

\[a = \frac{1}{\tau}\]  

is the inverse of the motor time constant

\[x(t)\]  

is horizontal position of the payload

\[\text{Fig. 1. A gantry crane}\]

\[\text{Fig. 2. Multiple rope anti-sway mechanism}\]
For:

\[ x_1 \] horizontal motion  
\[ x_2 \] vertical motion  
\[ x_3 \] swing angle  
\[ u_1 \] control via trolley drive motor  
\[ x_4 \] horizontal velocity  
\[ x_5 \] vertical velocity  
\[ x_6 \] swing velocity  
\[ u_2 \] control via hoist motor

The state variables of the crane, see fig. 4, control are [4]:

\[ \dot{x}_1 = x_4, \quad \dot{x}_2 = x_5, \quad \dot{x}_3 = x_6, \quad \dot{x}_4 = u_1 + 17x_1, \]
\[ \dot{x}_5 = u_2, \quad \dot{x}_6 = -\frac{1}{x_2}(u_1 + 27x_3 + 2x_5x_6) \]

\[ |x_4(t)| \leq 2.5, \quad |x_3(t)| \leq 1, \]
\[ |u_1(t)| \leq 2.8, \quad -0.8 \leq |u_2(t)| \leq 0.7, \quad (2) \]

minimize the cost criterion [6]:

\[ F(x,u) = \frac{1}{2} \int_0^9 \left[ x_1^2(t) + x_5^2(t) + a(u_1^2(t) + u_2^2(t)) \right] dt \quad (3) \]

with the Hamiltonian function [6]:

\[ H = \frac{1}{2}(x_1^2 + x_5^2 + 0.01(u_1^2 + u_2^2) + \lambda_1 x_1 + \lambda_2 x_5 + \lambda_3 x_4 + \lambda_4 x_5 + \lambda_5 x_6 + \lambda_6(u_1 + 17x_1) - \frac{1}{x_2}(u_1 + 27x_3 + 2x_5x_6)) \]
The ideal anti-sway system of a crane driver should have the following control strategy features:

- Minimize load swing achieving the maximal trolley speed,
- Minimize load swing on bringing the trolley to rest
- Small position errors are corrected by "creeping" the trolley
- Ideal trajectory for load swing
- Not add time to the theoretical minimum cycle
- Be effective during simultaneous trolley / gantry travel and hoisting
- Not be sensitive to changing ambient conditions such as wind
- Influence static load position as little as possible
- Have a minimum amount of components located on the head block
- Have low maintenance requirements
- Easy to integrate with micro motion (trim, slew horizontal fine positioning)
- Induce minimum secondary sway
- Prevent rotational sway
- Be effective when the crane is operated in the manual mode
- Less stops/starts for the travel machine, ie lower maintenance/fuel consumption
II. THE DYNAMICS OF LOAD SWAY - BASIC EQUATIONS

For a pendulum fixed to a trolley, assuming that the rope is rigid, the mass of the load is concentrated at one point and neglecting friction, the force equilibrium equation is:

\[ m\ddot{x}_r = -Sl_o g \sin \theta \]
\[ m\ddot{y}_r = Sl_o g \cos \theta \]

where:
- \( x_r \) is the position of the trolley
- \( l_o \) is the length of the rope (it can change)
- \( m \) is the mass of the load
- \( S \) is the force of the rope
- \( g \) is the acceleration of gravity
- \( \theta \) is the sway angle

\( a_x, a_y \) are the accelerations in the horizontal/vertical directions.

For linearization, we set \( \sin \theta = \theta \) and \( \cos \theta = 1 \):

\[ u(t) = \sum a_i \text{function}(t + t_i) \]  (7a)

For open loop system, we get:

\[ u(t) = \sum a_i \text{function}(t + t_i) \]  (7b)

\[ \dot{x}_r(t) = v_{\text{ref}}, \quad \theta(t) = 0, \quad t \geq t_{\text{ss}} \]

For closed loop control system, we get:

\[ u(t) = C_1(v_{\text{ref}} - \dot{x}_r(t)) + C_2(0 - \theta(t)) \]  (7c)

Auxiliary ropes add new (horizontal/vertical) forces to the system:

\[ F_x + ma_x = -Sl_o g \sin \theta \]
\[ F_y + ma_y = Sl_o g \cos \theta \]

The linearized state equation of the crane system, for \( F = \) the friction coefficient of the trolley, \( g = \) the gravitational acceleration; \( J = \) the moment of inertia of the load; \( l = \) the length of rope, is [8]:

\[
x(t) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
0 & -F/M & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & -F/ML & -g/L & 0 \\
\end{bmatrix} x(t) + \begin{bmatrix}
0 \\
1/M \\
0 \\
1/ML \\
\end{bmatrix} u(t)
\]

\[ L = J + ml^2/mL \]  (8).

The state \( x \) is defined by:

\[ x(t) = (r(t), \dot{r}(t), \theta(t), \dot{\theta}(t)) \] with initial state of linear control is \( x(0) = [1, 0, 0, 0] \)

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**Fig. 11. Patterns of crane velocity (case (d) allows change in rope length)**

**Fig. 12. Crane trolley control scheme**

**Fig. 13. Multivariable control system**

**Fig. 14. Horizontal position**

**Fig. 15. Anti sway control and swing angle**
1) **Nonlinear System**

\[ l(t) \ddot{\theta}(t) + 2l(t) \dot{\theta}(t) + g \sin \theta(t) = \cos \theta(t) u(t) \]

2) **Linear Time-Varying System**

For linear time varying system, if the sway angle \( \theta(t) \) is small, \( \sin \theta(t) = \theta(t) \) and \( \cos \theta(t) \approx 1 \), then

\[ l \dot{\theta}(t) + g \theta(t) = u(t) \]

3) **Linear Time-Invariant System**

\[ l \dot{\theta}(t) + g \theta(t) = u(t) \]

The equation has a unique analytical solution, especially if the acceleration of the trolley changes stepwise. The assumption for constant rope length is restrictive; it means that hoisting is not allowed during acceleration. The acceleration is based on the angle measurement and at the same time the actual velocity is driven towards the desired velocity. Various kinds of control schemes can be used in crane control, i.e:

- The **Rule-based control**, e.g. the swinging up formula is:
  \[ u = \text{sign}(x_4(|x_2| - \pi/2)) = \pm 1 \]
  \[ u = u_{old} + \text{sign}(u_{old}) \text{Friction} \]

- The **linear-quadratic (LQ) controller** is:

\[
\begin{align*}
\min J &= \int \{x'Qx + u'Ru\} \ dt \\
\text{subject to} & \quad x' = Ax + Bu
\end{align*}
\]

where:
- \( x_1 = \text{desired} - \text{measured position of the cart} \)
- \( x_2 = \text{desired} - \text{measured angle of the pendulum} \)
- \( x_3 = \text{desired} - \text{observed velocity of the cart} \)
- \( x_4 = \text{desired} - \text{observed pendulum angular velocity} \)

The optimal feedback gain vector \( K = [K_1, \ldots, K_4] \) is calculated such that the feedback law \( u = -Kx \); where \( x = [x_1, \ldots, x_4] \) minimises the cost function: \( J = \int \{x'Qx + u'Ru\} \ dt \) - where \( Q \) and \( R \) are the weighting matrices, subject to the state equation: \( x' = Ax + Bu \).

**IV. FUTURE WORK**

Future work includes control architectures (cascade control, sliding non linear control) obstacle avoidance, helicopter load transfer and ship mounted crane load manipulation, see fig. 19, 20, 21. In case of ship mounted crane load is subjected to oscillations, especially from a heavy ship to a lighter landing craft utility ship in high sea conditions.
states under different sea states, ship speed, ship heading with respect to the waves, see fig. 22. Problems are decomposed into two sub problems, to control the:

- ship motion due to sea waves (6 DOF: surge, sway, heave, yaw, pitch, roll)
- crane swing, by disturbances acting as accelerations on the boom tip

**V. CONCLUSIONS**

In this paper, we developed a multivariable control scheme to reduce the load swing while the load is simultaneously hoisted (or lowered) and transferred. The control problem for the original linear time-varying system has been reduced to the optimal control of a linear time-invariant system.

**REFERENCES**