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Energetic Aspects Referring to Electrical Drive Systems

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Abstract:- Certain aspects referring to the energy consumptions in the electrical drives with D.C. motors are presented. The possibilities of the decrease of the energy losses are emphasized; these possibilities appear especially in the transient period of the speed change. A simple structure for suboptimal control is proposed.

Keywords: - drive system, energy consumption, optimal control, suboptimal control.

1 Introduction

The aim of this paper is to present some energetic aspects regarding the electromechanical transient process of the electrical drive systems with D.C. motor and to emphasize the great possibilities of the reducing energy consumption in this case. There are many papers and books which present the energy consumption of the electrical drives and their optimal control, but we consider that it is not sufficient underlined the possibilities of the energy losses decrease.

Some previous results of the authors have indicated the possibilities of the diminution of the energy losses in the rotor winding up to 25…30% if the optimal control is applied (by comparison with the cascade control). This amount of the reduction is not exaggerated if we take into account that these losses are at least equal with the variation of the kinetic energy, as it will be indicate below. Since the goal is to emphasize the general energetic aspects and not an exact computing of the energy components, a simplified model will be considered. Mainly, the electromagnetic transient processes are neglected and therefore, the rotor current is adopted as control variable.

Starting from the results obtained on this basis, a simple structure for suboptimal control is proposed.

The main conclusions of this paper are valid for different types of the drive motors. For simplicity only the D.C. motors case is presented.

2 Energy Consumptions in the Transient Process

We shall consider the following model for an electrical drive with a D.C. motor [3]:

$$u(t) = Ri(t) + c_e\omega(t) \quad (1)$$

$$c_m i(t) = J\dot{\omega}(t) + m(t), \quad (2)$$

where u(t) and i(t) are the rotor voltage and current, respectively, ω(t) is the rotor speed, m(t) is the load torque, R is the rotor winding resistance, J is the inertia, and c_e, c_m are motor parameters. We suppose that m(t) = constant on the interval [0,T] of the transient process and the rotor inductance can be neglected.

From (1) and (2) we can establish the Laplace transforms for ω(t) and i(t):

$$\Omega(s) = \frac{1}{T_m s + 1}\left[\frac{1}{c_e}U(s) - \frac{R}{c_e c_m}M(s)\right] \quad (3)$$

$$I(s) = \frac{1}{T_m s + 1}\left[\frac{T_m}{R}sU(s) + \frac{1}{c_m}M(s)\right], \quad (4)$$

where $T_m = RJ / c_e c_m$ is the electromechanical time constant of the drive system.

The electrical power received from the electric supply is

$$p(t) = u(t)i(t) = p_J(t) + p_e(t), \quad (5)$$

where

$$p_J(t) = Ri^2(t) \quad (6)$$

is the Joule power loss and

$$p_e(t) = c_e\omega(t)i(t) = c_m i(t)\omega(t) = m_e(t)\omega(t) \quad (7)$$

is the electromagnetic power, which corresponds to the electromagnetic torque $m_e(t)$. Certain quasi constant losses (e.g. iron and mechanical losses, the losses of the brushes contacts) were neglected. In this case, the electromagnetic power corresponds to the mechanical power $p_M(t)$ delivered to the driven devices. This last one contains two components: one is the useful power $p_2=m\omega$, and the other is the power developed for acceleration.

With the mentioned assumptions, we can consider

$$p(t) = p_J(t) + p_M(t) \tag{8}$$

Corresponding to (8), the energy consumptions on the interval [0,T] is

$$W = W_J + W_M = \int_0^T p_J(t)dt + \int_0^T p_M(t)dt \tag{9}$$

We shall consider a non load motor (m=0); in this case, $p_M(t)$ is the power developed for acceleration. Our first goal is to establish the weight of the two terms in (9). For this purpose, we shall compute $\omega(t)$ and $i(t)$ for a step variation U of $u(t)$, using for instance (3) and (4):

$$\omega(t) = \frac{U}{c_e}(1 - e^{-t/T_m}) \tag{10}$$

$$i(t) = I_p e^{-t/T_m}, \tag{11}$$

where I_p is the starting current value.

In this case, the energy loss in the winding on the interval [0,T] is

$$W_J = \int_0^T Ri^2(t)dt = \frac{RI_p^2 T_m}{2}(1 - e^{-2T/T_m}) \tag{12}$$

The energy developed for acceleration is

$$W_a = \int_0^T m_e(t)\omega(t)dt = \int_0^T c_m i(t)\omega(t)dt.$$

$$W_a = c_m I_p \frac{U}{c_e} T_m \left(\frac{1}{2} - e^{T/T_m} + \frac{1}{2} e^{2T/T_m}\right). \tag{13}$$

The duration of the transient process is about $T=4T_m$. For this value of T, the exponentials in (12) and (13) can be neglected. Tacking into account that $c_e=c_m$ and $U=RI_p$, from (12) and (13), yields

$$W_J \simeq W_a \simeq \frac{U^2 T_m}{2R} = RI_p^2 \frac{T_m}{2}. \tag{14}$$

Therefore, for no load drive system, the energy developed for acceleration and the Joule energy loss are equal. For the non zero load torque, the rotor current increases and the weight of the energy loss is greater.

In conclusion, in the transient period of acceleration obtained for a step variation of the supplied voltage, the energy loss represents at least half of the energy consumption. Therefore, a suitable control of the rotor current will allow to diminish the Joule losses.

Let now consider a step variation of the load torque, for instance from zero to m_0. The current variation in this case is

$$i(t) = \frac{m_0}{c_m}(1 - e^{-t/T_m}) \tag{15}$$

If we neglect again the small values of the exponential, the energy loss results

$$W_J' \simeq \frac{Rm_0^2 T_m}{2c_m^2}.$$

One obtain for the rated torque

$$W_J' \simeq RI_n^2 \frac{T_m}{2}, \tag{16}$$

where I_n is the rated value of the current. Since $I_n \ll I_p$, from (14) and (16) it results $W_J' \ll W_J$. Therefore, the Joule losses are significantly smaller in the transient process caused by the variation of the load then for the transient process determined by the variation of the rotor voltage.

3 Optimal Control

In order to obtain a good behaviour of the system and a reduced energy consumption, it is recommended to adopt an optimal control, using a quadratic criterion

$$I = \frac{s_1}{2}[\omega_d - \omega(T)]^2 + \frac{1}{2}\int_0^T [q_1(\omega_d - \omega(t))^2 + q_2 i^2(t) + pu^2(t) + ru(t)i(t)]dt \tag{17}$$

The criterion (17) is used in the problems with free end-point. The first term penalizes the difference between the desired value ω_d and the final value $\omega(T)$. The first term in integral penalizes the mean transient error of the speed and the second one refers to the energy losses. The next term penalizes the great value of the control variable u(t) and the last one refers to the global energy consumption. In the problems with fixed end-point, $s_1 = 0$ and it is imposed to achieve $\omega(T)=\omega_d$.

Since our goal is to study the energetic aspects of the drive system control, we shall consider only the criteria in the form

$$I_J = \int_0^T Ri^2(t)dt \tag{18}$$

and

$$I_T = \int_0^T i(t)u(t)dt \tag{19}$$

Note that the optimal control is equivalent in the both mentioned cases if the load torque m is constant. Indeed, based on (1),

$$I_T = \int_0^T c_e\omega(t)i(t)dt + \int_0^T Ri^2(t)dt \tag{20}$$

The first integral in (20) can be expressed as

$$\int_0^T c_e\omega(t)i(t)dt = \int_0^T c_e\omega(t)\frac{1}{c_m}(m + J\frac{d\omega}{dt})dt = m[\alpha(T) - \alpha(0)] + \frac{J}{2}[\omega^2(T) - \omega^2(0)] \tag{21}$$

(α is the angular displacement). For a given m, the last expression is constant. Therefore, the difference between the criteria (18) and (19) is constant, and the optimal control leads to similar results in both cases, so that only the criterion (18) will be considered below.

The optimal control problem refers to the criterion (18) and the system (1), (2), with imposed terminal states $\omega(0)$ and $\omega(T)$.

The Hamiltonian [2], [3] of the problem is

$$H = Ri^2(t) + \lambda(t)\frac{1}{J}(c_m i(t) - m)$$

(λ is the co-state variable).

From the necessary condition $\partial H(.)/\partial i = 0$, we find $i(t)=const.=i_0$, and then

$$I_J = Ri_0^2 T \tag{22}$$

In this case, if $\omega(0)=0$,

$$\omega(t) = \frac{1}{J}(c_m i_0 - m)t \tag{23}$$

and

$$\omega_d = \omega(T) = \frac{1}{J}(c_m i_0 - m)T \tag{24}$$

If T is substituted from (24) in (22), we can find the optimal value of the current from the condition $\partial I_J / \partial i_0 = 0$:

$$i_0^* = 2\frac{m}{c_m}. \tag{25}$$

On the other hand, if we substitute i_0 from (24) in (22), we find the optimal T from the condition $\partial I_J / \partial T = 0$:

$$T^* = \frac{J\omega_d}{m} \tag{26}$$

The above expressions put in evidence some interesting aspects:

- the optimal current is constant (it has a double value of the steady-state current);
- the energy losses do not decrease if we adopt a current less then the value given by (25), because this implies to increase the time of the transient process;
- if the current i_0^* is adopted, the necessary time to achieve the desired speed is given by (26).
- The minimum energy loss is

$$I_J^* = 4RTm^2 / c_m^2 = 4\left(RJ / c_m^2\right)\omega_d m \tag{27}$$

- If the drive system starts with a constant voltage, the consumed energy in the transient period results from (9) and (14)

$$W = RT_m I_p^2 = RT_m I_n^2 \mu^2 ,$$

where $\mu = I_p / I_n$ (usually $\mu = 6...8$).

If we consider that $T \cong 4T_m$ then

$$W / I_J^* \cong 3$$

(depending on μ). Therefore, the energy consumption can be reduced by three times by comparison with the direct start of the drive.

Of course the direct voltage supply is rarely used, but the optimal control remains the most advantageous procedure by comparison with other methods. This aspect will be pointed in the next section.

4 Suboptimal Solution Implementation

The above presented results show a simple way for optimal control law implementation: the rotor current given by (25) has to be provided. This control implies to estimate the load torque m at the beginning of the optimization process [4].

Of course, the desired value i_0^* cannot be instantaneously obtained, because the current is not directly controlled, but by means of the rotor voltage. This fact introduces an unavoidable delay of the increase of the current. Therefore, the solution will be suboptimal, but the difference is not significant comparatively with the optimal solution if a suitable control loop is used.

The simplest control algorithm is the following:

- establish the current at the value given by (25), for $t < T^*$ given by (26);
- adopt $i(t) = m / c_m$ for $t \geq T^*$; this current value ensures the desired steady-state speed ω_d.

This open loop control can occurs errors if the estimation of the parameters and of the torque is not exact. More convenient is to adopt a closed loop control, based on the speed error $\varepsilon(t) = \omega_d - \omega(t)$. The main steps of the algorithm are:

(1^o) For $|\varepsilon(t)| \geq \delta$, $\delta > 0$, put $i(t) = i_0^*$.

(2^o) After the first moment for which $|\varepsilon(t)| < \delta$, adopt a suitable feedback linear control for the drive system. For instance, a PI controller can be used; the output of this controller establishes the reference for the rotor current.

In addition, certain restriction has to be introduced:

- In order to avoid a great length of the transient process, one adopts a limit value T_{lim} if the value T^* given by (26) exceeds T_{lim}. In this case, the imposed rotor current will result from (24):

$$i_0 = \frac{1}{c_m}\left(\frac{J\omega_d}{T_{lim}} + m\right) \tag{28}$$

- A limit value of the current i_{lim} will be adopted if the value given by (25) is very high.

Of course, the energy loss increase if one of the mentioned restrictions is activated, but these limitations have to be introduced.

The above algorithm was tested via numerical simulation. Some of the results are presented below. The results refer to a drive system with a D.C. motor having the following rated data: rotor voltage U=110 V, rotor current I= 1.3 A, rotor resistance R=3.1 Ω, motor constants c_e=0.58 Vs/rad, c_m=0.58 Nm/A, inertia J=0.028 Nms^2/rad, rated torque M=0.78 Nm.

The Fig 1…4 present the system behaviour for the imposed speed ω_d=25 rad/s and for different load torques (0.2, 0.5, 0.78, 1.3 Nm, respectively). In the second and third cases no restriction is activated. The Fig. 1 presents the case when the restriction $T \leq T_{lim}$ interferes ($T_{lim} = 2s$) and the Fig. 4 corresponds to the case when the current is limited ($i_{lim} = 2.3I \cong 3A$)

A change of the control law was performed in all cases for $t > T^*$ (or for $t > T_{lim}$), as it is indicated in the above algorithm.

A transient process of the current increase was considered in all cases.

The last figure presents the behaviour of the optimal system (for m=0.78 Nm) when the control variable is the rotor voltage u(t) and the electromagnetic transient process is not neglected. A criterion in the general form (17) was adopted, with the final time T=0.3s. A change of the control law was introduced for t > 0.95T, in order to maintain the desired steady-state value of the speed. The results are obtained based on the algorithm indicated in [5], [6]. One can remark that the shapes of curves i(t) and ω(t) are rather likewise for the suboptimal and optimal solution, respectively.

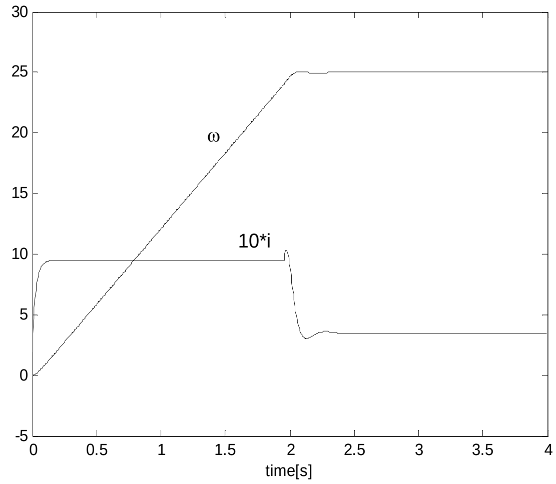

Fig. 1 The system behaviour for m=0.2Nm and $T<T_{lim}$

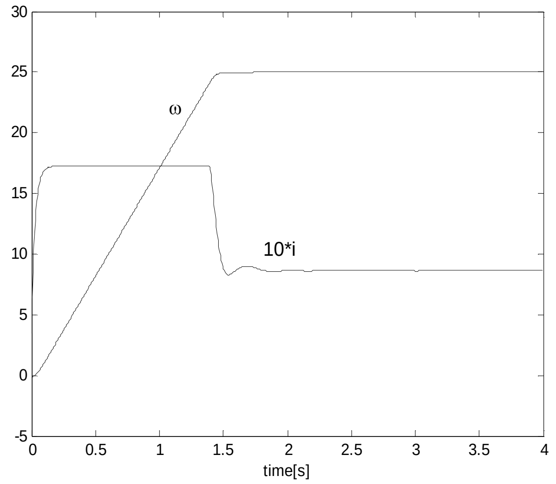

Fig. 2 The system behaviour for m=0.5Nm without restrictions

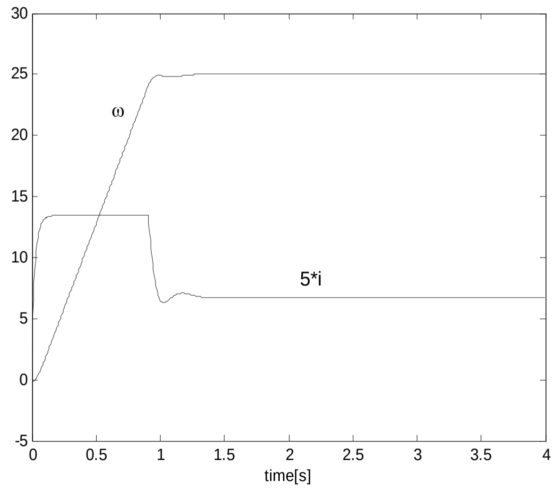

Fig. 3 The system behaviour for m=0.78Nm without restrictions

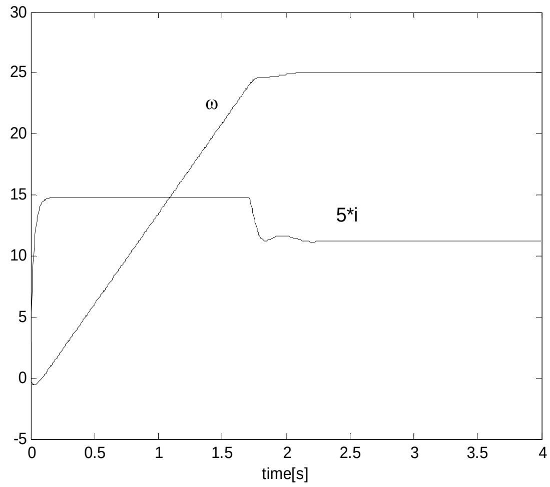

Fig. 4 The system behaviour for m=1.3Nm and $i<i_{lim}$

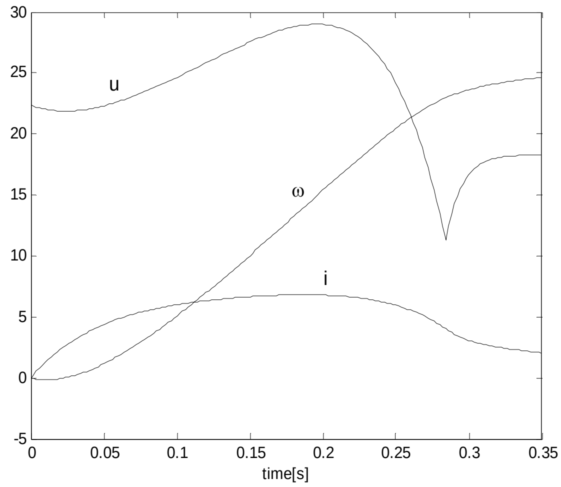

Fig. 5 The system behaviour in the optimal control case (m=0.78Nm)

The energy losses I_J on the interval [0,T] and J_t on the interval $[0,T_t]$ of the transient process were computed for the first four cases and are presented in the next table.

The computed energy I_J is smaller in each case with about 3 – 4% then ones resulted from (22) because of the transient increase of the current. These energy losses correspond to the minimum given by (27) for the second and third cases, when no restriction are activated, and they are greater then the minimum value (27) for the other situations.

m (Nm)	T (s)	T_t (s)	I_J (J)	I_t (J)
0.2	1.96	2.5	5,36	5.73
0.5	1.39	2	12.48	14.23
0.78	0.91	1.5	19.6	22.37
1.3	1.71	2.5	45.55	58.49

In the optimal control case, presented in the Fig. 5, the energy loss on the interval [0,T] is 28.53 J, and the total energy loss on the transient interval is I_t=29.98 J. These values are greater then ones obtained for the suboptimal control for the same value of the torque (m= 0.78 Nm) because the adopted criterion refers not only to energy losses, but involves other terms too. Also, a very small value for the final time (T=0.3 s) was adopted.

The energy consumption in this case is less with 25…30% as in the usual cascade control case [3], [4], depending on the weight matrices in the criterion.

5 Conclusions

An optimal control problem referring to an electrical drive version is studied. This case is approached as a linear quadratic optimal problem.

The winding energy losses represent a great amount of the total energy consumption of the drive system with D.C. motor in the transient process produced by a step variation of the rotor voltage.

A suitable control of the rotor current leads to a significant decrease of the energy losses.

A simple suboptimal algorithm is presented and a comparison with the optimal solution is performed.

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