Network Models to Assess PV Distributed Generation Effect on Voltage Profile in LV Distribution Networks

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Abstract – Great interest is posed on the connection of micro photovoltaic (PV) distributed generation (DG) units to LV distribution networks. Since the presence of grid-connected PV DG units is expected to grow in the future thanks to the new promotion measures recently introduced in the European Countries, it is necessary to investigate on how to assess the maximum amount of power that DG units can inject into distribution feeders without violating technical constraints.

The aim of the present work is to compare analytical and numerical network models as applied to estimate the penetration limits of PV DG given by voltage constraints on realistic LV distribution feeders.

The results provided by the comparison between the analytical method and the numerical simulations, carried out by Simulink® (Dynamic System Simulation for MATLAB®), proved that the analytical expressions used imply acceptable errors fully counterbalanced by the advantages of this approach in terms of clearness and simplicity of use for predicting DG penetration limits.

Key Words - Distributed Generation, Photovoltaic Systems, Power Quality, Network Models for Steady-State Analysis.

1. Introduction

In many Countries various government actions try to incentive the diffusion of renewable energy generation systems connected to electrical distribution networks [1]. For example, in Italy the Legislative Decree No. 387 of 29 December 2003 implements the European Directive No. 2001/77/EC on the promotion of electricity produced from renewable energy sources in the internal electricity market.

At low voltage (LV), in particular, the main interest is on the connection of micro photovoltaic (PV) distributed generation (DG) units ranging from a few kW up to about 30kW. In fact, photovoltaic is one of the most promising technology, which is also relatively easy to be integrated into buildings.

Since the presence of grid-connected PV DG units is expected to grow in the future, thanks to the new promotion measures introduced [2], it is necessary to investigate on how to assess the maximum amount of power that DG units can inject into distribution feeders. In practice, this amount is limited by technical issues [3], [4], [5] that at the LV level are essentially related to voltage and thermal constraints [6] (in general, PV generators are interfaced statically with the network and hence their contribution to the short circuit levels is negligible). Previous works by the Authors have dealt with the problem of assessing the slow voltage variations due to the presence of DG units in distribution networks and focused on voltage quality supplied to customers [7], [8].

In distribution networks operating in normal conditions and without grid-connected DG units the aforesaid voltage variations are only caused by load demand changes and they are compensated by automatic voltage regulation. Since DG changes the nature of distribution networks, existing voltage control can result into ineffective procedures. It is useful to evaluate what is the upper limit of PV power that can be injected by the DG units without modifying the voltage regulation system.

This can done by means of both analytical and numerical studies. The aim of the analytical models is to provide a tool that can be easily understood and used to describe the voltage profile of a LV distribution feeder with generation units by means of few fundamental parameters that influence the upper limit of the total power that can be injected by the DG units. To reach this goal the proposed analytical method is based on constant current models for loads and generators. This method is recalled in Sections 2 and 3 for a single power injection point (with one or more generation units). The typical case of three-phase radial LV distribution networks operating...
2. Voltage Drop Calculation in LV Distribution Feeders

Consider a LV feeder, whose length is \( L \), supplying \( N+1 \) load centres as shown in Fig. 1, where the laterals are represented by equivalent loads tapped at the load centres.

![Fig. 1. Generic distribution of lumped loads along the LV feeder](image)

Actual distribution networks are designed to minimize the influence of load variability on voltage at secondary substation transformer primary terminals. As a consequence primary terminals voltage is usually considered constant. Then, if the effect of the transformer impedance is neglected, secondary terminals voltage can be considered constant as well. In this work we have considered an ideal transformer to simplify the analytical study so as to have constant voltage at the beginning of the line \( (V_{LV}) \). However, \( V_{LV} \) is treated just as a parameter on which possible overvoltage depends too. If necessary, the variability of \( V_{LV} \) can be taken into account by means of an additional line stretch whose impedance is equal to the transformer internal one.

The voltage drop, \( \Delta V_i \), across the \( i \)-th stretch of line bounded by two consecutive load points (indicated by \( i-1 \) and \( i \)) is given by the following expression:

\[
\Delta V_i = V_{i-1} - V_i = \sqrt{3} \cdot r \cdot \left[ \sum_{j=1}^{N} (l_j \cdot I_{lj}) \right] = \sqrt{3} \cdot r \cdot l_i \cdot \left[ \sum_{j=1}^{N} I_{lj} \right]
\]  

\((1)\)

where: \( i = 1, 2, ..., N \); \( r \) (\( \Omega/\text{km} \)) is line resistance per phase per meter; \( l_i \) is the length of the \( i \)-th stretch of line.

In (1) the constant current model is used for loads, which are assumed operating at unity power factor (PF). The line is considered of constant section. This is a usual design requirements as often, in practice, the feeder can be reconfigured to backfeed loads, e.g. for post-fault reconfiguration.

Voltage at node \( k \) can be calculated as follows:

\[
V_k = V_{LV} - \sqrt{3} \cdot r \cdot \left[ \sum_{i=1}^{k} \left( z_j \cdot I_{lj} \right) + z_k \cdot \left( \sum_{j=k}^{N} I_{lj} \right) \right]
\]  

\((2)\)

where: \( k = 1, 2, ..., N \); \( V_{LV} \) is the voltage at the busbars of the secondary substation (SS).

Let us define \( z_j \) as:

\[
z_j = \begin{cases} 
0 & \text{if } j = 0 \\
I_l + I_2 + I_3 + ... + I_{l_j} & \text{if } j = 1, 2, 3, ..., N
\end{cases}
\]

The voltage at node \( k \), whose distance from the SS is \( z_k \), can be written as:

\[
V(z_k) = V_{LV} - \sqrt{3} \cdot r \cdot \left[ \sum_{j=0}^{k} (z_j \cdot I_{lj}) + z_k \cdot \left( \sum_{j=k}^{N} I_{lj} \right) \right]
\]

\((3)\)

with \( 0 \leq z_k \leq L \)

Defining \( \lambda_L(0,k-l) \) as the distance from the SS such that:

\[
\lambda_L(0,k-1) \cdot \sum_{j=0}^{k-l} I_{lj} = \sum_{j=0}^{k-l} (z_j \cdot I_{lj})
\]

and \( \sum_{l} (m,n) \) as the total current drawn by the loads connected to the nodes \( m, m+1, ..., n \):

\[
\sum_L(m,n) = \sum_{j=m}^{n} I_{lj}, \quad m, n = 0, ..., N
\]

expression (4) can be rewritten as:

\[
V(z_k) = V_{LV} - \sqrt{3} r \left[ \lambda_L(0,k-1) \sum_L(0,k-1) + z_k \sum_L(k,N) \right]
\]

\((7)\)

The above procedure allows to calculate \( V(z_k) \) replacing all the lumped loads with two equivalent loads only, \( \sum_L(0,k-1) \) and \( \sum_L(k,N) \), tapped, respectively, at the distance \( \lambda_L(0,k-l) \) and \( z_k \) from the SS.

3. Effect of a single current injection point on the voltage profile

This Section deals with changes in voltage profile modification in a LV distribution feeder due to the presence of one or more DG units that inject current \( (I_{gen}) \) into a single node \( (\text{gen}) \) of the feeder (Fig. 2).

DG units power factor (PF) is assumed equal to 1 since, usually, local power generation systems using the photovoltaic energy operate at unity PF [9], [10]. Loads and generators are represented by constant current models.
Including the effect of current \( I_{gen} \), injected into the feeder with reference to the case represented in Fig. 2, expressions (4) and (7) can be written as:

\[
V_{LV} = \sqrt{3} \cdot r \cdot \left[ \sum_{j=0}^{k} \left( z_j \cdot I_{Lj} \right) + z_k \sum_{j=k}^{N} I_{Lj} - z_k \cdot I_{gen} \right],
\]

\[0 \leq z_k \leq z_{gen}, \]

\[V_{LV} = \sqrt{3} \cdot r \cdot \left[ \sum_{j=0}^{k} \left( z_j \cdot I_{Lj} \right) + z_k \sum_{j=k}^{N} I_{Lj} - z_{gen} \cdot I_{gen} \right],
\]

\[z_{gen} < z_k \leq L\]  

(8)

Comparing expression (8) with (4) and (8') with (7), it can be concluded that voltage \( V_k \) increases due to the presence of DG because of the reduction in the current supplied by the SS.

Further, the voltage rise at the nodes upstream from the DG connection point \( 0 \leq z_k \leq z_{gen} \) is different from the one at the downstream nodes \( z_{gen} < z_k \leq L \). Then, defining the voltage variation \( AV_k \) at the k-th load point as the difference between (8') and (7), we obtain:

\[AV_k = \left\{ \begin{array}{ll}
\sqrt{3} \cdot r \cdot z_k \cdot I_{gen}, & 0 \leq z_k \leq z_{gen} \\
\sqrt{3} \cdot r \cdot z_{gen} \cdot I_{gen}, & 0 \leq z_k < z_{gen}
\end{array} \right. \]

(9)

Expression (9) shows that the voltage rise at the k-th upstream node is proportional to distance \( z_k \). This voltage rise reaches the maximum value at the generator connection point \( z_k = z_{gen} \). On the other hand, the voltage rise at the k-th downstream node is constant with the distance from the SS and equal to the aforesaid maximum value. This means that the voltage profile downstream from the generation point decreases with the same slope as in the case without DG. If current \( I_{gen} \) is such to determine a power flux inversion upstream from the generation point, i.e. \( I_{gen} \geq \Sigma_{\text{gen}}(N) \), the voltage profile has a local maximum at the generation point. If \( V(z_{gen}) \geq V_{LV} \) the local maximum is also the global one.

Considering expression (8') calculated for \( z_k = z_{gen} \), condition \( V(z_{gen}) \geq V_{LV} \) can be written as:

\[V_{LV} = \sqrt{3} \cdot r \cdot \left[ \lambda_L(0, \cdot gen-1) \cdot z_{gen} \cdot I_{gen} \right] \geq V_{LV}\]

(10)

and then

\[\lambda_L(0, \cdot gen-1) \cdot z_{gen} \cdot I_{gen} \geq z_{gen} \cdot I_{gen}\]

(11)

It can be concluded that \( V(z_{gen}) \geq V_{LV} \) when

\[I_{gen} \geq I^*_\text{gen}\]

(11')

where

\[I^*_\text{gen} = \lambda_{L_a}(0, \cdot gen-1) \cdot \Sigma_L(0, \cdot gen-1) \cdot \Sigma_L(\cdot gen, N) \]

(12)

and

\[\lambda_{L_a}(0, \cdot gen-1) = \frac{\lambda_L(0, \cdot gen-1)}{z_{gen}}\]

(13)

Expression (12) can be written as:

\[I^*_\text{gen} = [\lambda_L(0, \cdot gen-1) \cdot \alpha_{L_a} + \alpha_{LR}] \cdot I_{LTOT}\]

(12')

where

\[I_{LTOT} = \Sigma_L(\cdot , N)\]

(14)

\[\alpha_{LR} = \frac{\Sigma_L(\cdot , gen-1)}{I_{LTOT}}\]

(15)

\[\alpha_{L_a} = \frac{\Sigma_L(\cdot , gen, N)}{I_{LTOT}}\]

(16)

Note that in the performed calculations it is useful to normalise distances by \( z_{gen} \) or \( L \) according to which one is the most suitable in a given case. Normalisation by \( z_{gen} \) is indicated by subscript “n”, (e.g. \( \lambda_{La}(0, k) = \lambda_L(0, k)/z_{gen} \)); normalisation by \( L \) is indicated by superscript “\( L \)”, (e.g. \( \lambda_L(0, k) = \lambda_L(0, k)/L \)). On the grounds of the above definitions, the following expression holds:

\[\lambda_{La}(0, k) = \frac{1}{z_{gen}} \cdot \sum_{j=0}^{k} z_j \cdot I_{Lj} = \frac{1}{z_{gen}} \cdot \frac{\sum_{j=0}^{k} z_j \cdot I_{Lj}}{L} = \frac{1}{z_{gen}} \cdot \frac{\sum_{j=0}^{k} z_j \cdot I_{Lj}}{L} = \frac{1}{z_{gen}} \cdot \frac{\sum_{j=0}^{k} z_j \cdot I_{Lj}}{L} = \frac{1}{z_{gen} \cdot L} \cdot z_{gen} \cdot I_{LTOT} \]

(17)

From a physical point of view \( I^*_\text{gen} \) represents a current threshold such that, when current \( I_{gen} \) exceeds it, the voltage at the generation point, \( V(z_{gen}) \), exceeds the voltage at the busbars: the greater the current injected by the generators, the greater the voltage at the generation point. When condition (11') holds, it is interesting to calculate the maximum value of \( I_{gen} \).
referred to as $I_{\text{gen max}}$, given by (18), obtained assuming $V_L =$ equal to the upper voltage limit, $V_{\text{max}}$ (normally established by national standards):

$$ I_{\text{gen max}} = \frac{V_{\text{max}} - V_L}{\sqrt{3} \cdot r \cdot z'_{\text{gen}}} + \left[ L_n(0, \text{gen} - 1) \cdot \alpha_{LL} + \alpha_{LR} \right] \cdot I_{\text{TOT}} $$

(18)

which can be written as the sum of two terms:

$$ I_{\text{gen max}} = I_{\text{gen}}^0 + I_{\text{gen}}^* $$

(18')

where

$$ I_{\text{gen}}^* = \frac{V_{\text{max}} - V_L}{\sqrt{3} \cdot r \cdot z'_{\text{gen}}} L $$

(19)

The first term, $I_{\text{gen}}$, is a function of: line geometrical and electrical $(L, r)$ characteristics; distribution operating conditions $(L_L, r)$; normalised distance of the injection point from the busbars $(z'_{\text{gen}})$; the second term, $I_{\text{gen}}^*$, is a function of: value of the total current drawn by feeder loads $(I_{\text{TOT}})$; quantities related to load current distribution and position of the injection point $(\alpha_{LL}, \alpha_{LR} \text{ and } \alpha_{L}(0, \text{gen} - 1))$.

Thanks to (18) it is possible to highlight the dependence of $I_{\text{gen max}}$ on some important parameters, such as $V_L$, $z'_{\text{gen}}$ and $I_{\text{TOT}}$. It can be concluded that:

- $I_{\text{gen max}}$ decreases as $V_L$ increases:
  $$ V_L = V_{\text{max}} \Rightarrow \min_{V_L} \{ I_{\text{gen max}} \} = I_{\text{gen}} $$
  (20)

- $I_{\text{gen max}}$ decreases as $z'_{\text{gen}}$ increases:
  $$ z'_{\text{gen}} = 1, \text{i.e. } \text{gen} \equiv N $$
  $$ \min_{z'_{\text{gen}}} \{ I_{\text{gen max}} \} = \frac{V_{\text{max}} - V_L}{\sqrt{3} \cdot r \cdot L} + \lambda_L(0, N) \cdot I_{\text{TOT}} $$
  (21)

In particular, when $V_L = V_{\text{max}}$ and the DG units are connected at the end of the feeder $(z'_{\text{gen}} = 1)$, we obtain:

$$ \{ I_{\text{gen max}} \} = \lambda_L(0, N) \cdot I_{\text{TOT}} $$

(22)

- $I_{\text{gen max}}$ decreases as $I_{\text{TOT}}$ decreases:
  $$ I_{\text{TOT}} = 0 (I_{\text{gen}} = 0) $$
  $$ \Rightarrow \min_{I_{\text{TOT}}} \{ I_{\text{gen max}} \} = \frac{V_{\text{max}} - V_L}{\sqrt{3} \cdot r \cdot z'_{\text{gen}}} L $$
  (23)

From the above considerations it can be noted that the case of $I_{\text{gen max}}$ greater than $I_{\text{TOT}}$ is also possible. If distribution substations protections allow active power to flow from LV to MV, the value of $I_{\text{gen max}}$ is the one calculated by (18), otherwise the actual value of $I_{\text{gen max}}$ can be lower. In general, the possibility to inject power in LV distribution lines, besides voltage constraints is also limited by protections, line current carrying capacity, generation of harmonic currents, etc. These further limitations could sometimes be more restrictive than voltage constraints. However, their evaluation is not the aim of the present work.

It is useful to introduce another important parameter, $V'_{LV}$, which we define as the maximum value allowed of voltage $V_{LV}$ that does not cause overvoltages in the presence of a given penetration level $I_{\text{gen}}$ ($\geq I_{\text{gen}}$). This voltage value can be calculated by means of the following expression:

$$ V'_{LV} = V_{\text{max}} + \sqrt{3} \cdot r \cdot \left( I'_{\text{gen}} \cdot \lambda_{\text{gen}} \right) \cdot z'_{\text{gen}} $$

(24)

Expression (24) is very useful, for example, when the parameters of the voltage control system (such as the off-load MV/LV transformer turn ratio) have to be changed as a consequence of an increase in distributed generation.

The assumptions under which the analytical expressions have been developed, i.e. loads at unity PF and constant current models for loads and generators, lead to acceptable errors in the evaluation of DG penetration limit, as will be shown in Section 4.

### 4. Numerical application and simulation tool

In this Section the results provided by the analytical method presented will be compared to the ones obtained by a more accurate tool using data of actual LV distribution feeders. To do this the values provided by expression:

$$ P_{\text{gen max}} = I_{\text{gen max}} \cdot V_{\text{max}} $$

(25)

obtained from (18) multiplying by $V_{\text{max}}$ will be compared to those provided by numerical simulations using more accurate models such as constant impedance model for loads with PF=0.9 and constant power model for the PV generators with PF=1. The simulations have been performed by Simulink® and SimPowerSystems Tool in MATLAB® environment.

To perform the aforesaid comparison per cent error between PV DG penetration calculated by the analytical expressions ($P_{\text{gen max}, \text{An}}$) and the one calculated by numerical simulations ($P_{\text{gen max}, \text{Sim}}$) is defined as follows:

$$ e\% = \frac{P_{\text{gen max}, \text{An}} - P_{\text{gen max}, \text{Sim}}}{P_{\text{gen max}, \text{Sim}}} \cdot 100 $$

(26)

The analysis is carried out using data relative to two LV three phase distribution feeders (cable lines), typically used in Italian distribution system.

In Table 1 the cable line electrical and geometric characteristics are reported.
Table 1. Cable line characteristics

<table>
<thead>
<tr>
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<th>Feeder A</th>
<th>Feeder B</th>
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</thead>
<tbody>
<tr>
<td>Section [mm²]</td>
<td>35</td>
<td>95</td>
</tr>
<tr>
<td>Resistance [Ω/km]</td>
<td>0.97</td>
<td>0.383</td>
</tr>
<tr>
<td>Reactance [Ω/km]</td>
<td>0.11</td>
<td>0.074</td>
</tr>
<tr>
<td>Current carrying capacity (Iz) [A]</td>
<td>123</td>
<td>245</td>
</tr>
<tr>
<td>Length [km]</td>
<td>0.250</td>
<td>0.150</td>
</tr>
<tr>
<td>Cable</td>
<td>overhead</td>
<td>underground</td>
</tr>
<tr>
<td>Conductor</td>
<td>Al</td>
<td>Al</td>
</tr>
</tbody>
</table>

In both cases five load points and rectangular load distribution (\(\lambda_L^L = \lambda_L^L (0, N) = 0.5\)) are considered, as shown in Fig. 3.

![Fig. 3. LV distribution feeder with five load points, rectangular load distribution and single current injection point (\(z'_gen=0.75\))](image)

Three loading conditions have been considered (see Table 2):

Table 2. Feeders loading conditions

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<thead>
<tr>
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<tbody>
<tr>
<td>A</td>
<td>7.4</td>
<td>36.9</td>
<td>73.8</td>
</tr>
<tr>
<td>B</td>
<td>12.3</td>
<td>61.3</td>
<td>122.5</td>
</tr>
</tbody>
</table>

Fig. 4 shows the block diagram of the single-phase equivalent circuit representing the three-phase feeder implemented by Simulink®.

The block labelled “MV/LV substation” implements the secondary distribution substation, which supplies the LV cable line with its five nodes implemented by block “Feeder”. The block labelled “IMPPPT” represents the Maximum Power Point Tracker (MPPT), which is the part of the PV system control that allows the extraction of the maximum power from the PV array. The PF control and current reference generator for PV inverter is implemented in block “Local generation”. The power factor (PF) control is one of the most important issues in connecting PV generators to the utility grid because, as said before, they are usually operated at unity PF. The Phase Locked Loop (PLL) technique has been used for generating the current reference synchronised with the utility voltage in the PV power conversion system [11].

The PV system can be connected to each of the feeder nodes: DG 1 (\(z'_gen=0.25\)), DG 2 (\(z'_gen=0.5\)), DG 3 (\(z'_gen=0.75\)) and DG 4 (\(z'_gen=1\)) as shown in Fig. 4.

Figs. 5, 6, 7 and 8 show plots of the per cent error given by expression (26) as a function of \(V_{LV}\) for different feeder loading conditions. In the graphs the lowest voltage values are the ones at which thermal constraints are more restrictive than voltage constraints [6]. Consequently, it is meaningless to report the error values for voltages lower than that limit. The widest range of voltage \(V_{LV}\) at which overvoltage is prevailing over thermal constraints is obviously in the longest feeder for \(z'_gen=1\) (case A). Further, the range width is not much sensitive to feeder length when DG is connected close to the substation (\(z'_gen=0.25\)). In the cases examined the per cent error is negative for high \(V_{LV}\) values, while it is positive for the lower ones.
The absolute value of the per cent error is always lower than 10%. The positive errors are always lower than 2.5% for \( z'_{\text{gen}}=0.25 \), while they are lower than 10% for \( z'_{\text{gen}}=1 \). The negative ones are always lower than about 8%. The results of the analysis performed prove that the per cent errors are acceptable and, actually, compensated by the advantages of the analytical approach.

5. Conclusions

The voltage profile of a distribution feeder is modified when DG injects current into the line because of the decrease in the current flowing through feeder. This causes a rise in the voltage profile that may lead the voltage to exceed its maximum limit, normally established by Standards. Since this fact represents a hindrance to the penetration of DG units into distribution networks, it is useful to determine the limit value of the total power that can be injected into the line by DG without causing overvoltages.

In this paper analytical expressions have been presented in case of one or more generators injecting current into a single point. This approach is interesting because it allows to define analytical expressions easy to be implemented and used. This expressions shows the parameters affecting the upper limit of current/power that can be injected by the DG units. The results provided by the analytical method have been compared to the ones obtained by numerical simulations carried out by Simulink® (Dynamic System Simulation for MATLAB®) on realistic LV distribution feeders. This comparison proved that the analytical expressions used imply acceptable errors fully counterbalanced by the advantages of this approach in terms of clearness and simplicity of use in predicting DG penetration limits.

References:


