Homogeneity-Based Higher-Order Sliding Mode Controller Design for Permanent Magnet Linear Synchronous Motor

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Abstract: The permanent magnet linear synchronous motor (PMLSM) is sensitive to various disturbances such as the load disturbances, parameter perturbations, end effect and so on. To overcome this trouble, a new nonlinear robust controller using the homogeneity-based higher-order sliding mode control technique for the PMLSM is proposed. The detailed systematic controller design procedure is discussed. A digital-signal-processor (DSP) –based PMLSM position control system is implemented. The simulation and experimental results under different parameter and load variations are discussed and compared. The proposed control system shows good robustness and high accuracy in spite of the uncertainties, which confirms the theoretical analysis.

Key-Words: PMLSM, homogeneity, higher-order sliding mode, DSP

1 Introduction

Unlike rotary motors, it is not required for linear motors to indirectly couple mechanisms such as gear boxes, chains and screws, which will greatly reduce the effects of contact-types of nonlinearities and disturbances such as backlash and frictional forces [1]. The advantages of the PMLSM include simple mechanical construction, high speed, high acceleration and high motion precision. Therefore, the PMLSM is suitable for high-performance servo applications and has been used widely for the industrial robots, machine tools, semiconductor manufacturing systems, X-Y driving devices and so on [2, 3]. However, the PMLSM is sensitive to the load disturbances and parameter perturbations in the servo drive system because it is not equipped with auxiliary mechanisms such as gears or ball screws. In addition, the end effect makes the thrust force control more difficult [4]. How to compensate these disturbances which directly impose on the mover of the PMLSM and cause unsatisfying dynamic performance is very important in direct drive applications.

Due to the typical precision positioning requirements and low offset tolerance of their applications, the control of the PMLSM under the influence of disturbances is particularly challenging since the conventional PID control usually may not suffice in these application domains [5]. There has been considerable research on applications of advanced control schemes for the PMLSM. A disturbance suppression control system with the force feedforward action to suppress the effect of disturbances was presented in [6]. In such a system, the disturbances can be detected by the disturbance observer. Feedforward control is used in the force or torque controllers to obtain robustness. However, the inverse dynamic based disturbance observer cannot guarantee sufficient robustness for the servo drive system if the disturbances are large. The linearization method has been successfully used for the PMLSM [7]. This method, however, requires accurate parameters of the PMLSM and complex control procedures. In last few years, some research has focused on applications of the feedforward neural network (NN) for the PMLSM. In [8], an on-line trained fuzzy NN (FNN) controller was proposed to control a permanent magnet synchronous servo motor drive. However, the FNN’s application domain is limited to the static problem due to the feedforward network structure, and the weight updates of the feedforward NN do not utilize the internal information of the NN, and the function approximation is sensitive to the training data.

The standard sliding mode control technique in the variable structure control is a very effective nonlinear robust control approach. The basic idea is to force the state via a discontinuous feedback to move on a prescribed manifold [9-11]. However, the specific problem entailed by this technique is the chattering effect which influences the practical applications. This paper focuses on the application of the homogeneity-based higher-order sliding mode control technique for the PMLSM control system. By using this approach, the chattering effect is totally removed, and higher-order precision is provided whereas all the qualities of standard sliding mode are kept. Meanwhile, the homogeneity provides for the
highest possible asymptotic accuracy in the presence of the uncertainties [17]. To the authors’ best knowledge, this is the first time that the homogeneity-based higher-order sliding mode control algorithm is applied to the PMLSM position control system. Both the simulation and experimental results show the proposed controller has a good disturbance-rejection performance and tracks different position commands well compared with the conventional three-closed-loop PID controller. Thus, the effectiveness of the proposed system and correctness of the theoretical analysis are testified.

2 Mathematical Model of PMLSM
As shown in Fig.1, the PMLSM studied in this paper is a single-side flat motor which comprises a long stationary “secondary” and a moving short “primary”. The secondary is equipped with a sequence of Neodymium-Iron-Boron (NdFeB) permanent-magnet with a guidance rail and linear scale. The primary contains the core armature winding and Hall sensing elements. The electromagnetic thrust force is generated by the interaction between the secondary NdFeB magnet and magnetic field of AC windings in the mover driven by a pulse width modulation (PWM) voltage source inverter. The motion of the PMLSM is highly controllable as the electromagnetic thrust force is directly added to the mechanical system without coupling mechanism.

![Fig.1. PMLSM used for study](image)

The $d$-$q$ coordinate system is a “rotating” reference frame that moves at a synchronous speed. The voltage equations of the PMLSM can be described as follows [12]:

\[
\frac{d i_d}{d t} = \frac{R}{L} i_d + \frac{\pi}{\tau} v i_q + \frac{1}{L} U_d
\]
\[
\frac{d i_q}{d t} = \frac{R}{L} i_q - \frac{\pi}{\tau} v i_d - \frac{\psi \pi}{\tau L} v + \frac{1}{L} U_q
\]
\[
\frac{d v}{d t} = \frac{3n_p \psi}{2\tau M} i_q - \frac{B}{M} v - \frac{1}{M} (F_i + F_d)
\]

For the sake of terseness, we note that $X = (x_1, x_2, x_3, x_4)^T = (i_d, i_q, S, v)^T$, where $S$ is the linear position of the mover. Let $u$ denote the input $u = [u_1, u_2]^T = [u_d, u_q]^T$. The formalization of the parameters is stated as

$p_1 = -R/L$, $p_2 = \pi v / \tau$, $p_3 = 1/L$, $p_4 = -\psi \pi / \tau L$, $p_5 = 3n_p \psi / 2\tau M$, $p_6 = -B / M$, $F_i = F_i + F_d$.

Then the dynamic equation of the PMLSM can be rewritten as

\[
\dot{x}_1 = p_1 x_1 + p_2 x_2 x_4 + p_3 u_1
\]
\[
\dot{x}_2 = p_2 x_2 - p_3 x_1 x_4 + p_4 x_4 + p_5 u_2
\]
\[
\dot{x}_3 = x_4
\]
\[
\dot{x}_4 = p_6 x_4 - F_i / M
\]

The controller is designed to guarantee the robust performance in presence of parameters and load variations. The field-oriented-control is employed, namely, the reference value of the direct-axis primary current $x_d = 0$. The linear
position of the mover must track a reference trajectory \( x_{3d} \).

### 3 Proposition of Control Strategy for PMLSM

As we all know, the standard sliding mode features are high accuracy and robustness with respect to various internal and external disturbances. It may be implemented only if the relative degree of the constraint is 1, i.e. control has to appear explicitly already in the first total time derivative of the constraint function. Also, high-frequency control switching may cause the so-called chattering effect [13]. Some researchers relate the chattering behavior to the discontinuity of the sign function on the sliding variable. To overcome the problem, they suggest to replace the sign function in a small vicinity of the surface by a smooth approximation, which implies a small deterioration of accuracy and robustness [14, 15].

In recent years, an approach called “high-order sliding mode” has been proposed. Consider a smooth dynamic system with a smooth output function \( x \), and let the system be closed by some possibly-dynamical discontinuous feedback. Then provided that successive total time derivatives \( \sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)} \) are continuous functions of the closed-system state-space variables and the set \( \sigma = \dot{\sigma} = \ldots = \sigma^{(r-1)} = 0 \) is non-empty and consists locally of Filippov trajectories, the motion on the set \( \sigma = \dot{\sigma} = \ldots = \sigma^{(r-1)} = 0 \) is called \( r \)-sliding mode (\( r \)th order sliding mode). The \( r \)th derivative \( \sigma^{(r)} \) is mostly supposed to be discontinuous or non-existent. Almost all known higher-order sliding mode controllers possess specific homogeneous properties. The corresponding homogeneity of \( r \)-sliding controllers is called the \( r \)th-order sliding homogeneity in [16]. The homogeneity makes the convergence proofs of the higher-order sliding mode controllers standard and provides for the highest possible asymptotic accuracy in the presence of the noises, delays and discrete measurements [17].

#### 3.1 Homogeneity-Based Higher-order Sliding Mode Control

Consider a dynamic system of the form

\[
\dot{x} = a(x) + b(x)u, \quad y = \sigma(x(t)) \tag{9}
\]

where \( x \in \mathbb{R}^n \) is the state variable; \( u \in \mathbb{R} \) is control; \( \sigma \in \mathbb{R} \) is a measured output; the smooth functions \( a, b, \sigma \) are assumed unknown.; the dimension \( n \) can also be uncertain. The control objective is to make the output \( \sigma \) vanish in finite time and to keep \( \sigma = 0 \).

The output \( \sigma \) satisfies an equation of the form

\[
\sigma^{(r)} = h(x, t) + g(x, t)v, \quad g = \frac{\partial \sigma^{(r)}}{\partial v} \neq 0, \quad h = \sigma^{(r)} \bigg|_{t=0} \tag{10}
\]

where \( h \) and \( g \) are unknown smooth functions, and \( v \) is the actual control instead of \( u \) which is considered as an additional coordinate increasing the dimension of the initial state space to a unit. Suppose that the inequalities

\[
0 < K_m \leq g(x, t) \leq K_M, \quad |h(x, t)| \leq C \tag{11}
\]

hold for some \( K_m, K_M, C > 0 \). (10) and (11) imply the differential inclusion

\[
\sigma^{(r)} \in [-C, C] + [K_m, K_M]v \tag{12}
\]

A bounded feedback control

\[
v = \varphi(\sigma, \dot{\sigma}, \ldots, \sigma^{(r-1)}) \tag{13}
\]

is constructed such that all trajectories of (12), (13) converge in finite time to the origin \( \sigma = \dot{\sigma} = \ldots = \sigma^{(r-1)} = 0 \) of the \( r \)-sliding phase space. A differential inclusion \( \dot{x} \in F(x) \) is further called a Filippov differential inclusion if the vector set \( F(x) \) is non-empty, closed, convex, locally bounded and upper-semicontinuous [18].

**Definition 1.** A function \( f : \mathbb{R}^n \rightarrow \mathbb{R} \) (respectively, a vector-set field \( F(x) \subset \mathbb{R}^n \), \( x \in \mathbb{R}^n \) or a vector field \( f : \mathbb{R}^n \rightarrow \mathbb{R}^n \)) is called homogeneous of the degree \( q \in \mathbb{R} \) with the dilation \( d_x = (x_1, x_2, \ldots, x_n) \mapsto (\kappa^m x_1, \kappa^m x_2, \ldots, \kappa^m x_n) \), where \( m_1, \ldots, m_n \) are some positive numbers (weights), if for any \( \kappa > 0 \) the identity \( f(x) = \kappa^{-q} d^{-1}_x f(d_x x) \) holds (respectively, \( F(x) = \kappa^{-q} d^{-1}_x F(d_x x) \), or \( f(x) = \kappa^{-q} d^{-1}_x f(d_x x) \)). The non-zero homogeneity degree \( q \) of a vector field can always be scaled to \( \pm 1 \) by an appropriate proportional change of the weights \( m_1, \ldots, m_n \).

Note that the homogeneity of a vector field \( f(x) \) (a vector-set field \( F(x) \)) can be equivalently be defined as the invariance of the differential equation \( \dot{x} \in f(x) \) (differential inclusion \( \dot{x} \in F(x) \)) with respect to the combined time-coordinate transformation:

\[
G_x : (t, x) \mapsto (\kappa^p t, d_x x), \quad p = -q.
\]

**Property 1.** A differential inclusion \( \dot{x} \in F(x) \) (equation \( \dot{x} = f(x) \)) is further called globally uniformly finite-time stable at 0, if it is Lyapunov stable and for any \( R > 0 \) exists \( T > 0 \), such that any trajectory starting within the disk \( \|x\| < R \) stabilizes at zeros in the time \( T \).

**Property 2.** A differential inclusion \( \dot{x} \in F(x) \) (equation \( \dot{x} = f(x) \)) is further called globally uniformly asymptotically stable at 0, if it is Lyapunov stable and for
any \( R>0, \varepsilon >0, T>0 \) exists such that any trajectory starting within the disk \( \| x \| < R \) enters the disk \( \| x \| < \varepsilon \) in the time \( T \) to stay there forever. A set \( D \) is called retractable if \( d_{x}D \subset D \) for any \( \kappa <1 \).

**Property 3.** A homogeneous differential inclusion \( \dot{x} \in F(x) \) (equation \( \dot{x}=f(x) \)) is further called contractive, if there are 2 compact sets \( D_{1}, D_{2} \) and \( T>0 \) such that \( D_{2} \) lies in the interior of \( D_{1} \) and contains the origin, \( D_{1} \) is dilation-retractable, and all trajectories starting at the time 0 within \( D_{1} \) are localized in \( D_{2} \) at the time moment \( T \).

**Theorem 1.** Let \( \dot{x} \in F(x) \) be a homogeneous Filippov inclusion with a negative homogeneous degree \( -p \). Then properties 1, 2 and 3 are equivalent and the maximal settling time is a continuous homogeneous function of the initial conditions of the degree \( p \).

**Corollary 1.** The global uniform finite-time stability of homogeneous differential equations (Filippov inclusions) with negative homogeneous degree is robust with respect to homogeneous perturbations causing locally small changes of the equation (inclusion) graph.

**Definition 2.** Scaling the system homogeneity degree to \( -1 \), achieve that the homogeneity weights of \( t, \sigma, \dot{\sigma}, ..., \sigma^{(r-1)} \) are \( 1, r, r-1, ..., 1 \), respectively. This homogeneity is further called the r-sliding homogeneity. The inclusion (12), (13) and controller (13) are called r-sliding homogeneous if for any \( \kappa >0 \) the combined time-coordinate transformation \( G_{x}:(t,\sigma,\dot{\sigma},...,\sigma^{(r-1)})\mapsto(\kappa t,\kappa^{r}\sigma,\kappa^{r-1}\dot{\sigma},...\kappa^{r-1}\sigma^{(r-1)}) \) preserves the closed-loop inclusion (12), (13).

Transformation (14) transfers (12), (13) into

\[
\frac{d^{r}(\kappa^{m}\sigma)}{d\kappa^{m}t^{r}} \in [-C,C]+[K_{m},K_{M}]\varphi(\kappa^{r}\sigma,\kappa^{r-1}\dot{\sigma},...\kappa^{r-1}\sigma^{(r-1)})
\]

Hence, (13) is r-sliding homogeneous iff

\[
\varphi(\kappa^{r}\sigma,\kappa^{r-1}\dot{\sigma},...\kappa^{r-1}\sigma^{(r-1)}) = \varphi(\sigma,\dot{\sigma},...\sigma^{(r-1)})
\]

Such a homogeneous controller is inevitably discontinuous at the origin \((0,...,0)\), unless \( \varphi \) is a constant function. It is also uniformly bounded, since it is locally bounded and takes on all its values in any vicinity of the origin.

Let \( q \) be the least common multiple of 1, 2, ..., \( r \), and \( \beta_{i},...\beta_{r-1}, \) > 0. Define

\[
N_{i,r} = \left[ \left| \sigma^{(i)} \right|^{q} + \left| \sigma^{(i-1)} \right|^{q} + ... + \left| \sigma^{(r-1)} \right|^{q} \right]^{(r-i)/q}
\]

\[
\varphi_{i,r} = \text{sign} \ \sigma, \ \varphi_{i,r} = \text{sign}(\sigma^{(i)}) + \beta_{i}N_{i,r}\varphi_{i-1,r},
\]

Then \( v = -\alpha\varphi_{i-1,r}(\sigma,\dot{\sigma},...\sigma^{(r-1)}) \) defines the standard r-sliding controller [16]. The main drawback of the controller is some trajectory chattering during the transient caused by the complicated structure of the control discontinuity set. The output-feedback performance with noisy measurements is also problematic. Corollary 1 allows new controller structures to be produced transforming known homogeneous controllers. Define the homogeneous norm and the saturation function [17]

\[
N_{r} = N_{r,r} = \left[ \left| \sigma^{(i)} \right|^{q} + \left| \sigma^{(i-1)} \right|^{q} + ... + \left| \sigma^{(r-1)} \right|^{q} \right]^{1/q}
\]

sat\( (z,e) = \min \left[ 1, \max(-1,z/e) \right] \)

Let \( i = 1, ..., r-1 \). The construction is as follows:

\[
\phi_{i,r} = \text{sign} \ \sigma, \ \phi_{i,r} = \text{sat}\left[ (\sigma^{(i)}) + \beta_{i}N_{i,r}\varphi_{i-1,r} \right] / N_{r}^{r-i}, e_{i}
\]

Obviously \( \phi_{i,r} \) is homogeneous of the weight 0 and continuous everywhere except \( \sigma = \sigma_0 = ... = \sigma^{(r-1)} = 0 \). The controller

\[
v = -\alpha\phi_{i-1,r}(\sigma,\dot{\sigma},...\sigma^{(r-1)})
\]

ensures the finite-time convergence to the r-sliding mode \( \sigma = 0 \) with properly chosen \( \alpha, \beta_{i} \) and small \( e_{i} \). It can be shown that \( \beta_{i} \) and \( e_{i} \) can be chosen once for each \( r \), and only \( \alpha >0 \) is to be adjusted with respect to \( C,K_{m},K_{M} \).

### 3.2 Design of the Homogeneity-Based Second-Order Sliding Mode Controller for PMLSM

The content in this section is to design a MIMO homogeneity-based second-order sliding mode controller for the PMLSM. The aim is to force the direct-axis current \( x_{1} \) and linear position \( x_{3} \) to be the reference values \( x_{1d} \) and \( x_{3d} \), respectively. Take

\[
\sigma_{1} = x_{1} - x_{1d} = e_{1}
\]

Note that the relative degree of \( \sigma_{1} \) equals 1. Let \( e_{3} = x_{3} - x_{3d} \), and

\[
\sigma_{2} = \dot{e}_{3} + \lambda_{1}e_{1} + \lambda_{2}e_{3}
\]

where \( \lambda_{1} \) and \( \lambda_{2} \) are positive constant numbers such that \( H(z) = z + \lambda_{1}z + \lambda_{2}z \) is Hurwitz polynomial. The relative degree of \( \sigma_{2} \) is also 1. Based on (8), the first and second time derivatives of \( \sigma_{1} \) can be achieved as follows

\[
\dot{\sigma}_{1} = \dot{e}_{1} = p_{1}\dot{x}_{1} + p_{2}\dot{x}_{2}\dot{x}_{4} + p_{3}\dot{u}_{1} - \dot{\dot{x}}_{1d}
\]

\[
\ddot{\sigma}_{1} = p_{1}\ddot{x}_{1} + p_{2}\ddot{x}_{2}\dot{x}_{4} + p_{2}\ddot{x}_{2}\dot{x}_{4} - \ddot{x}_{1d} + p_{3}\ddot{u}_{1}
\]

\[
= p_{1}(p_{1}\dot{x}_{1} + p_{2}\dot{x}_{2}\dot{x}_{4} + p_{3}\dot{u}_{1})
\]

\[+p_{2}\ddot{x}_{2}(p_{1}\dot{x}_{1} + p_{2}\dot{x}_{2}\dot{x}_{4} + p_{3}\dot{x}_{4} + p_{3}\dot{u}_{2})\]
\[ + p_2 x_2 (p_3 x_2 + p_6 x_4 - F_L / M) - \ddot{x}_d + p_3 \dot{u}_1 = C_1(x) + D_1(x) \dot{u}_1 \]  
\[ = C_1(x) + D_1(x) \dot{u}_1 \]  
(26)

The first and second time derivatives of \( \sigma_1 \) can also be obtained as follows:

\[ \dot{\sigma}_2 = p_2 (p_3 x_2 - p_2 x_1 x_4 + p_4 x_4 + p_3 u_2) \]
\[ + (\dot{\lambda}_4 + \lambda_4) (p_8 x_2 + p_6 x_4 - F_L / M) \]
\[ - \dot{F}_L / M + \lambda_2 x_4 - \lambda_4 x_4 - \lambda_3 x_4 - \lambda_2 x_4 - \lambda_3 x_4 - \lambda_2 x_4 \]
\[ \dot{\sigma}_2 = -x_4 (\dot{\lambda}_4 + \lambda_4) + (p_8 p_5 + \dot{\lambda}_4) \]
\[ + \lambda_4 p_6 + \lambda_4 x_4 - \lambda_2 (p_3 x_2 + p_6 x_4 - F_L / M) \]
\[ - p_2 p_3 (x_1 x_4 + x_1 \dot{x}_1) - (\dot{\lambda}_4 + \lambda_4) \dot{F}_L / M + p_3 \dot{u}_2 \]
\[ = C_2(x) + D_2(x) \dot{u}_2 \]  
(27)

The generalized load force \( F_L \) is considered as a perturbation. Because the electromagnetic time constant is much smaller than the mechanical time constant, the variation of the generalized load force \( F_L \) is very slow compared with electrical variations. \( \dot{F}_L \) is supposed to be bounded as well as is two first time derivatives. \( C_{1N}, C_{2N}, D_{1N} = D_{2N} = p_{3N} = d_1 \) are the known nominal expressions by substituting the system nominal parameters into \( C_1(x), C_2(x), D_1(x)\) and \( D_2(x) \).

Let
\[ \dot{\sigma} = C(x) + D(x) [\dot{u}_1 \ 2 \dot{u}_2]^T \]  
(29)

where
\[ C(x) = \begin{bmatrix} C_1(x) \\ C_2(x) \end{bmatrix} = \begin{bmatrix} C_{1N} \\ C_{2N} \end{bmatrix} + \Delta C_1 + \Delta C_2 = C_N + \Delta C, \]
\[ D(x) = \begin{bmatrix} D_1(x) \\ D_2(x) \end{bmatrix} = \begin{bmatrix} d_N \\ d_N \end{bmatrix} + \begin{bmatrix} \Delta d_1 \\ \Delta d_2 \end{bmatrix} = D_N + \Delta D \]  
(30)

\( \Delta C \) and \( \Delta D \) contain all the uncertainties due to parameters and load force variations. Since \( p_{3N} \neq 0 \), the diagonal matrix \( D_N \) is reversible. However, the model (29) is uncertain. The concept of a feedback linearization technique is employed:

\[ [\dot{u}_1 \ 2 \dot{u}_2]^T = D_1^{-1} [-C_N + [v_1 \ v_2]^T] \]  
(31)

The actual input \( v = [v_1 \ v_2]^T \) is designed to stabilize the new system. Substitute (31) into (29), a new expression can be achieved

\[ \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix} = \begin{bmatrix} \dot{C}_1 \\ \dot{C}_2 \end{bmatrix} + \begin{bmatrix} \dot{D}_1 \\ \dot{D}_2 \end{bmatrix} [v_1 \ v_2] \]  
(32)

where
\[ \dot{C}_1 = \Delta C_1 - \frac{\Delta d C_{1N}}{d_N} \]
\[ \dot{C}_2 = \Delta C_2 - \frac{\Delta d C_{2N}}{d_N} \]
\[ \dot{D}_1 = \dot{D}_2 = 1 + \frac{\Delta d}{d_N} \]  
(33)

Thus, the MIMO problem is decoupled into a set of single-input problems which satisfy the requirements of homogeneity-based higher-order sliding mode control technique. Since the state variables and the perturbations of parameters are bounded, and under the assumption that \( |\Delta d| < |D_1| = |D_2| \), there exist positive constant numbers \( M_1, M_2, K_m \) and \( K_M \) so that
\[ |\dot{C}_1| < M_1 \]
\[ |\dot{C}_2| < M_2 \]
\[ 0 < K_m < \dot{D}_1 - \dot{D}_2 < K_M \]  
(34)

which is equivalent to (11).

The homogeneity approach to higher-order sliding mode design in the previous section is used to design the robust controller for PMLSM. Achieve \( r = 2, q = 2 \) and let \( \varepsilon = 0.2 \), thus,
\[ N_z = N_{2,2} = |(\sigma|^{1/2} + |\sigma|^1/2) | \]
\[ = \phi_{0,2} \]  
(35)

\[ \phi_{1,2} = \text{sat}[(\sigma|^{1/2} + |\sigma|^1/2) \sigma, 0.2] \]  
(36)

According to (22), the homogeneity-based 2-sliding mode controller is designed as
\[ v_j = -\alpha_j \text{sat}[(\sigma_j|^{1/2} + |\sigma_j|^1/2) \sigma], j = 1, 2 \]
\[ \alpha_j, j = 1, 2 \text{ is a chosen positive constant.} \]

3.3 Design of the Force Observer

Since the generalized load force \( F_L \) is unknown, it can be replaced by its estimated value in (31). Because the variation of \( F_L \) is very slow compared with electrical variations, \( F_L \) can be considered as a constant in a small time range. \( F_L \) and the linear speed \( v \) are chosen as new state variables. From (7), it can be obtained as
\[ \frac{d\xi}{dt} = H \xi + Aw \]  
(38)

where
\[ \xi = (F_L \ v)^T \]
\[ H = \begin{bmatrix} 0 & 0 \\ -1 & -B \\ M & M \end{bmatrix} \]
\[ A = \begin{bmatrix} 0 \\ 3\pi w \\ \frac{2\tau M}{M} \end{bmatrix} = \begin{bmatrix} c_r \\ a_r \end{bmatrix}, \]
\[ w = i_q \]

Obviously, this system can be observed. The observer can be designed as follows:
\[ \frac{d\hat{\xi}}{dt} = H \hat{\xi} + Aw + Q(v - E \hat{\xi}) \]  
(39)

where \( P = (q_l \ q_2)^T \) is a constant vector, and
4 Simulation and Experimentation

4.1 Simulation
The simulated results can be obtained by the MATLAB package. Tab.1 shows the simulation system parameters which are from the single-side flat PMLSM shown in Fig.1. The stroke length is 800mm. The maximum value of the quadrature-axis current permitted is 15A. To investigate the effectiveness of the proposed sliding mode control system, three simulation cases including a parameter variation and load disturbance are considered here. The PMLSM is in the condition of nominal system parameters without load at Case 1. At Case 2, the mass of mover is increased at 3 times the nominal value of the quadrature-axis current permitted is 15A. At Case 3, with nominal system parameters, a sudden load of 5.4kg is added at t=0.4s. The simulation results using the proposed scheme are compared with those using the field-oriented-control based three-closed-loop PID servo system of the PMLSM introduced in [12]. By applying the partial model matching method, the PID controller parameters can be solved as follows, the current loop proportional gain is \( K_{pC} = 4.5 \), the speed loop PID gains are \( K_I = 25.9, K_P = 2.3 \) and \( K_D = 0.1 \), the position loop proportional gain is \( K_{pP} = 19.2 \). For the proposed 2-sliding controller, controller parameters are selected as \( \lambda_1 = 3100, \lambda_2 = 200, \alpha_1 = 61000, \alpha_2 = 78000, q_1 = -1054 \) and \( q_2 = 75.6 \). Moreover, a second-order transfer function of the following form with the rise time of 0.06s is chosen as the reference model for a step command change: \( H(s) = \frac{6400}{s^2 + 180s + 6400} \), where \( s \) is the Laplace operator. The reference value of the direct-axis current is set as zero.

**TABLE 1  SYSTEM PARAMETERS OF PMLSM**

<table>
<thead>
<tr>
<th>Primary Winding Resistance</th>
<th>1.23Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>Quadrature-Axis Primary Inductance</td>
<td>8.41mH</td>
</tr>
<tr>
<td>Permanent Magnet Flux</td>
<td>0.55Wb</td>
</tr>
<tr>
<td>Mass of the Primary Part</td>
<td>10.6kg</td>
</tr>
<tr>
<td>Polar Pitch</td>
<td>30mm</td>
</tr>
<tr>
<td>Viscous Damping Coefficient</td>
<td>2Ns/m</td>
</tr>
</tbody>
</table>

In the simulation, a position step command with the 8mm-amplitude is given. The position responses and quadrature-axis current at three cases using the PID controller and homogeneity-based 2-sliding controller are shown in Fig.4 and Fig.5. As the parameters of the PID controller depend only on the nominal parameters of the drive system, the performance of the servo drive system is sensitive to the parameter variations and load disturbance in the system. As shown in Fig.4, the position tracking response of the square command can be satisfying by using a PID controller only at Case 1. At Case 2, the position overshoot phenomenon appears as shown in Fig.4(c) if there is a large parameter variation. When a sudden load is added at Case 3, the error between the position command and actual position is large as shown in Fig.4(e), and the tracking performance is unsatisfying. The position tracking responses using the proposed sliding mode controller at three cases are all satisfying as shown in Figs.5(a), (c) and (e). The robustness using the proposed controller is obvious compared with the PID controller. Meanwhile, it can be observed from the simulated results in Figs.4(b), (d) and (f) that there is no chattering in the quadrature-axis current by employing the homogeneity-based higher-order sliding mode control technique.
Fig.4. Simulated results using the PID controller (a) position response at Case 1 (b) quadrature-axis current at Case 1 (c) position response at Case 2 (d) quadrature-axis current at Case 2 (e) position response at Case 3 (f) quadrature-axis current at Case 3

Fig.5. Simulated results using the proposed sliding mode controller (a) position response at Case 1 (b) quadrature-axis current at Case 1 (c) position response at Case 2 (d) quadrature-axis current at Case 2 (e) position response at Case 3 (f) quadrature-axis current at Case 3

4.2 Experimentation

In the experimental studies, a 32-bit fixed-point microprocessor TMS320F2812 manufactured by Texas Instruments is used which has the advantages of high speed (150MIPS), 2 sets (4 channels) of QEP inputs, 2 sets (12 channels) of PWM outputs and 12 channels of 12-bit A/D converters (200ns conversion time). The C language is used for the control program. The DC link voltage value is 190V. The inverter legs are made of six insulated gate bipolar transistors (IGBTs). The PWM is implemented with a space-vector modulation technique with the switching frequency of 20kHz. The sampling frequencies of the phase current and position are 10kHz and 5kHz, respectively. A Heidenhain optical encoder with the resolution of 0.5 μm is equipped in the system as a position sensor. The block diagram of the hardware system is shown in Fig.6.

Fig.6. Block diagram of the hardware system

As mentioned above, a sudden load is added at about t=0.4s at Case 3 in the simulation. However, the accurate moment when load is added is difficult to be decided as delay exists in the practical operation. In the experimentation, a new case of Case 4 is considered. At this case, a load of 5.4kg is added at about t=2s, and a position step command is given at t=1.7s. A position step command with the 8mm-amplitude is given. The experimental results of the two controllers at Case 1, 2 and 4 under the step commands are shown in Fig.7 and Fig.8. Though the experimental results are similar to the simulated results, the minor difference between them is caused by the uncertainties of the real plant. As shown in Figs.7(a), (c) and (e), the dynamic performance of the PID servo drive system is sensitive to the parameter variation and load disturbance. Only the position tracking response of the square command can be satisfying at Case 1. The improvement of the tracking responses at three cases using the proposed sliding mode controller is obvious as shown in Figs.8(a), (c) and (e). It can also be observed from the experimental results in Figs.8(b), (d) and (f) that there is no control chattering in the quadrature-axis current by employing the homogeneity-based higher-order sliding mode control technique. In order to further testify the accuracy of the proposed controller, the position sinusoidal response at Case 2 is considered. The reference sinusoidal position with the 10mm-amplitude is shown in Fig.9(a). It can be viewed in Fig.9(b) that the linear position tracking error does
not exceed 12 μm, which can completely satisfy the experimental requirements. Fig. 9(d) displays the direct-axis current \( i_d \) which converges to zero. It can also be seen from Fig. 9(c) that chattering of the quadrature-axis current is removed by using the proposed controller.

Fig. 7. Experimental results using the PID controller under the step response (a) position response at Case 1 (b) quadrature-axis current at Case 1 (c) position response at Case 2 (d) quadrature-axis current at Case 2 (e) position response at Case 4 (f) quadrature-axis current at Case 4

Fig. 8. Experimental results using the proposed controller under the step response (a) position response at Case 1 (b) quadrature-axis current at Case 1 (c) position response at Case 2 (d) quadrature-axis current at Case 2 (e) position response at Case 4 (f) quadrature-axis current at Case 4

Fig. 9. Experimental results of the position sinusoidal response using the proposed controller (a) linear position reference (b) position tracking error (c) quadrature-axis current (d) direct-axis current

5 Conclusion
In this paper, the homogeneity-based higher-order sliding mode control technique is applied to the PMLSM control system. The systematic design methodology of the homogeneity-based 2-sliding controller is discussed. The proposed PMLSM position control system shows the satisfactory tracking performance of excellent robustness in spite of the uncertainties under different position commands. Therefore, the validity of the proposed control system is confirmed.

References:


