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Study on Reactive Power Optimization Problem Taking the Line Current as State Variable

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Abstract: -The math model of optimization problem is established by describing the electric power network equations as the hybrid form with node voltage and line current based on π equivalence circuit of power equipment, there the objective function is performed with a product of line current magnitude square and line resistance. It is indicated that Kuhn-Tacker optimal conditions are simple and convenient while the line current is considered as state variable in this paper. Finally, the case study is made by grads method at IEEE-30 system, it is explained that calculation efficiency of proposed method is higher than the method based on node voltage with direct expression due to the more information about state variable is included in the objective function.

Key-Words: - Node voltage; Line current; Optimal Power Flow; Reactive Power Optimization; Power Flow;

1 Introduction

The node voltage analysis is the main method in conventional power system application considering the nodal injective power and node voltage as variable, the line power is only used in reduced analysis for example in DC power flow calculation, but the line current is rarely taken into state variable account.

The line current method is proposed by Goswami in literatures [1]~[3] with the advantages which is able to treat with the mesh and quickly convergences considering the load as constant impedance model and the grounding branch of a line is ignored. It is feasible in the distribution network while these assumptions are generally unsuccessful in the transmission network. Therefore, there are some limits. The basic line model is established in [4,5] discarded the above terms by taking the nodal injected power as a voltage source and the impedance branch as link branch and the grounding branch as tree branch based on the π equivalence circuit, therefore the line analysis method is introduced for electric power network. The line current is actually line current while there is no line degradation, which can be considered as state variable, so the analysis in power network is more direct.

The optimal power flow model is best method to solve the reactive power optimization problem up to now included the interior point method[6,7] and the evolvement arithmetic[8]~[11] and other method[11] etc. The summarization is made in [12] and the shortcomings are pointed out that calculating efficiency must be ulteriorly improved and the dealing with inequality constraints is not very valid.

The math model of optimization problem is established by describing the electric power network equations as the hybrid form with node voltage and line current based on π equivalence circuit of power equipment, there the objective function is performed with a product of line current magnitude square and line resistance. It is indicated that Kuhn-Tacker optimal conditions are simple and convenient while the line current is considered as state variable in this paper. Finally, the case study is made by grads method at IEEE-30 system, it is explained that calculation efficiency of proposed method is higher than the method based on node voltage with direct expression due to the more information about state variable is included in the objective function.

2 The network equations

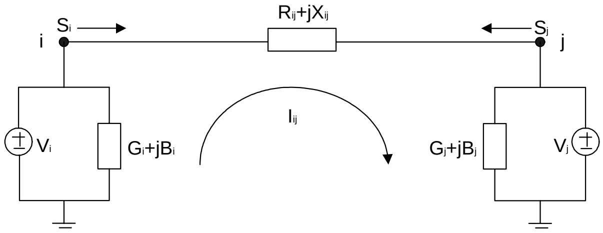

Fig. 1 Basic π Circuit

The above π-type equivalence circuit is the basic unit of power network analysis while the load is considered as a voltage source showed in figure 1.

In figure 1, $s_i = p_i + jq_i$ and $s_j = p_j + jq_j$ denote nodal injected power; $u_i = e_i + jf_i$ and $u_j = e_j + jf_j$ denote the node voltage; line current is $i_l = i_l^a + ji_l^r$; the subscript $i, j = 1,2,\cdots,N$ denote the node number; the subscript $l = 1,2,\cdots,L$ denotes the line number. Therefore, line current equation can be represented as below:

$$\dot{i}_l (R_{ij} + jX_{ij}) = \dot{u}_i - \dot{u}_j \tag{1}$$

It is obtained by expanding formula (1):

$$\begin{aligned} &i_l^a R_{ij} - i_i^r X_{ij} - e_i + e_j = 0 \\ &i_l^a X_{ij} + i_i^r R_{ij} - f_i + f_j = 0 \end{aligned} \tag{2}$$

However, the voltage at node i of the equivalent voltage source of the load branch is:

$$\overset{*}{u}_i = \frac{p_i - jq_j}{\sum\limits_{l\in i} i_{li} - u_i \sum\limits_{l\in i} (G_l + jB_l)} \tag{3}$$

It follows from (3) that:

$$\overset{*}{u}_i \sum_{l\in i} i_{li} - \overset{*}{u}_i u_i \sum_{l\in i} (G_l + jB_l) = p_i - jq_i \tag{4}$$

Above equation is represented as the plural form:

$$\begin{aligned} &(e_i - jf_i)(\sum_{l\in i} i_{li}^a + j\sum_{l\in i} i_{li}^r) - \\ &(e_i^2 + f_i^2)\sum_{l\in i} (G_l + jB_l) = p_i - jq_i \end{aligned} \tag{5}$$

Furthermore, it is obtained:

$$\begin{aligned} &e_i \sum_{l\in i} i_{li}^a + f_i \sum_{l\in i} i_{li}^r - (e_i^2 + f_i^2)\sum_{l\in i} G_l = p_i \\ &e_i \sum_{l\in i} i_{li}^r - f_i \sum_{l\in i} i_{li}^a - (e_i^2 + f_i^2)\sum_{l\in i} B_l = -q_i \end{aligned} \tag{6}$$

While the conductance G_l of the ground branch is ignored, it is chenged as follows:

$$\begin{aligned} &e_i \sum_{l\in i} i_{li}^a + f_i \sum_{l\in i} i_{li}^r = p_i \\ &e_i \sum_{l\in i} i_{li}^r - f_i \sum_{l\in i} i_{li}^a - (e_i^2 + f_i^2)\sum_{l\in i} B_l = -q_i \end{aligned} \tag{7}$$

As for node j, there is the same form of nodal voltage function with the node i. The equations (2) and (7) are the basic models of the power network.

3 Reactive power optimization problem

It is to minimize the network losses as the objective function of reactive power optimization problem:

$$z = \min \sum_{l=1}^{L} I_l^2 R_l \tag{8}$$

where $l = 1,2,\cdots,L$ denotes line number; I_l and R_l are the current magnitude and resistance of l-th line. The equality constraints are network equations, while the conductance of grounding branch is ignored they can be describe as following polar coordinates form:

$$V_i \cos\theta_i \sum_{l\in i} I_l \cos\phi_l + V_i \sin\theta_i \sum_{l\in i} I_l \sin\phi_l = p_i \tag{9}$$

$$\begin{aligned} &V_i \cos\theta_i \sum_{l\in i} I_l \sin\phi_l - V_i \sin\theta_i \sum_{l\in i} I_l \cos\phi_l \\ &- V_i^2 \sum_{l\in i} B_l = -q_i + q_{ci} \end{aligned} \tag{10}$$

where V_i, θ_i respectively are the voltage magnitude and angle of node i; ϕ_l is the current angle of line l; q_{ci} is the compensatory capacity in reactive power of node i. The above formulas can be change as:

$$\begin{aligned} &V_i \sum_{l\in i} I_l \cos(\phi_l - \theta_i) = p_i \\ &V_i \sum_{l\in i} I_l \sin(\phi_l - \theta_i) - V_i^2 \sum_{l\in i} B_l = -q_i + q_{ci} \end{aligned} \tag{11}$$

The line current variables also satisfy following equations by setting $R_l = R_{ij}, X_l = X_{ij}$:

$$\begin{aligned} &I_l \cos\phi_l R_l - I_l \sin\phi_l X_l - V_i \cos\theta_i + V_j \cos\theta_j = 0 \\ &I_l \cos\phi_l X_l + I_l \sin\phi_l R_l - V_i \sin\theta_i + V_j \sin\theta_j = 0 \end{aligned} \tag{12}$$

The inequality constraints of reactive power optimization problem are:

$$V_i^{\min} \le V_i \le V_i^{\max} \tag{13}$$

and:

$$q_{ci}^{\min} \le q_{ci} \le q_{ci}^{\max} \tag{14}$$

The reactive power optimization problem is composed with formulas (8), (11)~(14).

4 Kuhn-Tacker conditions

While the reactive power optimization problem is described as the form of nonlinear programming, the objective function is:

$$f(x, y) = 0 \tag{15}$$

The equality constraint is:

$$g(x, y) = 0 \tag{16}$$

The inequality constraint is:

$$h(x, y) \le 0 \tag{17}$$

where, x is the state variable; y is the control variable. The enlarged Lagrange function is:

$$L = f(x) + \alpha^T g(x,u) + \beta^T h(x,y) \tag{18}$$

where α, β are the Lagrange multipliers respectively corresponding equality and inequality constraints. The Kuhn-Tacker conditions are as:

$$L_x = f_x + \alpha^T g_x + \beta^T h_x = 0 \quad (19)$$

$$L_y = f_y + \alpha^T g_y + \beta^T h_y = 0 \quad (20)$$

$$g(x, y) = 0 \quad (21)$$

$$\beta^T h(x, y) = 0 \quad (22)$$

$$h(x, y) \le 0 \quad (23)$$

The scale of reactive power optimization problem is smaller while the line current is considered as the state variable compared with the case that line current and node voltage are as state variable, there are:

$$f_x = 2RI \quad (24)$$

where R is diagonal matrix with the line resistance elements; I is the line current vector; and:

$$f_y = 0 \quad (25)$$

To linearize the formulas (12) and (13), it can be obtained:

$$\begin{bmatrix} \Delta E \\ \Delta S \end{bmatrix} = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta I \\ \Delta V \end{bmatrix} + \begin{bmatrix} 0 \\ W \end{bmatrix} \Delta Y \quad (26)$$

where, ΔE is the line voltage error vector; ΔS is nodal injected power error vector; ΔI is the line current error vector; ΔV is the node voltage error vector; ΔY is the control variable error vector; H is block diagonal matrix with dimensions 2L×2L as following form:

$$\begin{bmatrix} R_l \cos\phi_l - X_l \sin\phi_l & -R_l I_l \sin\phi_l - X_l I_l \cos\phi_l \\ X_l \cos\phi_l + R_l \sin\phi_l & R_l I_l \cos\phi_l - X_l I_l \sin\phi_l \end{bmatrix} \quad (27)$$

N is the node-line incidence matrix with dimensions 2L×2N as following form:

$$\begin{bmatrix} -\cos\theta_i & V_i \sin\theta_i \\ -\sin\theta_i & -V_i \cos\theta_i \end{bmatrix} \quad (28)$$

J is a matrix that structure is same with the node-line incidence matrix with dimensions 2 N×2 L as following form:

$$\begin{bmatrix} V_i \cos(\phi_l - \theta_i) & -V_i I_l \sin(\phi_l - \theta_i) \\ V_i \sin(\phi_l - \theta_i) & V_i I_l \cos(\phi_l - \theta_i) \end{bmatrix} \quad (29)$$

L is a diagonal matrix with dimensions 2N×2N as following form:

$$\begin{bmatrix} \sum_{l \in i} I_l \cos(\phi_l - \theta_i) & V_i \sum_{l \in i} I_l \sin(\phi_l - \theta_i) \\ \sum_{l \in i} I_l \sin(\phi_l - \theta_i) - 2V_i \sum_{l \in i} B_l & -V_i \sum_{l \in i} I_l \cos(\phi_l - \theta_i) \end{bmatrix} \quad (30)$$

W is a block diagonal matrix with dimension 2N×N and its elements are as $\begin{bmatrix} 0 & 1 \end{bmatrix}^T$. It can be seen from formula (26):

$$\Delta E = H \Delta I + N \Delta V \quad (31)$$

$$\Delta S = J \Delta I + L \Delta V + W \Delta Y \quad (32)$$

If $\Delta S = 0$, the $\Delta V = -L^{-1}(J \Delta I + W \Delta Y)$ is obtained and introduced in formula (29), the line current corrective equation is:

$$\Delta E = (H - NL^{-1}J)\Delta I - NL^{-1}W \Delta Y \quad (33)$$

where $g_x = H - NL^{-1}J$ is the Jacob matrix for line current analysis; the $g_y = -NL^{-1}W$ with dimensions 2L×N.

5 Calculation

The Newton and grads method can be used to solve the Kuhn-Tacker conditions expressed by formulas (18)~(22). The grads method is used in this paper due to explain the efficiency of proposed arithmetic. The processes are as:

1) To set $k = 0$ and give the initial value of control variable vector y^k;

2) To calculate power flow according to formula (21);

3) To adjust that node voltage is whether or not violated, the penitentiary multiplier β^k must be determine if yes; otherwise to continue;

4) To perform coefficient matrixes and calculate Lagrange multiplier α^k according to formula (19);

5) To set $\Delta y^k = g_y \alpha^k$ (because of that f_y, h_y are equal to 0) and determine the corresponding control variable error vector, the iterations are ended if enough smaller; otherwise to correct the control variable:

$$y^{k+1} = y^k + S^k \Delta y^k \quad (34)$$

where the S^k is iteration step, which can be determine by one-dimensional searching technology. It is to adjust the control variables is whether or not violated by formula (14), the $\Delta y_i^k = 0$ if corresponding control variable is violated at node i, then to set $k = k + 1$ and turn to step 2);

The following points must be explained in above processes:

1) The node voltage analysis method is still used to calculate power flow in step 2) above, then the line current can be obtained according to formula (12) and the matrixes H, N, J, L also are get. The network states are either obtained by combining the formula (11) with (12), but the scale of problem is

enlarged and works are increased;

2) The penitentiary multiplier β^k can be solved by probe method and interior point method, there is not explained in detail;

3) The calculation of h_x must be regarded due to the node voltage is just related in formula (13). While the control variable error is ignored, it can be seen from formula (33) that is as:

$$\Delta V = -L^{-1} J \Delta I \tag{35}$$

viz.:

$$h_x = \Delta V / \Delta I = -L^{-1} J \tag{36}$$

The matrixes h_x, g_x, g_y can be expediently obtained because of the block diagonal form in matrix L.

6 Case study

The case study is made at IEEE-30 system by using grads method due to indicate the efficiency of proposed in this paper. The calculating results are listed in table 1 with the node data and table 2 with the line data.

Tab.1 The Results in Nodes

Node No.	Magnitude of Node Voltage	Angle of Node Voltage
1	1.0325	-4.69514
2	1.0913	-6.2897
3	1.0883	-7.96668
4	1.02742	-11.4393
5	1.01806	-12.6595
6	1.02923	-11.8209
7	1.0058	-8.98669
8	1.00828	-8.04644
9	1.023	-6.47197
10	1.05301	-10.0464
11	1.05719	-9.76528
12	1.04303	-10.8103
13	1.04622	-10.7799
14	1.04164	-10.9733
15	1.05318	-10.4868
16	1.04755	-10.8221
17	1.0636	-8.13097
18	1.05184	-11.2833
19	1.05604	-10.116
20	1.02008	-6.90728
21	1.04854	-10.6507
22	1.0338	-2.73349
23	1.02776	-5.62431
24	1.06526	-9.13676
25	1.05059	-10.2218
26	1.05442	-10.5013
27	1.04465	-11.0428
28	1.06307	-9.97491
29	1.02309	-6.49742
30	1.05	0

Tab.2 The Results in Lines

Line No.	Magnitude of Line Current
1	2.96097
2	1.03308
3	0.0757143
4	0.24069
5	0.469304
6	0.36855
7	1.83489
8	1.147
9	1.27351
10	1.08376
11	0.147711
12	0.143271
13	0.304861
14	0.564292
15	0.0563208
16	0.608776
17	0.0770484
18	0.0824186
20	0.0414895
19	0.479188
26	0.919257
21	0.0757026
22	0.133785
23	0.130145
24	0.450157
25	0.123161
27	0.0395628
28	0.601978
29	3.80608
30	0.113142
31	0.455993
32	0.0642532
33	0.0309831
34	0.311054
35	0.0233962
36	0.0106204
37	0.0766863
38	0.0827966
39	0.180353
40	0.242551
41	1.13922

The 5 iterations are needed to solve the reactive power optimization problem by grads method based on the line current variable, the network losses are listed in table 3. In table 4, the calculating results by different methods are showed. The calculating effects with the line current variable are better than the case with node voltage variable by grads method and approaches to the calculating results by Newton method with node voltage variable. It is explained that efficiency of the proposed method in this paper is high due to the information in f_x appeared in formula (19) is much more.

Tab.3 The Network Losses

Iter.	1	2	3	4	5
Net. Los.	0.0710068	0.0707587	0.070587	0.0704896	0.0704643

Tab.4 Comparing with Other Arithmetic

Methods	Iterations	Network	Compensatory	Compensatory

		Losses	Capacity at Node 10	Capacity at Node 24
1	5	0.0704643	0.22275	0.145011
2	6	0.0704835	0.198811	0.105563
3	5	0.0704647	0.216341	0.132754

In table 4, the 1 denotes ‘Grads Method with Line Current Variable’ and the 2 denotes ‘Grads Method with Node Voltage Variable’ while the 3 ‘Newton Method with Node Voltage Variable’.

7 Conclusion

The objective function is directly expressed as the network losses while the line current is considered as state variable by describing the reactive power optimization problem to optimal power flow form. It is indicated that calculating efficiency of the proposed method in this paper is quoteworthy seen from the calculating results due to the variety in the objective function value is sensitive while the state variable changes.

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