Study on Reactive Power Optimization Problem Taking the Line Current as State Variable

Zhang Jinsong¹  Wang Chengmin¹ Zhang Gong², and Hou Zhijian¹
¹.Department of Electrical Engineering
Shanghai Jiaotong University,  ².EPRI of China
Shanghai, Huashan Road, No. 1954,  Beijing, Qinghe Xiaoting
China

Abstract: -The math model of optimization problem is established by describing the electric power network equations as the hybrid form with node voltage and line current based on \( \pi \) equivalence circuit of power equipment, there the objective function is performed with a product of line current magnitude square and line resistance. It is indicated that Kuhn-Tacker optimal conditions are simple and convenient while the line current is considered as state variable in this paper. Finally, the case study is made by grads method at IEEE-30 system, it is explained that calculation efficiency of proposed method is higher than the method based on node voltage with direct expression due to the more information about state variable is included in the objective function.

Key-Words: - Node voltage; Line current; Optimal Power Flow; Reactive Power Optimization; Power Flow;

1 Introduction
The node voltage analysis is the main method in conventional power system application considering the nodal injective power and node voltage as variable, the line power is only used in reduced analysis for example in DC power flow calculation, but the line current is rarely taken into state variable account.

The line current method is proposed by Goswami in literatures [1]~[3] with the advantages which is able to treat with the mesh and quickly convergences considering the load as constant impedance model and the grounding branch of a line is ignored. It is feasible in the distribution network while these assumptions are generally unsuccessful in the transmission network. Therefore, there are some limits. The basic line model is established in [4,5] discarded the above terms by taking the nodal injected power as a voltage source and the impedance branch as link branch and the grounding branch as tree branch based on the \( \pi \) equivalence circuit, therefore the line analysis method is introduced for electric power network. The line current is actually line current while there is no line degradation, which can be considered as state variable, so the analysis in power network is more direct.

The optimal power flow model is best method to solve the reactive power optimization problem up to now included the interior point method[6,7] and the evolvement arithmetic[8]~[11] and other method[11] etc. The summarization is made in [12] and the shortcomings are pointed out that calculating efficiency must be ulteriorly improved and the dealing with inequality constraints is not very valid.

The math model of optimization problem is established by describing the electric power network equations as the hybrid form with node voltage and line current based on \( \pi \) equivalence circuit of power equipment, there the objective function is performed with a product of line current magnitude square and line resistance. It is indicated that Kuhn-Tacker optimal conditions are simple and convenient while the line current is considered as state variable in this paper. Finally, the case study is made by grads method at IEEE-30 system, it is explained that calculation efficiency of proposed method is higher than the method based on node voltage with direct expression due to the more information about state variable is included in the objective function.

2 The network equations

Fig. 1 Basic \( \pi \) Circuit

The above \( \pi \)-type equivalence circuit is the basic unit of power network analysis while the load is considered as a voltage source showed in figure 1.
In figure 1, \( s_i = p_i + jq_i \) and \( s_j = p_j + jq_j \) denote nodal injected power; \( u_i = e_i + jf_i \) and \( u_j = e_j + jf_j \) denote the node voltage; line current \( i_l = i_{li} + ji_{lj} \); the subscript \( i, j = 1,2,\ldots, N \) denote the node number; the subscript \( l = 1,2,\ldots, L \) denotes the line number. Therefore, line current equation can be represented as below:

\[
i_l(R_l + jX_l) = u_l - u_j
\]  

(1)

It is obtained by expanding formula (1):

\[
i_l^a R_l - i_l^a X_l - e_l + e_j = 0
\]  

(2)

\[
i_l^a X_l + i_l^a R_l - f_l + f_j = 0
\]  

(2)

However, the voltage at node \( i \) of the equivalent voltage source of the load branch is:

\[
u^*_i = \sum_{l=1}^L i_{li} - u_i \sum_{l=1}^L (G_l + jB_l)
\]  

(3)

It follows from (3) that:

\[
u^*_i = u^*_i + u_i \sum_{l=1}^L (G_l + jB_l) = p_i - jq_i
\]  

(4)

Above equation is represented as the plural form:

\[
(e_i - jf_i)\left(\sum_{l=1}^L i_{li}^2 + j\sum_{l=1}^L i_{lj}^2\right) -
(e_i^2 + f_i^2)\left(\sum_{l=1}^L G_l + jB_l\right) = p_i - jq_i
\]  

(5)

Furthermore, it is obtained:

\[
e_i \sum_{l=1}^L i_{li}^2 + f_i \sum_{l=1}^L i_{lj}^2 - (e_i^2 + f_i^2)\sum_{l=1}^L G_l = p_i
\]  

(6)

\[
e_i \sum_{l=1}^L i_{lj}^2 - f_i \sum_{l=1}^L i_{li}^2 - (e_i^2 + f_i^2)\sum_{l=1}^L B_l = -q_i
\]  

(7)

While the conductance \( G_l \) of the ground branch is ignored, it is changed as follows:

\[
e_i \sum_{l=1}^L i_{li}^2 + f_i \sum_{l=1}^L i_{lj}^2 = p_i
\]  

\[
e_i \sum_{l=1}^L i_{lj}^2 - f_i \sum_{l=1}^L i_{li}^2 - (e_i^2 + f_i^2)\sum_{l=1}^L B_l = -q_i
\]  

(7)

As for node \( j \), there is the same form of nodal voltage function with the node \( i \). The equations (2) and (7) are the basic models of the power network.

3  **Reactive power optimization problem**

It is to minimize the network losses as the objective function of reactive power optimization problem:

\[
z = \min \sum_{l=1}^L I_{li}^2 R_l
\]  

(8)

where \( l = 1,2,\ldots, L \) denotes line number; \( I_l \) and \( R_l \) are the current magnitude and resistance of \( l \)-th line. The equality constraints are network equations, while the conductance of grounding branch is ignored they can be describe as following polar coordinates form:

\[V_i \cos \theta_i \sum_{l=1}^L I_l \cos \phi_l + V_i \sin \theta_i \sum_{l=1}^L I_l \sin \phi_l = p_i
\]  

(9)

\[V_i \cos \theta_i \sum_{l=1}^L I_l \sin \phi_l - V_i \sin \theta_i \sum_{l=1}^L I_l \cos \phi_l
\]  

(9)

\[-V_i^2 \sum_{l=1}^L B_l = -q_i + q_{cl}
\]  

(10)

The line current variables also satisfy following equations by setting \( R_l = R_j, X_l = X_j \).

\[I_l \cos \phi_i X_j - I_j \sin \phi_j X_i - V_i \cos \theta_j + V_j \cos \theta_i = 0
\]  

(11)

\[I_i \cos \phi_i X_i + I_j \sin \phi_j R_i - V_i \sin \theta_j + V_j \sin \theta_i = 0
\]  

(12)

The inequality constraints of reactive power optimization problem are:

\[V_i^\text{min} \leq V_i \leq V_i^\text{max}
\]  

(13)

and:

\[q_{cl}^\text{min} \leq q_{cl} \leq q_{cl}^\text{max}
\]  

(14)

The reactive power optimization problem is composed with formulas (8), (11)–(14).

4  **Kuhn-Tacker conditions**

While the reactive power optimization problem is described as the form of nonlinear programming, the objective function is:

\[f(x,y) = 0
\]  

(15)

The equality constraint is:

\[g(x,y) = 0
\]  

(16)

The inequality constraint is:

\[h(x,y) \leq 0
\]  

(17)

where, \( x \) is the state variable; \( y \) is the control variable. The enlarged Lagrange function is:

\[L = f(x) + \alpha^T g(x,u) + \beta^T h(x,y)
\]  

(18)
where $\alpha, \beta$ are the Lagrange multiplies respectively corresponding equality and inequality constraints. The Kuhn-Tacker conditions are as:

\[
L_x = f_x + \alpha^T g_x + \beta^T h_x = 0 \tag{19}
\]
\[
L_y = f_y + \alpha^T g_y + \beta^T h_y = 0 \tag{20}
\]
\[
g(x, y) = 0 \tag{21}
\]
\[
\beta^T h(x, y) = 0 \tag{22}
\]
\[
h(x, y) \leq 0 \tag{23}
\]

The calculation of reactive power optimization problem is smaller while the line current is considered as the state variable compared with the case that line current and node voltage are as state variable, there are:

\[
f_x = 2RI \tag{24}
\]

where $R$ is diagonal matrix with the line resistance elements; $I$ is the line current vector; and:

\[
f_y = 0 \tag{25}
\]

To linearize the formulas (12) and (13), it can be obtained:

\[
[\Delta E] = \begin{bmatrix} H & N \\ J & L \end{bmatrix} \begin{bmatrix} \Delta I \\ \Delta V \end{bmatrix} + \begin{bmatrix} 0 \\ W \end{bmatrix} \Delta Y \tag{26}
\]

where, $\Delta E$ is the line voltage error vector; $\Delta S$ is nodal injected power error vector; $\Delta I$ is the line current error vector; $\Delta V$ is the node voltage error vector; $\Delta Y$ is the control variable error vector; $H$ is block diagonal matrix with dimensions $2L \times 2L$ as following form:

\[
\begin{bmatrix}
R_i \cos \phi_i - X_i \sin \phi_i & -R_i I_i \sin \phi_i - X_i I_i \cos \phi_i \\
X_i \cos \phi_i + R_i \sin \phi_i & R_i I_i \cos \phi_i - X_i I_i \sin \phi_i
\end{bmatrix} \tag{27}
\]

$N$ is the node-line incidence matrix with dimensions $2L \times 2N$ as following form:

\[
\begin{bmatrix}
-V_i \sin \theta_i & \cos \theta_i \\
-V_i \cos \theta_i & \sin \theta_i
\end{bmatrix} \tag{28}
\]

$J$ is a matrix that structure is same with the node-line incidence matrix with dimensions $2N \times 2L$ as following form:

\[
\begin{bmatrix}
V_i \cos(\phi_i - \theta_i) - V_i I_i \sin(\phi_i - \theta_i) \\
V_i \sin(\phi_i - \theta_i) & V_i I_i \cos(\phi_i - \theta_i)
\end{bmatrix} \tag{29}
\]

$L$ is a diagonal matrix with dimensions $2N \times 2N$ as following form:

\[
\begin{bmatrix}
\sum_{i} B_{ii} I_i \cos(\phi_i - \theta_i) + V_i \sum_{i} I_i \sin(\phi_i - \theta_i) \\
\sum_{i} B_{ii} I_i \sin(\phi_i - \theta_i) - 2V_i \sum_{i} B_{ii} I_i \cos(\phi_i - \theta_i)
\end{bmatrix} \tag{30}
\]

$W$ is a block diagonal matrix with dimension $2N \times N$ and its elements are as $[0 \ 1]^T$. It can be seen from formula (26):

\[
\Delta E = H\Delta I + N\Delta V \tag{31}
\]
\[
\Delta S = J\Delta I + L\Delta V + W\Delta Y \tag{32}
\]

If $\Delta S = 0$, the $\Delta V = -L^{-1}(J\Delta I + W\Delta Y)$ is obtained and introduced in formula (29), the line current corrective equation is:

\[
\Delta E = (H - NL^{-1}J)\Delta I - NL^{-1}W\Delta Y \tag{33}
\]

where $g_x = H - NL^{-1}J$ is the Jacob matrix for line current analysis; the $g_y = -NL^{-1}W$ with dimensions $2L \times N$.

### 5 Calculation

The Newton and grads method can be used to solve the Kuhn-Tacker conditions expressed by formulas (18)–(22). The grads method is used in this paper due to explain the efficiency of proposed arithmetic. The processes are as:

1. To set $k = 0$ and give the initial value of control variable vector $y^k$;

2. To calculate power flow according to formula (21);

3. To adjust that node voltage is whether or not violated, the penitentiary multiplier $\beta^k$ must be determine if yes; otherwise to continue;

4. To perform coefficient matrixes and calculate Lagrange multiplier $\alpha^k$ according to formula (19);

5. To set $\Delta y^k = g_y \alpha^k$ (because of that $f_y, h_y$ are equal to 0) and determine the corresponding control variable error vector, the iterations are ended if enough smaller; otherwise to correct the control variable:

\[
y^{k+1} = y^k + S^k \Delta y^k \tag{34}
\]

where the $S^k$ is iteration step, which can be determine by one-dimensional searching technology. It is to adjust the control variables is whether or not violated by formula (14), the $\Delta y^k = 0$ if corresponding control variable is violated at node $i$, then to set $k = k + 1$ and turn to step 2);

The following points must be explained in above processes:

1. The node voltage analysis method is still used to calculate power flow in step 2) above, then the line current can be obtained according to formula (12) and the matrixes $H, N, J, L$ also are get. The network states are either obtained by combining the formula (11) with (12), but the scale of problem is
enlarged and works are increased;

2) The penitentiary multiplier $\beta^k$ can be solved by probe method and interior point method, there is not explained in detail;

3) The calculation of $h_x$ must be regarded due to the node voltage is just related in formula (13). While the control variable error is ignored, it can be seen from formula (33) that is as:

$$\Delta V = -L^{-1} J \Delta t$$

viz.:

$$h_x = \frac{\Delta V}{\Delta t} = -L^{-1} J$$

The matrixes $h_x, g_x, g_y$ can be expediently obtained because of the block diagonal form in matrix $L$.

6 Case study
The case study is made at IEEE-30 system by using grads method due to indicate the efficiency of proposed in this paper. The calculating results are listed in table 1 with the node data and table 2 with the line data.

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The 5 iterations are needed to solve the reactive power optimization problem by grads method based on the line current variable, the network losses are listed in table 3. In table 4, the calculating results by different methods are showed. The calculating effects with the line current variable are better than the case with node voltage variable by grads method and approaches to the calculating results by Newton method with node voltage variable. It is explained that efficiency of the proposed method in this paper is high due to the information in $f_x$ appeared in formula (19) is much more.

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7 Conclusion
The objective function is directly expressed as the network losses while the line current is considered as state variable by describing the reactive power optimization problem to optimal power flow form. It is indicated that calculating efficiency of the proposed method in this paper is quoteworthy seen from the calculating results due to the variety in the objective function value is sensitive while the state variable changes.

References: