Performance Analysis of Reed-Solomon Coded Spectral Amplitude Encoding OCDM System

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Abstract: - Performance of Reed-Solomon (RS) coded spectral amplitude encoding (SAE) optical code division multiplexing (OCDM) system is evaluated in the presence of optical beating interference (OBI). It is seen that RS coding can significantly suppress OBI. The number of active users can be doubled. A system with 20×1Gb/s users can be realized at a received effective optical power of -40dBm. Moreover, it is found that Hadamard code and *m*-sequence are preferable to modified quadratic congruence (MQC) code in the RS coded system although MQC outperforms Hadamard code and *m*-sequence in the uncoded system [6].

Key-Words: - SAE/OCDM, fiber Bragg gratings, OBI, Reed-Solomon code

1 Introduction

A vision of ultra high-speed optical access network becomes clearer in the recent years. In the access environment, passive optical network (PON) is the preferred configuration thanks to its flexibility, easy maintenance and low cost. For the physical layer, there are currently two options: time and wavelength division multiplexing (i.e. TDM and WDM, respectively). In TDM-based network, the total throughput is limited by the electronic processing (about 10 Gb/s). Moreover, all users are required to operate at the aggregate speed and in synchronism with each other. Therefore, the per-user speed in TDM-based is limited, and TDM-based system is potentially expensive when the aggregate speed increases. As the demand of ultra high-speed connections (Gb/s per user) is expected to appear in very near future, WDM is being considered as a candidate for the ultra high-speed optical network. However, WDM is still expensive for use in the access environment. As researchers are looking for alternative options, optical code division multiplexing (OCDM) emerges as an attractive candidate. Its attractions include asynchronous access, large capacity, scalability, improved security and passive optical network (PON) compatibility.

In the OCDM network, users are distinguished by unique optical codes. Optical encoding can be done in either time or frequency domain. The time domain encoding is spectral inefficient hence not suitable for ultra high-speed network. The spectral efficient frequency domain encoder can use either spectral amplitude or phase to carry signal. Compared to the phase encoding system, spectral amplitude encoding OCDM (SAE/OCDM) is cheaper since a coherent optical source is not required [1],[2]. With the recent introduction of single-beam en/decoding schemes based on fiber Bragg gratings (FBG) [5], SAE/OCDM becomes even more attractive thanks to its practical size and affordable cost.

As a broadband laser source is used, optical beating interference (OBI) between waves with closed wavelengths is a critical degrading factor in frequency encoding system [3],[4],[6] and [8]. Investigations show that at most 11 ×1Gb/s users can be supported in SAE/OCDM system in the presence of OBI [3],[4], i.e. SAE/OCDM becomes less competitive than WDM in the access network environment where a large user set is preferred.

To combat with OBI, Reed-Solomon (RS) code is proposed for SAE/OCDM system in this paper. In these days, thanks to an availability of commercial

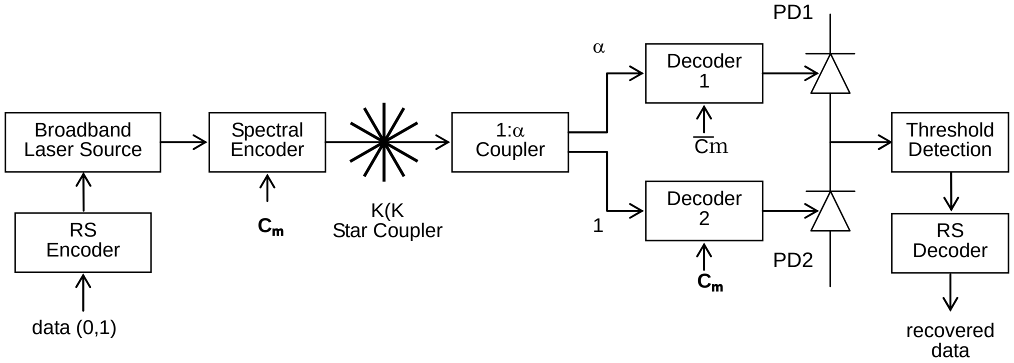

Figure 1: Block diagram of RS coded SAE/OCDM system

hardware for en/decoding up to 10 Gb/s bit rate, a use of channel coding becomes practical in ultra high-speed optical networks. We show that the number of active users can be doubled and receiver's sensitivity can be significantly improved. In our analysis, we use a similar system with that in [1],[6]. Besides a study on effect of RS code, we also present a more generalized analysis approach where both primary and secondary OBIs are taken into consideration.

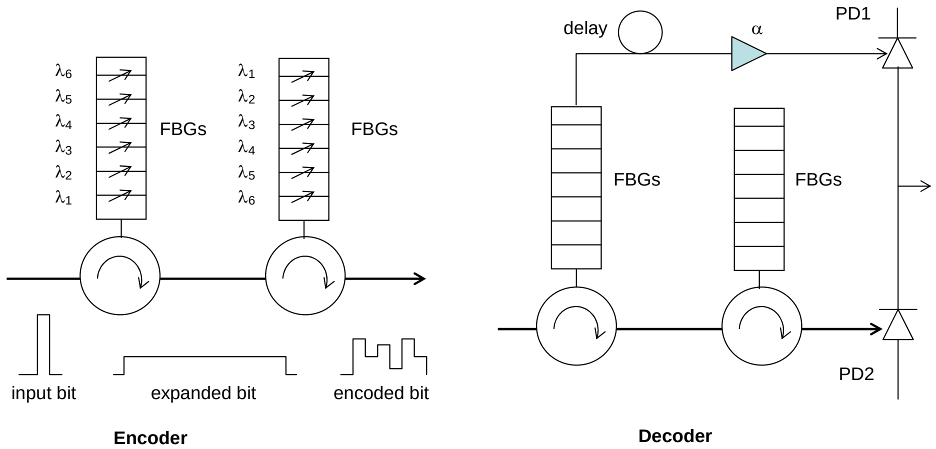

Figure 2: Spectral amplitude encoder/decoder using FBGs

2 System Descriptions

Fig. 1 shows a block diagram of RS coded SAE/OCDM system. Unipolar data (0,1) is first encoded at the RS encoder. RS coded bit is used to modulate a broadband laser source to produce on-off keying (OOK) pulse. The spectral amplitude encoder/decoder can be realized by a pair of FBGs as shown in Fig. 2 [5]. At the encoder, when a pulse is incident on the first FBGs, its wavelength components ($\lambda_1, \lambda_2, ..., \lambda_N$) are dispersed in time and the reflected pulse is temporally expanded. The reflected pattern is determined by a N-chip signature code denoted by a vector $C_m = \{c_{m,1}, c_{m,2}, ..., c_{m,N}\}$ with $c_{m,i} \in \{0,1\}$ for $i \in \{1...N\}$. Wavelength components are re-synchronized and the original pulse is reconstituted in the second FBGs. Signal pulse is then transmitted into a single optical fiber where K active users are connected via a set of $K \times K$ passive star coupler. At the receiver, there are two FBGs that are identical to those of the encoder but connected in reverse. The spectral components (i.e. equivalent to code C_m corresponding to the encoded pulse are reflected in the first FBGs then recombined at the second one resulting in decoded pulse in the lower branch (decoder 2). The complementary spectral components (i.e. equivalent to $C_m = \{c_{m,1}, c_{m,2}, ..., c_{m,N}\}$) at the other output of the first FBGs are delay-compensated and incorporated into decoded pulse in the upper branch (decoder 1). Decoded pulses in both branches are detected by a balanced detection structure. When signal ratio between the first and second branch $\alpha = \lambda / (\omega - \lambda)$, where ω and λ are weight and in-phase cross correlation of the spectral amplitude code, multiple access interference (MAI) can be completely cancelled [1],[2], and [6]. Finally detected signals are decoded by the RS decoder to decide the transmitted data.

3 Uncoded BER Derivation

Several code sets can be used for SAE/OCDM system, including *m*-sequence, Hadamard code and modified quadratic congruence (MQC) code [1],[4] and [6]. Each of code set can be represented by its length, weight and in-phase cross correlation (N, ω, λ). In *m*-sequence, $\omega = (N+1)/2$ and $\lambda = (N+1)/4$; weight and in-phase cross correlation of Hadamard code are $N/2$ and $N/4$, respectively. In MQC code, $\lambda = 1$ and for an odd prime p, we have code length $N=p^2+p$ and weight $\omega=p+1$. Constructions of these codes can be found in [1],[4] and [6]. Letting C_m and C_n be two code vectors, correlation between these two vectors can be expressed as

$$R_{C_m,C_n} = \sum_{i=1}^{N} c_{m,i} c_{n,i} = \begin{cases} \omega & \text{if } m=n \\ \lambda & \text{if } m \neq n \end{cases} \qquad (1)$$

The correlation between C_m and $\overline{C_n}$, $R_{C_m,\overline{C_m}} = \omega - R_{C_m,C_n}$. In the following analysis, it is assumed that average effective optical powers from all users are equal and denoted as P_s. Signals from all users have the same polarization state (worst case) and identical spectral width. Bit synchronism for all users is also assumed. Denote optical frequency and phase noise offsets between k-th and j-th user as $\Delta v_{k,j}$ and $\phi_{k,j}(t)$, the photocurrents on detection (responsivity = R) from PD1 and PD2 can be expressed respectively as [7]

$$\begin{aligned} I_1(t)/R = {} & \underbrace{\alpha P_1 \sum_{i=1}^{N} \overline{c}_{1,i}}_{\text{Data}} + \underbrace{\sum_{k=2}^{K} \alpha P_k \sum_{i=1}^{N} \overline{c}_{1,i}}_{\text{MAI}} \\ & + \underbrace{2\sum_{k=2}^{K} \sqrt{\alpha^2 P_1 P_k \sum_{i=1}^{N} \overline{c}_{1,i}} \cos\left[\Delta v_{1,k} t + \phi_{1,k}(t)\right]}_{\text{Primary OBI}} \\ & + \underbrace{2\sum_{k=j+1}^{K} \sum_{j=2}^{K} \sqrt{\alpha^2 P_k P_j \sum_{i=1}^{N} \overline{c}_{1,i}} \cos\left[\Delta v_{k,j} t + \phi_{k,j}(t)\right]}_{\text{Secondary OBI}}, \end{aligned} \qquad (2)$$

$$\begin{aligned} I_2(t)/R = {} & \underbrace{P_1 \sum_{i=1}^{N} c_{1,i}}_{\text{Data}} + \underbrace{\sum_{k=2}^{K} P_k \sum_{i=1}^{N} c_{1,i}}_{\text{MAI}} \\ & + \underbrace{2\sum_{k=2}^{K} \sqrt{P_1 P_k \sum_{i=1}^{N} c_{1,i}} \cos\left[\Delta v_{1,k} t + \phi_{1,k}(t)\right]}_{\text{Primary OBI}} \\ & + \underbrace{2\sum_{k=j+1}^{K} \sum_{j=2}^{K} \sqrt{P_k P_j \sum_{i=1}^{N} c_{1,i}} \cos\left[\Delta v_{k,j} t + \phi_{k,j}(t)\right]}_{\text{Secondary OBI}}, \end{aligned} \qquad (3)$$

where $P_k = \frac{P_s}{N} d_k \sum_{i=1}^{N} c_{k,i}$ for $k \in \{1...K\}$. From (1)-(3), we can see that MAI can be completely cancelled by the balanced detection structure. The signal-to-noise ratio thus can be determined as

$$SNR = \frac{I_{\text{Data}}^2}{I_{\text{OBI}}^2 + I_{\text{Th}}^2 + I_{\text{Sh}}^2} \qquad (4)$$

where $I_{\text{Data}}^2 = d_1 R \frac{P_s}{N} \omega$, receiver's thermal noise $I_{\text{Th}}^2 = \frac{4k_B TB}{R_L}$, where k_B is Boltzman's constant, T is the temperature, B is bit rate and R_L is the load resistance. I_{OBI}^2 and I_{Sh}^2 are photocurrents caused by OBI, receiver's shot noise, respectively.

Assuming binary symmetric channel (BSC), from (1)-(3), OBI and shot noise can be derived as

$$I_{\text{Sh}}^2 = eBR \frac{P_s}{N} [\omega + 2\lambda(K-1)] \qquad (5)$$

$$\begin{aligned} I_{\text{OBI}}^2 = {} & \frac{1}{2} \frac{B}{B_{opt}} \frac{R^2 P_x^2}{N^2} \omega (K-1)[\omega + (K-2) \times \\ & \times \frac{\lambda^2}{\omega - \lambda} + (K-2)\lambda] \end{aligned} \qquad (5)$$

where B_{opt} is the optical bandwidth used (B_{opt} = 2.5THz is used in this paper). The uncoded BER then can be derived using Gaussian approximation as $P_u = \frac{1}{2} Q\sqrt{SNR/8}$ ($Q(.)$ denotes a Q function) for the system using MQC code and $P_u = \frac{1}{2} Q\sqrt{SNR/2}$ for *m*-sequence or Hadamard code systems (as pulse is sent when data bit is zero in these systems) [9].

4 Coded System Analysis and Discussion

Data bit in RS coded SAE/OCDM system is first coded into (n,k) codewords where n is number of m-bit symbols ($n \leq 2^{m-1}$). These symbols are sent by SAE/OCDM transmitter using OOK pulse sequences. In RS coded SAE/OCDM system, any combination of $t = (n - k)/2$ symbol errors can be corrected. The decoded BER for RS coded system is given by [9]

$$P_{rs} \leq \left(\frac{2^{m-1}}{2^m - 1}\right) \sum_{j=t+1}^{n} \frac{j+t}{n} \binom{n}{j} p_s^j (1-p_s)^{n-j} \qquad (7)$$

where p_s is the probability of symbol error, under an assumption of independent channel errors we have $p_s = 1-(1-P_u)^m$. As RS code reduces channel bit rate with a code rate of k/n, for a fair comparison we increase the bit rate of the coded system (by a ratio of n/k) so that the real bit rate can be the same as that of the uncoded system.

Figures 3 and 4 show the BER of the uncoded and RS coded system versus the effective optical power from each user and the number of active users. Hadamard code (132,66,33) and MQC code (132,12,1) are used (m-sequence has similar performance with that of Hadamard code). For RS code: RS(63,35) and RS(127,63) are used. Fig. 3 shows performance versus P_s of a SAE/OCDM system with 20×1Gb/s users. It is seen that although the error floor of MQC is better than that of Hadamard code in the uncoded system [6], Hadamard code offers better performance with about 10dB gain compared to MQC code when RS code is used. As shown, the system with Hadamard code has BER of 10^{-9} at P_s=-40dBm while P_s = -30dBm in the one with MQC code. As a matter of fact, we find in our analysis that MQC code can outperform Hadamard code when P_s >-26dBm. Nevertheless, that result is not important as BER >> 10^{-9} is not practically necessary.

In Fig. 4, we further analyze BER versus the number of active users when each user bit rate is 1Gb/s and P_s=-30dBm. It is seen that the RS coded system can double the number of active users (22 to 25 users compared to about 10 users in the uncoded system). At this relatively high power level, MQC code can offer better performance than that of Hadamard code as the number of active users increase, however the difference is not major. Hadamard code (or m-sequence) is thus preferable in RS coded SAE/OCDM system. Although more FBGs may be required due to Hadamard's (or m-sequence) higher code weight, 10dB power gain is substantial, especially in access environment where high coupler loss is unavoidable.

5 Conclusion

We have theoretically evaluated the performance of RS coded SAE/OCDM system using FBGs. A generalized analysis method that considers both primary and secondary OBI in SAE/OCDM system is used. The result shows that RS code can significantly improve the performance of SAE/OCDM system. The number of active users can be doubled. The required optical power is as low as -40dBm for the system with 20×1Gbp/s users. In addition, we find that RS-coded SAE/OCDM system that uses Hadamard code (or m-sequence) can offer 10dB gain compared to the one with MQC code in the operation range (i.e. BER=10^{-9}).

As WDM is PON incompatible and still expensive, our study shows that SAE/OCDM can be a considerable candidate for the next generation ultra high-speed access network (after the present TDM-PON). However, we are aware that additional OBI countermeasures should be considered in order to accommodate more users and increase the bit rate in the SAE/OCDM system.

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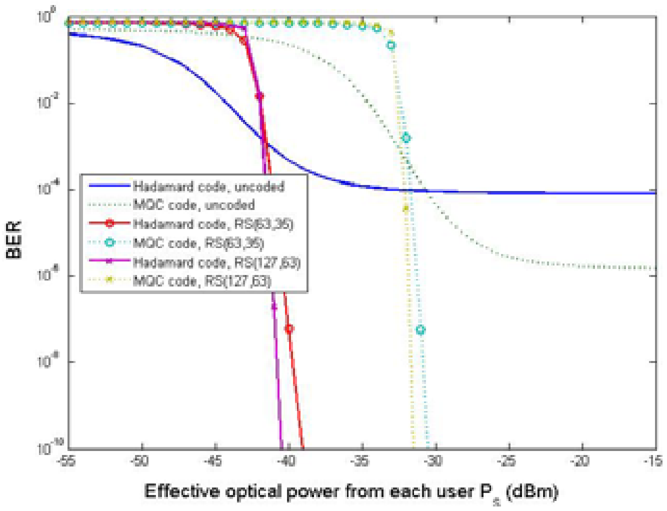

Figure 3: BER versus the effective power from each user. Number of users 20×1Gb/s

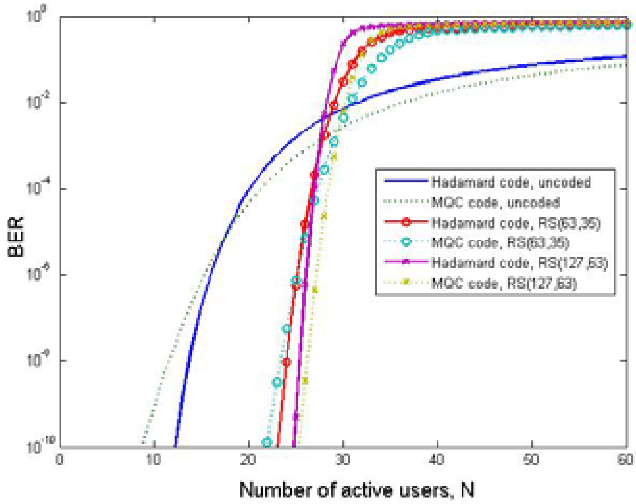

Figure 4: BER versus the number of users. Effective power from each user: -30dBm, user speed: 1Gb/s.