Long-Term Cumulative Distribution Modeling of Tropospheric Scintillation for the Earth-Satellite Links in the 40/50 GHz band

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Abstract: In this paper, we have obtained a model for the prediction of scintillation fade depth, scintillation intensity, and cumulative distribution of its intensity, in the 40/50 GHz band. By applying a regression analysis to the experimental data collected at Madrid, a new model is proposed. Moreover, the prediction accuracy of the our new model is evaluated using the experimental results from Spino d’Adda, Italy.

Keywords: earth-satellite link, radiowave propagation, tropospheric scintillation, fade, cumulative distribution.

1 Introduction

In order to satisfy the growing demand for the long distance communications, during the last decades, the exploitation of satellites for communication purposes has increased considerably. As the C-band (4/6 GHz) is already congested, and the Ku-band (12/14 GHz) is filling up rapidly, recently, interest focused on the utilization of higher bands (e.g. Ka and V bands) [1].

The performance of the satellite systems operating in the Ka and V bands essentially depends on the propagation characteristics of the transmission medium, i.e. troposphere. Some of the most important tropospheric propagation effects are attenuation due to rain, depolarization, gas absorption and scintillation [2,3].

In general, the impact of rain on communication systems is predominant. Scintillation, however, becomes important for low-margin systems that operating at high frequencies and low elevation angles. It has been observed that, at high frequencies and for low elevation angles, scintillation may contribute as much as rain, or more, to the total fade measured. This is especially true for low margin systems [4]. Therefore, it is necessary to model the scintillation phenomena for earth-satellite links.

This paper has been organized as follows; first, a review of tropospheric scintillation theory is presented, then, current models already proposed for scintillation fade, scintillation enhancement and its intensity, are introduced. Finally, according to the experimental data, and applying a regression method, the models are extracted and then compared to elsewhere data and current existing models.

2 Theory

The tropospheric turbulence yields time-varying modifications of the refractive index and thus affect propagation of radiowaves on earth-space paths by generating random amplitude, phase and angle of arrival fluctuations, called scintillation [4,5]. It is generally assumed that the fluctuations of signal level due to scintillation for short-term periods (i.e. up to several minutes) is stationary and follows a guassian distribution [6-8]. However, for longer time periods the variance of fluctuation is not more constant and follows its own pdf. Two distribution for long-term standard deviation or variance of scintillation is propsed. The first, is
gamma distribution and is appropriate for moderate scintillation intensities [9,10]. The second, is log-normal distribution and is proper for vigorous scintillation intensities [7,11]. Most of the models try to relate the parameters of these distributions to the link and meteorological parameters (e.g. beacon frequency, elevation angle, humidity, temperature).

In order to calculate, theoretically, the signal fadings (negative fluctuations around the mean signal level) and enhancements (positive fluctuations around the mean signal level) due to scintillation, we must use the following conditional integral equation [7]:

$$
\int_{-\infty}^{\infty} \int_{\chi_{\sigma}}^{\infty} p(\chi|\sigma) p(\sigma) d\sigma d\chi
$$

where $\chi_{\sigma}$ is a threshold value and $p(\chi|\sigma)$ is the conditional pdf of $\chi$ for given $\sigma$. Different models that already proposed for scintillation fade and enhancement, derived from (1) and experimental results.

3 Current Models

In this section, the current models proposed for scintillation intensity, fade and enhancement are presented.

3.1 Karasawa model

Based on the experimental results at Yamaguchi, Japan, at an elevation angle of 6.5°, frequencies of 11.5 and 14.2 GHz, during the year 1983, Karasawa presented a prediction model for signal standard deviation due to scintillation (scintillation intensity) as following [9]:

$$
\sigma_{pre} = 0.0228 \sigma_n f^{0.45} g(D_e) / \sin^{1.3} \theta 
$$

where $\sigma_n = 0.15 + 5.2 \times 10^{-3} N_{wet}$ (3)

and $N_{wet}$ is the wet term of ground refractivity index, $f$ is the beacon frequency (GHz), $\theta$ is the elevation angle of slant path, and $g(D_e)$ is given by:

$$
g(D_e) = \begin{cases} 
1.0 - 0.7 \left( \frac{D_e}{\sqrt{\lambda z}} \right) & 0 \leq \frac{D_e}{\sqrt{\lambda z}} \leq 1.0 \\
0.5 - 0.2 \left( \frac{D_e}{\sqrt{\lambda z}} \right) & 1.0 < \frac{D_e}{\sqrt{\lambda z}} \leq 2.0 \\
0.1 & 2.0 < \frac{D_e}{\sqrt{\lambda z}} 
\end{cases} 
$$

where $\lambda$ is the wavelength in m, $D_e$ is the effective antenna diameter ($D_e = D \sqrt{\eta}$), and $\eta$ is radiation efficiency of antenna), and $z$ is the distance of the turbulent part of the path, given by:

$$
z = \frac{2h}{\sin^2 \theta + 2h/a_e + \sin \theta} 
$$

According to (1), and assuming a gaussian distribution for short-term fluctuations of signal level and a gamma distribution for scintillation intensity, Karasawa obtained the following expression for the scintillation enhancement:

$$
y = \sigma_{pre} (-0.06 \log^3 p - 0.08 \log^2 p - 1.25 \log p + 2.67) 
$$

(0.01% ≤ $p$ ≤ 50%)

As the observed scintillation fade is larger than its enhancement, and guassian distribution results in symmetrical expressions for fade and enhancement, Karasawa fit a curve to the experimental data and proposed following expression for scintillation fade:

$$
y = (-0.06 \log^3 p + 0.07 \log^2 p - 1.7 \log p + 3)\sigma_{pre} 
$$

(0.01% ≤ $p$ ≤ 50%)

Karasawa model was tested in several sites and found to be valid for 7 to 14 GHz frequency range and elevation angles from 4° to 30°.

3.2 ITU-R model

The International Telecommunication Union-Radiocommunication sector, ITU-R, proposed a model that is mostly derived from the Karasawa model. They present the following model for prediction of scintillation intensity [12]:

$$
\sigma = (3.6 \times 10^{-3} + 1.03 \times 10^{-4} N_{wet}) f^{7/12} g(D_e) / \sin^{1.2} \theta 
$$

and $g(D_e)$ is given by [15]:

$$
g(D_e) = \sqrt{3.8637(x^2 + 1)^{11/12} \sin \left( \frac{11}{6} \arctan \frac{1}{x} \right) - 7.0835x^{5/6}} 
$$

$\approx \sqrt{1 - 7.0835x^{5/6}}$ for $x << 1$

and $x = 0.0584kD_e^2 / z$. 

where $x$ is the wavelength in m, $k$ is the effective antenna diameter ($k = D \sqrt{\eta}$), and $\eta$ is radiation efficiency of antenna, and $z$ is the distance of the turbulent part of the path, given by:

$$
z = \frac{2h}{\sin^2 \theta + 2h/a_e + \sin \theta} 
$$

According to (1), and assuming a gaussian distribution for short-term fluctuations of signal level and a gamma distribution for scintillation intensity, Karasawa obtained the following expression for the scintillation enhancement:

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(0.01% ≤ $p$ ≤ 50%)

Karasawa model was tested in several sites and found to be valid for 7 to 14 GHz frequency range and elevation angles from 4° to 30°.
ITU-R apply the (7) for scintillation fade and did not propose any model for scintillation enhancement. This model is applicable for frequencies between 7- to 20-GHz and elevation angles from 4° to 32°.

### 3.3 Ortgies models

By analyzing the experimental data derived from Olympus satellite measurements at Darmstadt, Germany, using the beacons at 12.5, 20, and 30 GHz, Ortgies presented two model for prediction of scintillation distribution parameters. He considered a log-normal pdf for long-term distribution of scintillation intensity with parameters of $\mu$ and $s$ which are mean and standard deviation of $\ln \sigma_x^2$, respectively. Ortgies found that the parameter $s$ is independent of link and meteorological parameters and is approximately 1.01. However, he presented two model for prediction of $\mu$. The first, links $\mu$ to wet term of refractivity index, $N_{\text{wet}}$, and called Ortgies-N model and given by [13]:

$$\mu = \ln(g(D_e)\cdot \sin(\theta)^{-2.4}\cdot f^{-1.21}) - 13.45 + 0.0462\cdot N_{\text{wet}}$$

(10)

While, the second, relate $\mu$ to $T(\text{°C})$ and called Ortgies-T model and is as following [14]:

$$\mu = \ln(g(D_e)\cdot \sin(\theta)^{-2.4}\cdot f^{-1.21}) - 12.5 + 0.0865\cdot T$$

(11)

Ortgies assumed a normal pdf for the short-term distribution of signal level fluctuations and a log-normal pdf for the long-term distribution of scintillation intensity, and applied the conditional integral (1) for calculating the cumulative distribution of signal amplitude. At last, the following simplified integral equation obtained for the long-term cdf of scintillation amplitude (by using the ERFC(.) function definition):

$$p(X \geq X_0) = \frac{1}{\sqrt{2\pi}\cdot s \cdot \sigma_x} \exp\left(\frac{(\ln \sigma_x^2 - \mu)^2}{2s^2}\right) d\sigma_x$$

(12)

According to experimental results from different sites, Ortgies models found to be reliable for elevation angles from 6.5° to 30° and frequencies of 8 to 20 GHz.

### 3.4 Otung model

Based on the scintillation measurements obtained at Sparsholts, U.K, over a one-year period using the Olympus satellite 19.77 GHz beacon, Otung proposed a model for scintillation variables (e.g. intensity, fade and enhancement) [10]. The Otung model for scintillation is very similar to ITU-R model and only the elevation dependence term is $\sin(\theta)^{-11/12}$ instead of $\sin(\theta)^{-12}$.

For scintillation fade and enhancement, Otung fit curves to the experimental data and found the following relation for them. The scintillation fading is given by:

$$X_{-a} = 3.6\sigma_{\text{pre}} \exp\{-9.5 \times 10^{-4} - [0.4 + 0.002p]\cdot \ln(p)\}$$

(13)

and the signal enhancement is as following:

$$X_{+a} = 3.17\sigma_{\text{pre}} \exp\{-0.0359 p - [0.272 - 0.004 f]\cdot \ln(p)\}$$

(14)

Otung model is valid for 20/30 GHz frequency range and elevation angles higher than 10°.

### 4 Analysis and Results

#### 4.1 Scintillation intensity prediction model

By frequency dependence analysis for 40/50 GHz band, according to the experimental results at Spino d’Adda, Italy, using the Italsat beacons at 39.6 and 49.5 GHz, Van de Kamp found the exponent of frequency scaling factor of scintillation intensity equal to 0.86 [16]. For elevation angle and meteorological dependency, we utilize the theoretical term of $\sin(\theta)^{-11/12}$. Therfore, the model that we apply to predict the scintillation intensity is:

$$\sigma_x = (5.71 \times 10^{-4} + 4.98 \times 10^{-5}) \cdot f^{-0.86} \cdot \sin(\theta)^{-11/12} \cdot g(D_e)$$

(15)

Now, we use the experimental results obtained at Madrid, Spain, over a one-year period, using the Italsat satellite beacon at 49.5 GHz, with an elevation angle of 40°. The diameter of receiving antenna is 1.2 m.

#### 4.2 CDF of the scintillation fluctuations

We have used the expression $\sigma_{\text{pre}}, \eta(p)$ for the formulation of curve fitting in which $\sigma_{\text{pre}}$ is the
predicted scintillation intensity and $\eta(p)$ is a cubic polynomial in $\log(p)$.

Figure 1 shows the experimental cumulative data of scintillation fade and enhancement at 49.5 GHz and their least-square fit curves.

So, the expression for signal fading is:

$$X_{-a} = \sigma_{pre} (-0.051 \log(p)^3 + 0.29 \log(p)^2 - 1.78 \log(p) + 2.67)$$

(16)

And the scintillation enhancement is:

$$X_{+a} = \sigma_{pre} (0.04 \log(p)^3 + 0.3 \log(p)^2 - 1.8 \log(p) + 2.6)$$

(17)

For the cdf of scintillation fade, R-square statistics and rms error are 0.996 and 0.009095, respectively. For the cdf of scintillation enhancement, those are, respectively, 0.994 and 0.01054.

4.3 Annual CDF of the scintillation intensity

Figure 2 shows the data of cumulative distribution of scintillation standard deviation at 49.5 GHz and the best fitting curve.

So, the relation for CDF of scintillation intensity becomes as following:

$$\sigma_a = \sigma_{pre} (0.006 \log(p)^3 + 0.21 \log(p)^2 - 1.2 \log(p) + 2.22)$$

(18)

In the case of annual cdf of scintillation intensity, those quality parameters are respectively, 0.9977 and 0.01342.

4.4 Worst month CDF of the scintillation intensity

Figure 3 shows the experimental data of the cumulative probability of worth month scintillation intensity and the least-square fit curve.

The worst month distribution is the synthetic peak envelope of monthly cumulative distribution obtained by selecting at each time percentage the maximum value of the scintillation intensity exceeded in 12 month. Therefore, The expression for worst month cdf is obtained as:
\[
\sigma_{wm} = \sigma_{pre} \left( 0.05 \log(p)^3 + 0.27 \log(p)^2 - 1.9 \log(p) + 3.5 \right)
\] 

For worst month cdf of scintillation intensity, the R-square statistics and rms error are 0.9933 and 0.01476, respectively. Thus, for all the obtained expressions for cumulative distribution of scintillation variables, R-square statistics is approximately 0.999 and the rms error is lower than 0.015, and this shows the accuracy of the fitting process.

5 Comparision

In this section, we will compare our proposed model for scintillation fade depth with current models and available elsewhere experimental data obtained at Spini d’Adda, Italy, using the beacon of 39.6 GHz. The elevation angle of the slant path is 37.8°, and the receiving antenna has a diameter of 3.5 meters and 0.64 radiation efficiency. The comparison between experimental fade data at 40 GHz, proposed model and current models are shown in Figure 4.

As it is clearly seen, for time percentages lower than 1%, the models of Karasawa and ITU-R underestimate the measurement results, while another models overestimate the data. The models of Ortgie-T and Ortgies-N have a better performance in this range of time percentages. In order to have a more obvious observation, we consider a window of Figure 4 in the range of time percentages between 1% to 50% and fades from 0-dB to 0.2-dB, which is shown in Figure 5.

As it is clearly seen, all of the current models underestimate the measured fade in the practical range of higher than 1%, while the new proposed model is in good agreement with data. Also, according to Figure 5 we can say that, among the current existing models, the Ortgies-T model predicts the scintillation fade better than others and ITU-R and Otung model have a similar behaviour.

6 Conclusion

New models for the long-term cumulative distribution of signal fluctuations due to scintillation in the V-band, are presented. We obtained models for annual cdf of scintillation fading, enhancements, and intensity. Meanwhile, worst month cdf of scintillation intensity is also obtained.

For extracting these models, we used the experimental data gathered at Madrid station, using the beacon 49.5 GHz, with elevation angle of 40° and the antenna of 1.2 meters diameter, and then, applied a regression analysis to these data.

To show the accuracy of the proposed model, we compared our model with the measurements results at 40 GHz and observed its good performance in comparison with other current models. For the time percentage higher than 1%, all the current models underestimated the experimental fade, and among them, only the ortgies-T model found to be better than others. Therefore, at higher frequencies, it is suggested to utilize the new
proposed model for the prediction of scintillation fade in designing the earth-satellite links.

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References


