A New Modeling of Inductive Sensors for Current Measuring at High Voltage

HASHEM ORAEE MIRZAMANI, AZIM LOTFJOU CHOOBARI, Mohammad Hadi Varahram
Department of Electrical Engineering
Sharif University of Technology
Sharif University, Tehran
IRAN

Abstract: - Inductive sensors are used in electric equipment like current measuring insulators and metal detectors. Modeling of these sensors will raise their accuracy and consequently will lead to developing of high precision current measuring devices. The idea of expressing the voltage induced in the sensor in terms of current passing through the line and geometrical parameters is the goal of this study. Inductive sensors with rectangular cross section have been studied and the results show good conformity with previous works Changes in dimensions like varying the diameter of current carrying cable will cause big error in current sensing if the modeling is not considered.

Key-Words: - Inductive Sensor, Modeling, Current Measuring

1 Introduction

Inductive sensors have been used for many years in electric networks [1, 2 and 3] . The size of the coil and its distance from the center of the conductor (in this paper, the conductor is the high voltage current carrying line) determine the accuracy of the sensor [4].

These sensors are designed for a predetermined distance from the centre of the conductor, but different cable sizes reduce their accuracy. The most remarkable point at using the inductive sensors is their simple design and implementation.

2 Modeling of the coil

At this work we assume rectangular shape for the cross section of the coil and we don't converse about the other available figures. At fig.1 the overal view of the coil in vicinity of a conductor is depicted.

For modeling the coil all dimensions of should be known. By this we can determine the voltage in term of current passing through the conductor.

The below allocations should be considered according to fig.1 when solving calculating the voltage of the coil.

D#	Diameter of the conductor#
I#	Effective current passing through the conductor#
d#	Diameter of the wire wounded round the coil#
N#	Total number of the turns round the coil#
N1#	Total number of the turns round the coil at one layer#
N2#	Total number of layers#
Ri#	The half of internal height of the coil (hi)#
Ro#	The half of external height of the coil (hi)#
A#	Nearest distance between the coil and the conductor#

According to fig.2:

$$R_o - R_i = n_2 d \tag{1}$$

$$n_1 = \frac{W}{d} \text{ or } W = n_1 d \tag{2}$$

Indeed,

$$n_1 \times n_2 \cong n \tag{3}$$

and

$$\Rightarrow \frac{R_o - R_i}{d} . \frac{W}{d} = n \Rightarrow (R_o - R_i).W = n.d^2 \tag{4}$$

Then,

$$\Rightarrow R_0 = \frac{n.d^2}{W} + R_i \tag{5}$$

Current Carrying Conductor

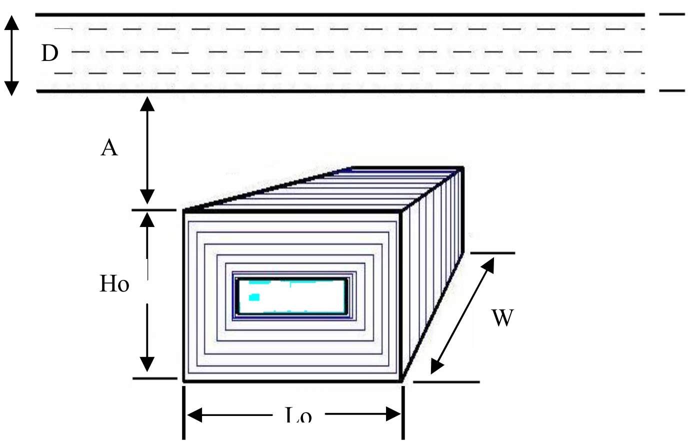

Fig.1 perspective view of an inductive coil near an overhead line

So, by considering that the total turns of the coil to be known, the outer height of the coil can be calculated.

Furthermore, the outer width of the coil can be written in term of known parameters:

$$l_0 = l_i + 2n_2 d \tag{6}$$

3 Determination of the voltage in the coil

There are three different assumptions for solving the problem and for calculating the voltage of the coil in term of known parameters.

These assumptions come from the fact that the coil dimensions are comparable with other dimensions like the distance between the coil and conductor or not.

3-1 First assumption

Here, one can assume that the coil dimensions are too small in comparing with distance between the coil and conductor. According to this, it can be inferred that the flux density at all turns are equal.

Flux density at the center of the coil is:

$$B = \frac{\mu_0 i(t)}{2\pi r} \tag{7}$$

In which,

# μ_0	Permeability of vacuum air (4π×10-7)
# B	Flux dnsity at the center of the coil
#	Distance between the center of the conductor and coil

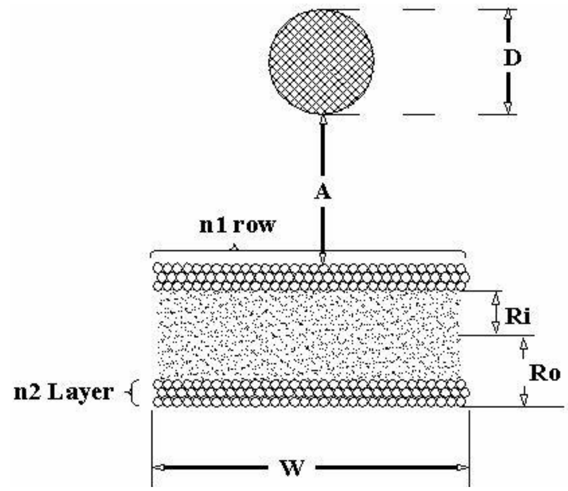

Fig.2 side view of the current sensor

"r" at relation (7) is calculated as below:

$$r = \frac{D}{2} + A + n_2 d + R_i \tag{8}$$

The coil is assumed to be very small. Then we can write:

$$R_i, n_2 d \langle\langle A \tag{9}$$

And consequently

$$r = \frac{D}{2} + A \tag{10}$$

Concluding from equations (7, 8, 9):

$$B = \frac{\mu_0 i(t)}{2\pi\left(\frac{D}{2} + A\right)} \tag{11}$$

In addition, the effective induced voltage is:

$$V(rms) = n.B'.CSA \tag{12}$$

In that

# CSA	Cross sectional area of the coil
# B'	Effective derivative of flux density passing through the coil

In that the CSA is equal to:

$$CSA = 2R_i \times l_i = h_i \times l_i \tag{13}$$

So, the induced voltage can be written as below:

$$V(rms) = n.h_i \times l_i . \frac{\mu_0}{2\pi\left(\frac{D}{2} + A\right)} . \widehat{I} \quad (14)$$

And about the current of the conductor:

$$\widehat{I} = \omega \times I = 2\pi f \times I \quad (15)$$

In recent equation, f is the frequency of current.

Using equations 13, 14, 15 the voltage of the coil can be rewritten as below:

$$V(rms) = \left(\frac{\mu_0 .n.h_i .l_i . f}{\left(\frac{D}{2} + A\right)} \right) .I \quad (16)$$

From relation (16) it will be understood that:

$$I \alpha V(rms)\left(\frac{D}{2} + A\right) \quad (17)$$

The recent equation has been considered as a general relation between the voltage induced in the coil and the current passing through the conductor [3].

3-2 Assumption two

Here assume that the length of the coil is small enough, but the height of the coil window is comparable with diameter of the conductor and distance between the conductor and coil. By this assumption the flux density is not constant and its value changes in the entire window of the coil.

$$B(h) = \frac{\mu_0 i(t)}{2\pi h} \quad (18)$$

In that "h" is the distance between the centers of the conductor and a point inside the coil.

The total flux passing through one turn is:

$$\Rightarrow \phi = \int B(h).ds = \int_{h_1}^{h_2} B(h).l_i .dh = \frac{\mu_0 i(t).l_i}{2\pi} \ln\left(\frac{h_2}{h_1}\right) \quad (19)$$

In relation (19) h1, h2 are the nearest and farthest distance of the coil to the conductor which are determined as below:

$$h_1 = \frac{D}{2} + A \quad (20)$$

And

$$h_2 = h_1 + 2R_i = \frac{D}{2} + A + h_i \quad (21)$$

Using relations from 18 to 21 the voltage of the coil can be written as below:

$$V(rms) = (\mu_0 .n.l_i . f .I)\ln\left(\frac{\frac{D}{2} + A + h_i}{\frac{D}{2} + A}\right) \quad (22)$$

The recent equation for the voltage induced in the coil has been used by some authors [4] for current measuring at high voltage over head lines.

3-3 Assumption three

At this assumption there will be no approximation and all calculations are done as they appear at the problem. At this condition the effect of coil length and its thickness are involved when solving the problem. As it is obvious, the cross sectional area of all turns are not equal. In addition the electromagnetic lines at two sides of the coil are not perpendicular to the surface of the coil. These lines make an angle between the 0˚ to 90˚ by the abscissa of the coil.

Though in this assumption we eliminate any approximation, some errors take place at the corners of the coil window where it has been rounded. For better understanding of this problem see the fig.3 in that shaded area depict the indispensable error in measuring of current.

This kind of errors can be considered by experimental results and making a look up table between current and voltage induced in the coil.

Calculating of the voltage induced in one turn

Calculating of the voltage induced in one turn can be done by looking to fig.4.

According to fig.4, the flux density at a turn located at distance ax far from the center of the coil is:

$$B(a_x, h_x) = \frac{\mu_0 I}{2\pi R_x}.\cos(\alpha_x) \quad (23)$$

$$\cos(\alpha_x) = \frac{h_x}{R_x} = \frac{h_x}{\sqrt{(h_x^{\ 2} + a_x^{\ 2})}} \quad (24)$$

So, for the voltage of the turn we can write:

$$dV_t(a_x) = \frac{d\phi_t(a_x)}{dt} \quad (25)$$

From relations 24 and 25 it will be understood that:

$$\phi_t(a_x) = \int_{h_1}^{h_2} B(a_x, h_x).ds \quad (26)$$

Fractional amount of cross sectional area is equal to:

$$ds = l.dh \quad (27)$$

“l” in this relation is the width of the turn.

Using relations (26, 27) the total flux passing through the turn will be determined:

$$\phi_t(a_x) = \frac{\mu_0.l.i(t)}{4\pi}.\ln\left(\frac{a_x^{\ 2} + h_2^{\ 2}}{a_x^{\ 2} + h_1^{\ 2}}\right) \quad (28)$$

The relation (28) depicts the total flux passing through the turn. h1 and h2 are the nearest and the farthest perpendicular distance of the turn from the conductor.

Using relations (25 and 28) the voltage induced in the specified turn can be obtained.

$$V_t(a_x) = \frac{\mu_0.l.f.I}{2}.\ln\left(\frac{a_x^{\ 2} + h_2^{\ 2}}{a_x^{\ 2} + h_1^{\ 2}}\right) \quad (29)$$

Calculating of the voltage induced in a layer

Layer: a serious of turns with equal cross sectional area.

If we integrate the relation (29) across the length of the coil, the total voltage induced at one layer can be found (see fig. 2). This work can be accomplished by multiplying the density of turns across the length to the voltage induced in one turn (relation (29)) and integrating of the result.

The density of turns across the length of the coil is:

$$n_p = \frac{n_1}{W} \quad (30)$$

When substituting N1 from relation (2) in the relation (3) the below conclusion is found:

$$n_p = \frac{1}{d} \quad (31)$$

The small amount of voltage for one layer according to relation (29) is:

$$dV(a_x, h_1, h_2)_l = n_p V_t(a_x).da_x \quad (32)$$
$$= \frac{\mu_0.n_p.l.f.I}{2}.\ln\left(\frac{a_x^{\ 2} + h_2^{\ 2}}{a_x^{\ 2} + h_1^{\ 2}}\right).da_x$$

By integrating of above relation across the coil length (for interval of 0 to w) the induced voltage at one layer can be found:

$$V_l = \frac{\mu_0.n_p.l.f.I}{2}.\left(W.\log\left(\frac{W^2 + 4h_2^{\ 2}}{W^2 + 4h_1^{\ 2}}\right) + k'\right) \quad (33)$$

And

$$k' = \left(4h_2.tg^{-1}\left(\frac{W}{2h_2}\right) - 4h_1.tg^{-1}\left(\frac{W}{2h_1}\right)\right) \quad (34)$$

in which h1 and h2 are defined like relaion (28) for one layer.

Calculating of total voltage induced in the coil

Currently if l, h1 and h2 are defined as variables of the problem, the total voltage induced in the coil can be found.

$$h_1 = \frac{D}{2} + A + n_2.d - x \quad (35)$$

,

$$h_2 = h_1 + 2R_i + 2x = \frac{D}{2} + A + n_2.d + h_i + x \quad (36)$$

And

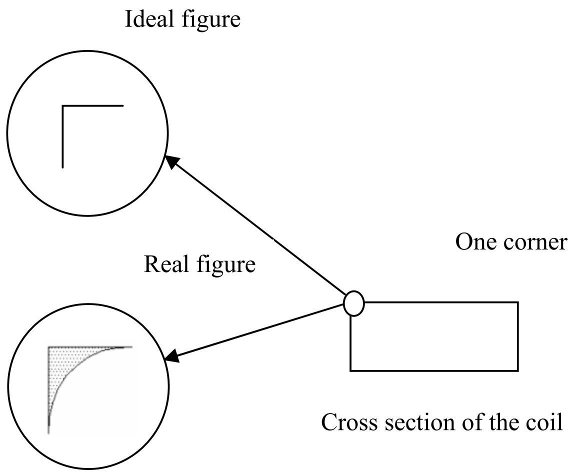

Fig.3 error in current measurement

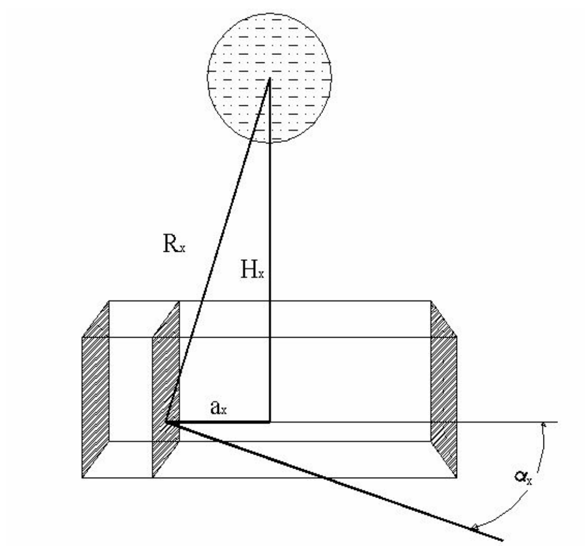

Fig.4 three dimensional view of the coil

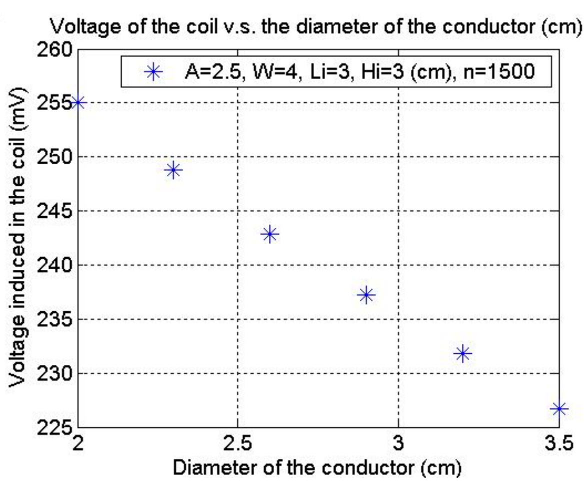

Fig.5 Coil voltage vs. changes in the conductor diameter

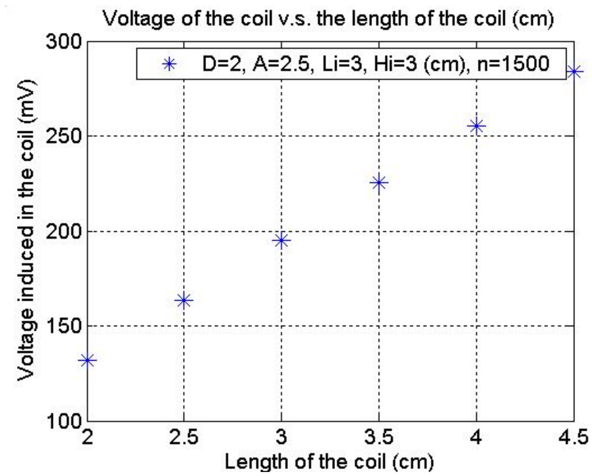

Fig.6 Coil voltage vs. changes of the coil length

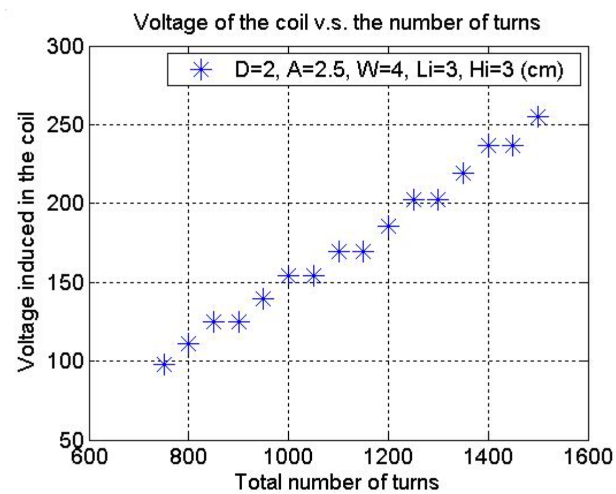

Fig.7 Coil voltage vs. changes in the number of turns

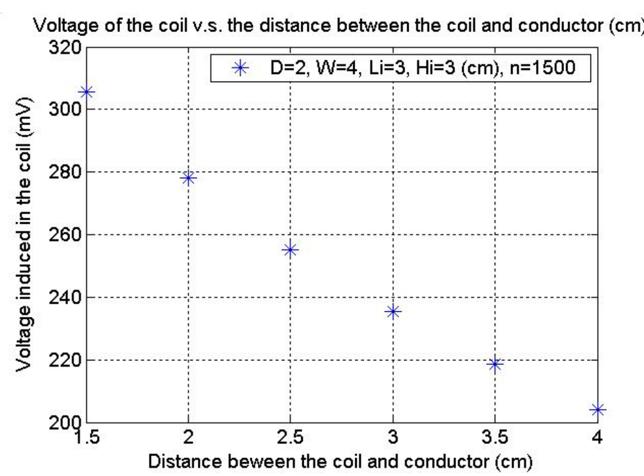

Fig.8 Coil voltage vs. changes in "A"

$$l = l_{in} + 2x \quad (37)$$

At relation (35), (36) and (37) the variable x is the distance from internal section of the coil to the expected layer.

Using relations (34 to 37), for small amount of voltage it can be written:

$$\partial V_l(rms) = n_p.V_l(x).dx \quad (38)$$

As, it was mentioned “x” change from 0 to n2×d.

$$\Rightarrow V(rms) = \int \partial V_l(rms) = \int_0^{n_2.d} n_p.V_l(x).dx \quad (39)$$

Evaluating of the above relation make the exact value of voltage induced in the coil known.

Here, some different solutions of the above relation have been presented (figs.5 to 8). These curves show that how different parameters affect the coil voltage and consequently cause error at measuring current. For instance, many authors [4] ignored the effect of changes of the conductor diameter on the induced voltage, but from fig.5 it would be understand that using the device for different network with different cable sizes can lead to an error 10%.

Indeed, most of the time the coil length is considered to be small enough in comparing with other dimensions, but here we noticed that (fig.6) the changes in coil length can double the voltage induced in the coil.

As it was predictable, the voltage induced in the coil is related directly to the number of the turns (fig.7).

Another good result of this work is the decreasing of the coil voltage linearly with increasing of the distance between the coil and conductor (fig.8).

Though changes in other parameters like coil internal width affect the output voltage of the coil, study of the effect of these parameters can be done similarity with the above work.

3. Conclusion

Recently, inductive coils are used widely at high voltage lines for current measuring purposes.
In this paper we modeled inductive sensors for current measuring purposes. Modeling of these sensors will lead to error reduction of these sensors and consequently will improve the operation of measuring and controlling devices using it. The main goal of this paper was to express the voltage induced in the inductive coil in term of known parameters.

4. Acknowledgements

Here, we acknowledge the Niroo Research Institute (NRI) Co. in IRAN for its corporation and funding of the project.

References:

[1] T. Marks, “Power Line Post Insulator with Dual Inductor Current Sensors’, US Patent 5,124,642 , Jun. 1992.

[2] T. Beling, “Power Distribution System and Means for Control Thereof”, US Patent 4,700,123, Oct. 1987.

[3] P. Falkowski et al., “Line Sensor with Corona Shield”, US Patent 5,252,913, Oct. 1993.

[4] E. Nishiyama, et. al., “Development of Portable Equipment for Monitoring Currents and Voltage of Power Transmission Lines”, Kummamoto National University of Tech., Japan, 2002