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A Simplified Analytical Approach for Efficiency Evaluation of the Weaving Machines with Filling Break Tolerance

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Abstract: – A machine interference problem is treated in this paper. For a group of weaving machines with filling break tolerance, served by one or more weavers, two indicators must be evaluated: the efficiency of the weaving machines and the percentage of working time for the weavers. Two different approaches are available: analytical approach, based on Markov chains, and simulation. A reduced Markov model able to evaluate with accuracy the indicators previously defined is proposed in this paper. A case study in which analytical and simulation results are compared demonstrates the effectiveness of this simplified analytical approach.

Key–Words: Weaving process, Machine interference problem, Markov chains, Stochastic Petri nets.

1 Introduction

The weaving process is a discrete event one because the warp yarns and the filling yarn break off at random instants. In case a weaving machine (loom) is down, because a yarn has broken, a weaver (loom operator) must remedy the broken yarn and than start up the loom again. We can say that a loom is a system with repair. The problem of allocation of looms to the weavers is a very important one in a large weaving mill. Two conflicting aspects must be taken into account: the loom efficiency (losses caused by interference) and the percentage of working time (work loading) for the weavers. The prediction of the loom efficiency, and the weaver work loading, when a group of weaving machines are allocated to one ore more weavers is a machine interference problem. The problem of allocation of looms in weaving is widely dealt with in textile literature, both from a theoretical and a practical point of view (see for example [1]). In this work we focus on the machine interference problem for the looms with filling break tolerance.

The analytical approach of machine interference problem is based on the queueing theory. The standard model is a Markov chain. If a Markov chain has s states and only steady–state probabilities are required, s linear equations must be solved. The method to be applied is simple in essence, but we must have in view the complexity of Markov models [2]. For any sizable practical problem s becomes very large and the solution time becomes very long, so that, the classical approach is difficult to apply.

When the Markov chain is very large, the two approaches available to deal with this problem are to either tolerate the largeness or avoid it. In this work, a largeness avoidance technique for an approximate evaluation of the two indicators previously defined is proposed.

The simulation is a complementary approach for machine interference problem [3]. A simulation program based on a stochastic coloured Petri net has been used in order to validate the analytical results.

The remainder of this paper is organized as follows. In Section 2 the interference problem concerning the weaving process is defined in details. Section 3 presents an example to show that a clasical Markov model is difficult to apply. In Section 4 a reduced Markov model able to evaluate with accuracy the efficiency of the weaving machines is proposed. Section 5 presents a case study in which analitical and simulation results are compared.

Finally, some conclusions are drawn regarding this work.

2 Problem Formulation

Consider m identical looms carrying out a weaving process completely known from a statistical point of view, and r weavers working together to serve them (usually, r=1). The looms work with two packages for each filling colour (an active package and a spare one) for a higher efficiency. Thus, in case the filling yarn between active package and prewinder breaks off, an automatic switch selects the spare package and avoids a stop of the loom. We say that the spare packages ensure a filling break tolerance for the weaving process.

For a stochastical modelling of a weaving process, six primary random variables have been considered:

- Time to break off a warp yarn – let λ_W be the warp breakage rate;
- Time to break off the filling yarn into the shed – let λ_F be the breakage rate into the shed;
- Time to break off the filling yarn between active package and prewinder – let λ_{PP} be the breakage rate between active package and prewinder;
- Time to remedy a warp breakage – let μ_W be the remedying rate of warp breakages;
- Time to remedy a breakage into the shed – let μ_F be the remedying rate of weft breakages;
- Time to remedy a breakage between package and prewinder – let μ_{PP} be the remedying rate of yarn breakages between package and prewinder.

We have to estimate the efficiency of the looms (*EF*) and the percentage of working time for the weavers (*WL*) in the case in which all these random variables are exponentially distributed and the parameters λ_W, λ_F, λ_{PP}, μ_W, μ_F, μ_{PP} are known.

Assumtions:

- A weaving process is in a steady–state condition.
- A weaving machine is either up or down, with no partial or intermediate states.
- All break events are stochastically independent.

3 Example of Clasical Approach

Consider the simple case in which only one weaving machine is allocated to a weaver (m=1, r=1). Taking into account the three kinds of breaks previously defined in Section 2, the reliability logic model illustrated in Fig. 1 is considered. For the weaving process the following states are possible:

- S_0 – The loom is running, no yarn breaks exist (the initial state);
- S_1 – While the loom is running, the weaver works to remedy a broken yarn between a spare package and prewinder;
- S_2 – The loom is down and the weaver works to remedy a warp breakage;

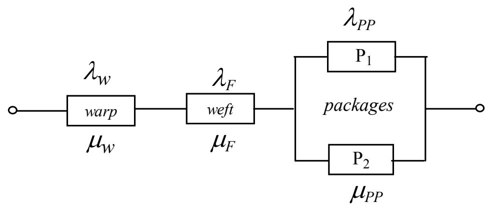


Fig. 1 – Reliability model for a weaving machine.

- S_3 – The loom is down and the weaver works to remedy a filling brakage into the shed;
- S_4 – The loom is down and the weaver works to remedy a warp broken yarn. A yarn between a spare package and prewinder is also broken, but this remedying has been temporary suspended.
- S_5 – The loom is down and the weaver works to remedy the filling yarn into the shed. A yarn between a spare package and prewinder is also broken, but this remedying has been suspended.
- S_6 – The loom is down because both packages are unavailable. The weaver works to remedy a breakage between package and prewinder.

The Markov chain describing the weaving process is presented in Fig. 2. The matrix **M** given in Fig. 3 presents the transition rates between states. Location (i, j) in matrix **M**, $i \neq j$, comprisis the transition rate from state j to state i. The value of location (i, i) is equal to the sum taken with minus of the transition rates in column i.

Let p_i be the steady–state probability of state S_i, $i \in \{0, 1, 2, 3, 4, 5, 6\}$. To determine the steady–state probabilities $p_0, p_1, ..., p_6$, the set of linear equations (1) must be solved, in which $\mathbf{P}=[p_0, p_1, ..., p_6]^T$, and $\mathbf{Z}=[0, 0, ..., 0]^T$.

$$\begin{cases} M \cdot P = Z \\ p_0 + p_1 + \cdots + p_6 = 1 \end{cases} \tag{1}$$

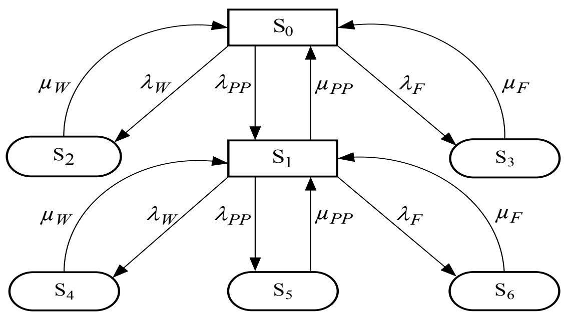


Fig. 2 – Markov chain for a weaving machine.

$$\begin{pmatrix} -(\lambda_{PP}+\lambda_W+\lambda_F) & \mu_{PP} & \mu_W & \mu_F & 0 & 0 & 0 \\ \lambda_{PP} & -(\mu_{PP}+\lambda_{PP}+\lambda_W+\lambda_F) & 0 & 0 & \mu_W & \mu_{PP} & \mu_F \\ \lambda_W & 0 & -\mu_W & 0 & 0 & 0 & 0 \\ \lambda_F & 0 & 0 & -\mu_F & 0 & 0 & 0 \\ 0 & \lambda_W & 0 & 0 & -\mu_W & 0 & 0 \\ 0 & \lambda_{PP} & 0 & 0 & 0 & -\mu_{PP} & 0 \\ 0 & \lambda_F & 0 & 0 & 0 & 0 & -\mu_F \end{pmatrix}$$

Fig. 3 –Transition matrix **M** of Markov chain presented in Fig. 2.

Thus, the set of equations

$$\begin{cases} -(\lambda_W+\lambda_F+\lambda_{PP})p_0+\mu_{PP}p_1+\mu_W p_2+\mu_F p_3=0 \\ \lambda_W p_0-\mu_W p_2=0 \\ \lambda_F p_0-\mu_F p_3=0 \\ \lambda_W p_1-\mu_W p_4=0 \\ \lambda_{PP}p_1-\mu_{PP}p_5=0 \\ \lambda_F p_1-\mu_F p_6=0 \\ p_0+p_1+\cdots+p_6=1 \end{cases} \qquad (2)$$

leads to the steady –state probabilities

$$p_0=\frac{1}{1+(\rho_W+\rho_F+\rho_{PP})(1+\rho_{PP})}\ , \text{ and}$$

$$p_1=\rho_{PP}p_0\ , \qquad (3)$$

where $\rho_W=\dfrac{\lambda_W}{\mu_W}, \rho_F=\dfrac{\lambda_F}{\mu_F}$, and $\rho_{PP}=\dfrac{\lambda_{PP}}{\mu_{PP}}$.

With these steady–state probabilities,

$$EF=(p_0+p_1)\cdot 100\,\%\ , \text{ and } \ WL=(1-p_0)\cdot 100\,\%\ .$$

Finally,

$$EF=\frac{1+\rho_{PP}}{1+(\rho_W+\rho_F+\rho_{PP})(1+\rho_{PP})}\cdot 100\,\%\ . \qquad (4)$$

For the case in which one weaver serves two weaving machines, the Markov chain comprises 46 states being quite complicated [2]. But, one weaver serves usually up to ten weaving machines, when the Markov chain has hundreds of states. We conclude that a classical approach for exact evaluation of machine efficiency is difficult to apply taking into account the complexity of Markov chains.

3 Simplified Analytical Approach

In order to simplify the analytical approach of the machine interference problem, a reduced stochastical model of a weaving machine is proposed. Thus, the following two random variables are introduced to describe the weaving machine as a system with repair:

- Time to stop the weaving process, regardless of the reason – a warp or a filling breakage, and
- Time to remedy a weaving machine.

Let λ be the stop rate, and μ be the remedying rate of the weaving machine. A reduced reliability model for a weaving machine is shown in Fig. 4.

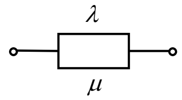
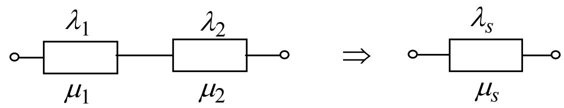
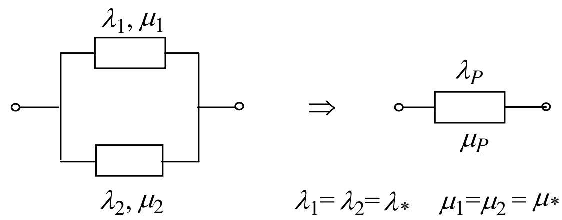


Fig. 4 – A reduced reliability model.

But, how can be determined the stop and the remedying rate, λ and μ, for this reduced model? On the initial reliability model presented in Fig. 1, serial and parallel transformations can be applied, as illustrated in Fig. 5. In order to determine equivalent parameters for serial and parallel transformations, the Markov chain presented in Fig. 6 and Fig. 7, respectively, must be considered.



a) serial transformation



b) parallel transformation

Fig. 5 – Equivalent models.

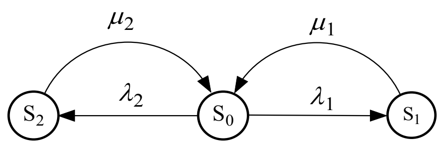


Fig. 6 – Markov chain for serial model.

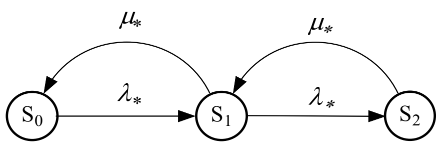


Fig. 7 – Markov chain for paralle model.

a) *Serial transformation*

Based on Markov chain presented in Fig. 6, the efficiency is equal to $EF = p_0 \cdot 100\,\%$. On the other hand, $EF = \dfrac{1}{1+\dfrac{\lambda_s}{\mu_s}} \cdot 100\,\%$. It follows that λ_S and μ_S are given by

$$\lambda_S = \lambda_1 + \lambda_2 \text{, and } \mu_S = \frac{\lambda_1 + \lambda_2}{\dfrac{\lambda_1}{\mu_1} + \dfrac{\lambda_2}{\mu_2}} . \quad (5)$$

b) *Parallel transformation*

Based on Markov chain presented in Fig. 7, the efficiency is equal to $EF = (p_0 + p_1) \cdot 100\ \%$. On the other hand, $EF = \dfrac{1}{1+\dfrac{\lambda_P}{\mu_P}} \cdot 100\,\%$. It follows that λ_P and μ_P are given by

$$\lambda_P = \frac{\lambda_*^2}{\lambda_* + \mu_*} \text{, and } \mu_P = \mu_* . \quad (6)$$

Parameters λ and μ, can be obtained by applying Eqs. (5) and (6) to the reliability model presented in Fig. 1, so that

$$\lambda = \lambda_W + \lambda_F + \frac{\lambda_{PP}^2}{\lambda_{PP} + \mu_{PP}} \text{, and} \quad (7)$$

$$\mu = \frac{\lambda_W + \lambda_F + \dfrac{\lambda_{PP}^2}{\lambda_{PP} + \mu_{PP}}}{\dfrac{\lambda_W}{\mu_W} + \dfrac{\lambda_F}{\mu_F} + \dfrac{1}{\mu_{PP}} \cdot \dfrac{\lambda_{PP}^2}{\lambda_{PP} + \mu_{PP}}} . \quad (8)$$

For the case in which one weaver serves one loom (m=1, r=1), the machine efficiency can be calculated by equation

$$EF = \frac{1 + \rho_{PP}}{1 + (\rho_W + \rho_F)(1 + \rho_{PP}) + \rho_{PP}^2} \cdot 100\,\% , \quad (9)$$

that is identical with Eq.(3) previously defined in Section 2. In other words, for the simple case in which the weaver serves only one weaving machine the reduced Markov chain is absolutely valid.

For the general case in which m weaving machines are allocated to r weavers, the reduced Markov chain is presented in Fig. 8. Variable u denotes the number of weaving machines up. The transition matrix of this Markov chain is presented in Fig. 9. As shown in [4], where this Markov model is analyzed, the steady–state probabilities are given by the following equations, where $\rho = \dfrac{\lambda}{\mu}$ and $\rho_* = \dfrac{\rho}{r}$.

$$p_i = p_0 \frac{\rho^i}{i!} \prod_{k=0}^{i-1} (m-k), i = 1, 2, ..., r. \quad (10)$$

$$p_i = p_r \rho_*^{i-r} \prod_{k=r+1}^{i} (m-k+1), i = r+1, ..., m . \quad (11)$$

$$p_0 = \frac{1}{1 + \sum_{i=1}^{r} \left(\dfrac{\rho^i}{i!} \prod_{k=0}^{i-1} (m-i) \right) + p_r \sum_{i=r+1}^{m} \left(\rho_*^{i-r} \prod_{k=r+1}^{i} (m-k+1) \right)} \quad (12)$$

The efficiency of the weaving machines is given by

$$EF = \left(p_0 + \sum_{i=1}^{m-1} \left(\frac{m-i}{m} \cdot p_i \right) \right) \cdot 100\% . \quad (13)$$

Note that Eq. (13) gives approximate results for the following reason. Even that all primary random variables previously defined in Section 2 have exponential distribution laws, the time to remedy a weaving machine has a hiper–exponential distribution, as shown in Fig. 10. Comparing with an exponential law with parameter μ, the variance is greater in this case. From the queueing theory, we know that the machine interference time depends both on the mean and the variance of time to remedy. For this reason, a factor of correction based on the coefficient of variation must be applied for an accurate evaluation of the machine interference time [5, pp. 415]. The following notations are introduced:

- Tr_m – The mean remedying time;
- Ti_m – The mean interference time;
- Td_m – The mean down time of the weaving machine because of a breakage;
- md_m –The mean number of machines down in a certain time.

The following equations can be written:

$$md_m = \sum_{i=1}^{m} (i \cdot p_i) \ (\textit{see Fig. 8}) \quad (14)$$

$$Td_m = Ti_m + Tr_m \text{, where } Tr_m = \frac{1}{\mu} . \quad (15)$$

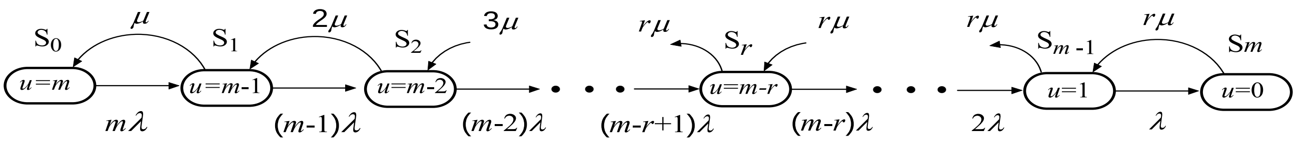


Fig. 8 – Reduced Markov chain for *m* weaving machines served by *r* weavers.

$$\begin{pmatrix}
-m\lambda & \mu & 0 & \cdot & 0 & 0 & 0 & \cdot & 0 & 0\\
m\lambda & -(m-1)\lambda-\mu & 2\mu & \cdot & 0 & 0 & 0 & \cdot & 0 & 0\\
0 & (m-1)\lambda & -(m-2)\lambda-2\mu & \cdot & 0 & 0 & 0 & \cdot & 0 & 0\\
0 & 0 & (m-2)\lambda & \cdot & 0 & 0 & 0 & \cdot & 0 & 0\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
0 & 0 & 0 & \cdot & (r-1)\mu & 0 & 0 & \cdot & 0 & 0\\
0 & 0 & 0 & \cdot & -(m-r+1)\lambda-(r-1)\mu & r\mu & 0 & \cdot & 0 & 0\\
0 & 0 & 0 & \cdot & (m-r+1)\lambda & -(m-r)\lambda-r\mu & r\mu & \cdot & 0 & 0\\
0 & 0 & 0 & \cdot & 0 & (m-r)\lambda & -(m-r-1)\lambda-r\mu & \cdot & 0 & 0\\
0 & 0 & 0 & \cdot & 0 & 0 & (m-r-1)\lambda & \cdot & 0 & 0\\
\cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot\\
0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & r\mu & 0\\
0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & -\lambda-r\mu & r\mu\\
0 & 0 & 0 & \cdot & 0 & 0 & 0 & \cdot & \lambda & -r\mu
\end{pmatrix}$$

Fig. 9 – Transition matrix of Markov chain presented in Fig. 8.

$$Td_m = \frac{md_m}{\overline{\lambda}} \quad (\textit{Little} \text{ formula [6]}), \tag{16}$$

where

$$\overline{\lambda} = \sum_{i=0}^{m} (m-i)\lambda p_i \tag{17}$$

denotes the mean stop rate in a group of *m* weaving machines. It follows that

$$Ti_m = \frac{1}{\lambda} \frac{\sum_{i=1}^{m} (i \cdot p_i)}{\sum_{i=0}^{m} (m-i) p_i} - \frac{1}{\mu}. \tag{18}$$

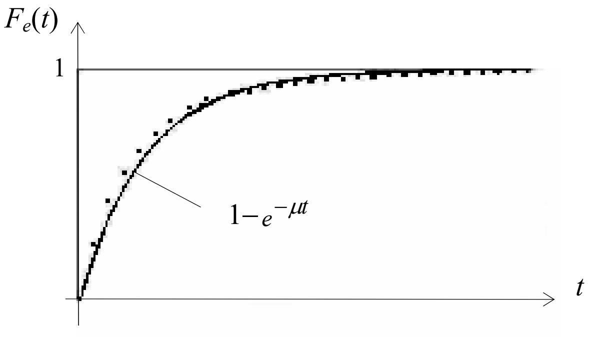


Fig. 10 – Empirical distribution for time to remedy.

Let c_v be the coefficient of variation for time to remedy a weaving machine, obtained by simulation. As shown in [1, pp.170], the estimation of Ti_m can be improved by applying a correction factor, so that

$$Ti_m^* = Ti_m \frac{1+c_v^2}{2}. \tag{19}$$

It follows that

$$md_m^* = (Ti_m^* + \frac{1}{\mu})\overline{\lambda} \tag{20}$$

and, finally,

$$EF^* = \left(1 - \frac{md_m^*}{m}\right) \cdot 100 \ \%. \tag{21}$$

Symbol * is used to denote the estimations when the correction factor is considered.

The weaver work loading can be obtained by the following equation.

$$WL = \left(\sum_{i=1}^{r-1} \frac{i}{r} p_i + \sum_{i=r}^{m} p_i \right) \cdot 100\%. \tag{22}$$

5 Case Study

To demonstrate the effectiveness of the reduced model presented in the previous section, analytical and simulation results are compared in this case study. The simulation model we have used is based on a coloured Petri net as presented in [7].

Consider a weaving process on a loom described by the following parameters:

- λ_W = 4.77 warp breakages/h;
- λ_F = 2.05 filling breakages into the shed/h;
- λ_{PP} = 1.37 yarn breakages between package and prewinder/h;
- μ_W = 58.88 warp remedies/h;
- μ_W = 220.05 filling remedies/h;
- μ_{PP} = 43.20 yarn remedies between package and prewinder/h.

Table 2 presents analytical results – given by Eqs. (21) and (22) – and simulation results, regarding the machine efficiency and the percentage of working time for the weavers. Note the good accordance between analytical and simulation results.

Many cases in which *m* weaving machines are served by *r* weavers have been considered. Note that, for example, when three weavers serve together

Table 1 – Anaytical and simulation results (expressed as %).

		Machine efficiency (*EF*)		Percentage of working time (*WL*)	
		analytical results	simulation results	analytical results	simulation results
r=1	*m*=1	91.63	91.62	8.37	8.38
	m=2	90.16	90.18	16.63	16.64
	m=3	89.02	89.06	24.74	24.71
	m=4	87.89	87.82	32.69	32.66
	m=5	86.69	86.65	40.43	40.44
	m=6	85.35	85.39	47.93	47.89
r=2	*m*=6	90.03	90.07	25.04	25.08
	m=8	89.59	89.57	33.28	33.31
	m=10	89.02	89.07	41.40	41.43
	m=12	88.30	83.36	49.35	49.39
r=3	*m*=9	90.16	90.19	25.10	25.11
	m=12	89.96	89.98	33.41	33.39
	m=15	89.67	89.70	41.66	41.69
	m=18	89.21	89.25	49.80	49.84

eighteen weaving machines, the machine efficiency is significantly greater than when one weaver serves six weaving machines.

6 Conclusions

This paper deals with a machine interference problem and presents a simplified analytic method based on a reduced Markov chain. For a group of *m* weaving machines served by *r* weavers, relationships for evaluating with accuracy the machine efficiency and the percentage of working time for the weavers are given.

A realistic case study, where analytical and simulation results are compared, demonstrates the effectiveness of this simplified analytical approach.

We assume that all random variables describing the weaving process are exponentially distributed. But, as shown in [1, pp.169], in many cases it is necessary to consider a normal or gamma distribution for remedying times. This is the first limitation of our work. A normal or gamma distribution for remedying times will be the subject of upcoming papers.

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