Using neural networks to establish the structural controllability and observability

R. PEREIRA          J.C. QUADRADO
DEEA, ISEL, Rua Conselheiro Emidio Navarro, 1950-072 LISBOA
CAULT, Av. Rovisco Pais, 1049-001 LISBOA
PORTUGAL

Abstract: - In this paper the usage of neural networks together with bondgraphs allowing the determination of the structural controllability and observability, is presented. The usage of bondgraphs allows a preliminary interpretation that enables the conclusion drawing about the structural characteristics, regardless of the numerical parameters of the system. The artificial neural networks are used to incorporate this knowledge again without the need for a formal mathematical model of the system. A mechanical suspension system is used as a case study.

Key-Words: - Neural networks, bondgraphs, controllability, observability, mechanical suspension

1 Introduction
Descriptor systems result from a convenient and natural modelling process. The Lagrange equations of constrained mechanical systems, which are well known in classical mechanics[1], as well as the bondgraphs, are actually also in the descriptor form. Bondgraphs are a representation of engineering systems by special conventional signs and symbols which are a very powerful technique for modelling systems destined to perform computer simulations. Bondgraphs were developed at MIT for the simulation of antiaircraft guns and missiles with electric drives and hydraulic control systems. They were created by Henry Paynter and further developed by Dean Karnopp and Ronald Rosenberg [2]. Originally, bondgraphs were designed for hydraulic control systems and hydrostatic transmissions, which have mechanical and electric subsystems. These mixed domains, with the bondgraphs can be represented in a common language. At the same time the bondgraphs represent algorithmic information, which facilitates the passage to computer programming [2].
The dual concepts of controllability and observability are fundamental in the control system theory [3]. The use of mathematical models to establish controllability and observability characteristics of drives is well known. Nevertheless it is possible to get information of structural controllability and observability of actuators without numerical calculation. Controllability and observability are fundamental in control drives. In section 4 is shown an expeditious way to determine structural controllability and observability of a mechanical suspension, by inspection of the bondgraphs causal connections.
The preliminary interpretation of bondgraphs allows to easily extracting conclusions about structural characteristics (just depending on the model structure and model elements, excluding the dependency on the numerical parameters) of actuators, despite of the possibility to show that results vary with some sets of parametric values given. However, the advantage of this formulation is that it allows choosing sensors and drives during the control project of an actuator.
The linear continuous descriptor systems under consideration have the form expressed in (1) and (2).

\[
\frac{d}{dt}[x] = A[x] + B[u] \quad (1)
\]

\[
[y] = C[x] + D[u] \quad (2)
\]

An artificial neural network is an information-processing system that as certain performance characteristics in common with biological neural networks. Artificial neural networks have been developed as generalizations of mathematical models of human cognition or neural biology, based on assumptions that:
- Information processing occurs at many simple elements called neurons;
- Signals are passed between neurons over connection links;
- Each connection link has an associated weight, which in a typical neural net, multiplies the signal transmitted;
- Each neuron applies an activation function (usually nonlinear) to its net input (sum of weighted input signals) to determine output signal.
A neural network is characterized by its pattern of connections between the neurons (called its architecture), its method of determining the weights on the connections (called training, or learning algorithm), and its activation junction.

A neural net consists of a large number of simple processing elements called neurons, units, cells or nodes. Each neuron is connected to other neurons by means of direct communication links, each with an associated weight. The weights represent information being used by the net to solve a problem. Neural nets can be applied to a wide variety of problems such as storing and recalling data of patterns, classifying patterns, performing general mappings from input patterns to output patterns, grouping similar patterns, or finding solutions to constrained optimization problems.

Each neuron as an internal state called its activation or activity level, which is a function of the inputs it as received. Typically, a neuron sends is activation as a signal to several other neurons. It is important to note that a neuron can send only one signal at a time, although that signal is broadcast to several other neurons[4].

The bondgraphs can use artificial neural networks to extract the referred knowledge.

2 Controllability
Controllability is defined as the capacity to control variables or states of a system. The classic numerical method to determine the controllability of a system is based on the determination of the order of the controllability matrix, [C]. The controllability matrix is defined in (3).

\[
[C] = [B \ AB \ ... \ A^{n-1}B] \quad (3)
\]

The order of the controllability matrix depends on the numerical values of the parameters, consequently does not determine the robustness of the control strategy. This method also does not allow to identify variables of the system that are not controllable.

An alternative numerical method is based on the feedback of the inputs, where the eigenvalues of matrix [A]-[B][F] are used. ([F] is a gain matrix generated randomly) instead of the [A] eigenvalues. This approach allows the determination of the controllability of each variable (eigenvalues). This method also depends on the parameters, so it cannot be considered as a robust measurement of the controllability. The structural controllability based in bondgraphs is by definition a robust measure therefore does not depend on the parameters of the system.

For the bondgraph expressed system to be structurally controllable, it is necessary to fulful two necessary conditions:

1. Each element of causal storage (I and C) in a bondgraph, with integral causality, must have at least a causal path binding it to a control source (SE or SF);
2. Each element of causal storage (I and C) in a bondgraph, with integral causality can differential causal (when differential causality is given) when applies to the model in the differential causality form without violating any of the junctions causality norms.

The condition 1) is assigned by attainment condition, whereas condition 2) refers to the structural dimension.

If condition 2) is reached then the order of the matrix [A] is n and the system is controllable through an only actuator whose localization is determined by the attainment condition 1) and other project considerations. If 2) is invalid then the [A] matrix order is of q < n. In this situation, for the model be controllable, is necessary, p = n-q actuators, being its localization determined by the attainment condition 1) and its participation determined by condition 2)[5].

3 Observability
Observability is defined as the capacity to observe variables or states of a system. The classic numerical method to determine the observability of a system requires the order of observability matrix calculation. The observability matrix [O] is defined in (4).

\[
[O] = \begin{bmatrix}
C \\
CA \\
\cdot \\
\cdot \\
CA^{n-1}
\end{bmatrix} \quad (4)
\]

The matrix observability order depends on parametrics numerical values, which are not a robustness observability measurement strategy. This method also does not identify which of the system variables are not observed. An alternative numerical method uses the output feedback, where the eigenvalues of matrix [A]-[F][B] replace the matrix [A] eigenvalues. This approach is highly dependent on the parameters and consequently is not a robust observability measure.

The robust structural observability can be obtained as long as the following conditions are satisfied:
1. Each element of storage with integral causality (I and C) in a bondgraph expressed model must have at least a causal path that binds it to an observer (elements C activated by effort or I activated by the flow, also assigned for De and Df, respectively)

2. Each element with integral causality storage (I and C) in a bondgraph expressed model, in the integral causality form, can be differential causal (differential causality attributed) when the differential causality is attributed to the model without violating the junctions causality norms. If to some integrally causal elements in an integral causality form, cannot be applied the differential causality in a differential causality form, then if the replacement of some or all observers jointly with the activation (I activated by flow in C activated by effort and vice-versa) accepts the differential causality, then this condition is kept. If the order of the matrix is n, then an only observer, conveniently placed (determined by the attainment condition) it is enough to warrant a total observability of the system.

If the matrix [A] order is n, then only one observer, conveniently located (determined by attainment condition 1) is enough to warranty total system observability.

If matrix [A] order is q, then p = n-q additional observers are needed to have observability assured. Their location can be determined by the attainment and project conditions.[5]

4 Case Study

This paper’s case study refers to a mechanical suspension shown in figure 1.

![Fig. 1 – Mechanical suspension](image)

Considering that the artificial neural network can model this system in two layers, the extracted model will have the same number of layers. It is important to notice that the conversion from artificial neural networks to bondgraphs it is not structural. Each connection is converted into a “1” junction with an inertial element. The remaining elements of the artificial neural network are converted into a “0” junction with a capacitor and resistive element connected by an “1” junction.

The bondgraph of this descriptor system is presented in figure 2 and its controllability and observability are determined.

![Fig. 2 – Bondgraph of a mechanical suspension](image)

The velocity variable controllability of the upper mass using the source is attainable. The graphical proof is shown in figure 3, where a causality path between SF and the flux considered (v) is shown.

![Fig. 3 – Velocity controllability](image)

The observability of the lower mass velocity with an observer (a velocity sensor) located in the upper mass represented on fig. 1 is also possible as shown in figure 4, where is possible to observe the causality path between the referred velocity and sensor.
In this case study all the elements of the bondgraph considered are necessary.

5 Conclusions

In this paper the usage of artificial neural networks allows the design of the bondgraphs based on the proposed conversion method. The usage of this method allows the determination of the structural controllability and observability. The usage of bondgraphs complements the artificial neural networks knowledge extraction without the need for a formal mathematical model of the systems. The case study is a good example for the implementation of this method. The method is still in the stage of preliminary implementation. Further work is needed to validate this method to different systems.

References:
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