CMOS Fuzzification Circuits for Linear Membership Functions

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Abstract: - The subject of the study was hardware implementations of fuzzy controllers as CMOS analog devices on the base of implementation of fuzzy inference rules as multi-valued logic functions using summing amplifiers as building blocks. Earlier a functional completeness of summing amplifier with saturation in an arbitrary-valued logic was proven that gave a theoretical background for analog implementation of fuzzy controllers. In previous works it was suggested to put into accordance to input and output linguistic variables of a controller description logical values of some multi-valued variables and instead of fuzzification and defuzzification procedures to apply piecewise-linear approximation between variable logical levels. In the case when derived logical levels were not evenly distributed in the range of an analog signal change it was suggested to increase the number of logical levels to reach even distribution. Such approach leaded to decreasing controller accuracy. This paper suggested instead of increasing logical levels to use special devices transforming analog input variables into multi-valued variables with even distribution of logical levels and output multi-valued variables into analog view. For these devices the names fuzzifier and defuzzifier were kept. The paper illustrated a design example for real industrial fuzzy controller and provided SPICE simulation results of its functioning.

Key-Words: - Fuzzy Logic, Fuzzy Controller, Multi-Valued Logic Functions, Summing Amplifier with Saturation, Fuzzy Inference, Fuzzification, Defuzzification.

1 Introduction

In contrast with traditional approach to fuzzy controllers implementation based on explicit fuzzification, fuzzy inference, and defuzzification procedures [1–3], which usually implemented as software for the special type of controllers, a fuzzy controller hardware implementation as an analog device has advantages of higher speed, lower power consumption, smaller die area etc.

In the previous works [4–9] we proposed and described the method of synthesis and designing analog fuzzy controllers on the base of summing amplifiers with saturation and gave the appropriate examples. In these publications the functional completeness of such amplifiers in an arbitrary valued logic was proven.

Actually in [4–9] functions of multiple-valued logic were specified by tables of fuzzy rules over linguistic variables. In these works it was implicitly assumed that the values of the analog signals, which correspond to linguistic variables, were evenly distributed in the range of an analog signal change. Otherwise by artificial means the number of linguistic variables can be increased that leads to growing the implementation complexity. In this paper the procedure of input analog variables transformation into multiple-valued variables with evenly distributed logical levels is suggested.

This procedure in some sense is analogous to the procedure of fuzzification in fuzzy control. Because of using a fuzzy controller description for implementation of this controller as a multiple-valued logical function we will use the same term "fuzzification" for the suggested procedure of logical levels equalization for input variables. First of all we would like to examine more attentively the fuzzification procedure for the case of linear membership functions or membership functions, which sufficiently simply can be represented as piecewise-linear, and to propose sufficiently simple universal method.

2 Fuzzification Procedures

We use the standard determination of a membership function. The membership function determines for each value of an analog[1] variable X the weight of the corresponding linguistic variable b.

$$w_b = F(b, X); \ 0 \le w_b \le 1. \qquad (1)$$

In Fig.1 the simplest example of membership functions is given.

"The membership function is a graphical representation of the magnitude of participation of each input. It associates a weighting with each of the inputs that are processed, define functional overlap between inputs, and ultimately determines an output response. The rules use the input membership

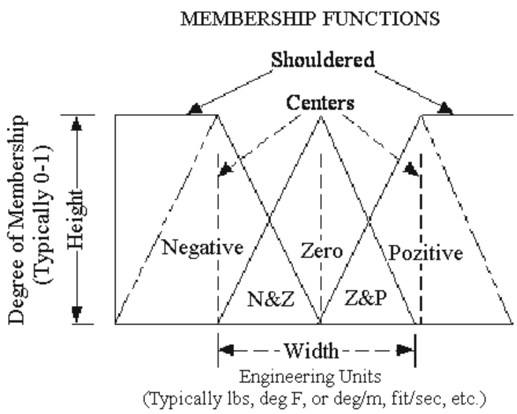

Figure 1. The simplest type of membership functions.

values as weighting factors to determine their influence on the fuzzy output sets of the final output conclusion. Once the functions are inferred, scaled, and combined, they are defuzzified into a crisp output which drives the system. There are different memberships functions associated with each input and output response. Some features to note are: SHAPE - triangular is common, but bell, trapezoidal, haversine and, exponential have been used (More complex functions are possible but require greater computing overhead to implement.); HEIGHT or magnitude (usually normalized to 1); WIDTH (of the base of function); SHOULDERING (locks height at maximum if an outer function. Shouldered functions evaluate as 1.0 past their center); CENTER points (center of the member function shape); OVERLAP (N&Z, Z&P, typically about 50% of width but can be less).[2]

Fig.1 illustrates the features of the triangular membership function which is used in the fallowing example.

The procedure of fuzzification and the construction of the corresponding diagram we will examine based on example of the fuzzy Controller for a Container Crane, membership functions for which are given in Fig.2.[3]

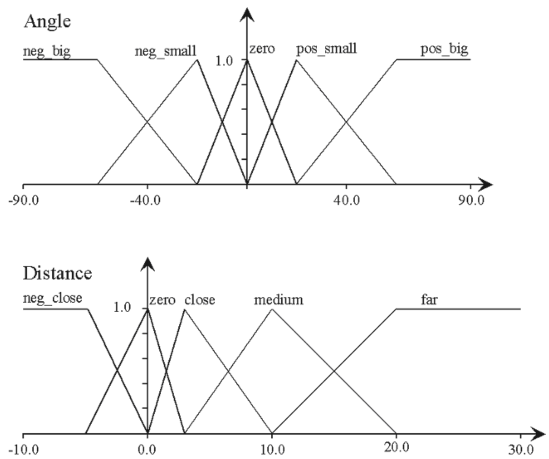

Figure 2. Membership functions for the Fuzzy Controller of a Container Crane.

We will assume, without disrupting the generality of reasoning, that with a change of the angle within the limits $-90^{\circ} \div +90^{\circ}$ and of the distances in the limits (-10 ÷ +30) *yards* the corresponding analog voltages vary within the limits (0÷3.5)*V*.[4]

Table 1 determines the function of fuzzi-fication for the piecewise-linear membership functions, since linearity of these functions

[1] In principle the input variable can be digital.

[2] Citation is taken from "Fuzzy Logic – an Introduction", part 4, by Steven D. Kaehler, http://www.seattlerobotics.org/encoder/mar98/fuz/fl_part4.html

[3] http://www.fuzzytech.com/bilder/pdf_cir.gif

[4] The source voltage of the circuit is 3.5*V*.

Angle membership functions **Table 1**

linguistic variable	neg_big	neg_small	zero	pos_small	pos_big
angle voltage	-90°÷-60° (0÷0.583)V	-20° 1.361V	0° 1.75V	20° 2.139V	60°÷90° (2.917÷3.5)V
logic values voltage	-2 0V	-1 0.875V	0 1.75V	+1 2.625V	+2 3.5V

gives the possibility to connect the points of logical values by straight lines. The corresponding function is given in Fig.3.[5]

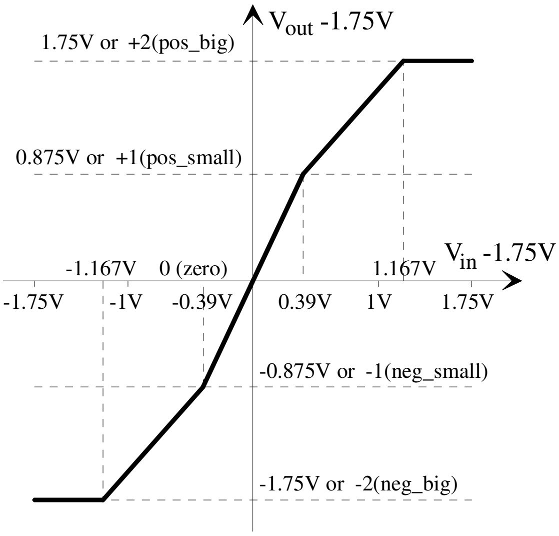

Figure 3. Piecewise-linear function for fuzzification of the variable “Angle”.

For the realization of function $V_{out} = F_1(V_{in})$ given in Fig.3 we should realize three auxiliary functions represented in Fig.4, whose sum with saturation on the levels ± 1.75V determines the fuzzificated input function for fuzzy inference part[6] of devices. In Fig.4 the angle α and values of functions $\varphi_j(\alpha)$ are represented in positive and negative voltages.

As the basic elements for the fuzzifier implementation we will use, as into [1-5], summing amplifiers with saturation, whose behavior is described as:

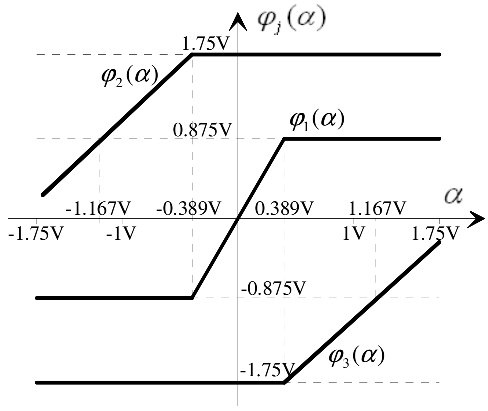

Figure 4. Component functions for the function represented in Fig.3.

$$Y = S(X,\Omega,\eta) = \begin{cases} +a & \text{if } -a \ge \sum_{j=0}^{n} \omega_j \cdot x_j - \eta \\ -(\sum_{j=0}^{n} \omega_j \cdot x_j - \eta) & \text{in other cases} \\ -a & \text{if } \sum_{j=0}^{n} \omega_j \cdot x_j - \eta \ge +a \end{cases}$$

where Y – output voltage; X – vector of x_j; x_j– j^{th} input voltage; Ω– vector of ω_j; $\omega_j = R_0 / R_j$ – weight of j^{th} input; η - permanent shift (threshold); ± a – levels of saturation.

The ways of forming the component functions, given in Fig.4, and the output function $F_1(\alpha)$ are shown below:

$$\begin{aligned} &\varphi_1(\alpha) = -0.5S(4.5\cdot\alpha); \\ &\varphi_2(\alpha) = -S(1.125\cdot\alpha + 2.19); \\ &\varphi_3(\alpha) = -S(1.125\cdot\alpha - 2.19); \\ &F_1(\alpha) = S(-\varphi_1(\alpha) - \varphi_2(\alpha) - \varphi_3(\alpha)) = \\ &\mathrm{S}(0.5S(4.5\cdot\alpha) + S(1.125\cdot\alpha + 2.19) + \\ &S(1.125\cdot\alpha - 2.19)). \end{aligned} \quad (1)$$

For the completion of the fuzzifier design there remains only to determine the values of

[5] In Fig.3 along the axes the variations of voltages from the average (equilibrium) point of summing amplifier are plotted.

[6] Really this part will be implemented as multi-valued logic function.

Distance membership functions **Table 2**

linguistic variables	neg_close	zero	close	medium	far
distance voltage	≤ -5 yards ≤ 0.4375V	0 yards 0.875V	3 yards 1,1375V	10 yards 1.75V	≥ 20 yards ≥ 2.625V
logic values voltage	-2 0V	-1 0.875V	0 1.75V	+1 2.625V	+2 3.5V

the input resistances of summing amplifiers and to conduct checking by SPICE simulation. It was done and we ensured that the fuzzifier implementation (1) is correct.

It seems that on the basis of the given implementation of the function $F_1(\alpha)$ it would be possible to complete presentment leaving the formulation of algorithm to the reader. However, from our point of view, it makes sense to consider the additional example, membership functions for which are given in Fig.2 for the variable "Distance". This example is characterized, first, by the asymmetry of the measured distance (-10 ÷ +30)*yards*[7] and, second, by the explicit asymmetry of the positions along the distance axis of the membership functions centers. For this case fuzzification is determined by table 2.[8]

Corresponding function $V_{out}(V_{in})$ is given in Fig.5. For implementation of this function $V_{out} = F_2(V_{in})$ it is necessary to realize four auxiliary functions, whose sum with saturation on the levels ±1.75V will give the desired result. The auxiliary functions are given in Fig.6. Their values and value of the variable d ("distance") are represented in negative and positive voltages.

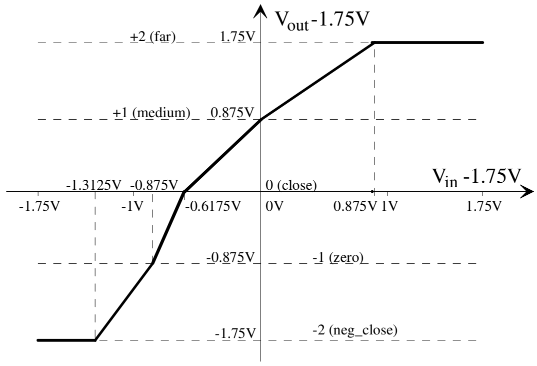

Figure 5. Piecewise-linear function for fuzzification of the variable "Distance".

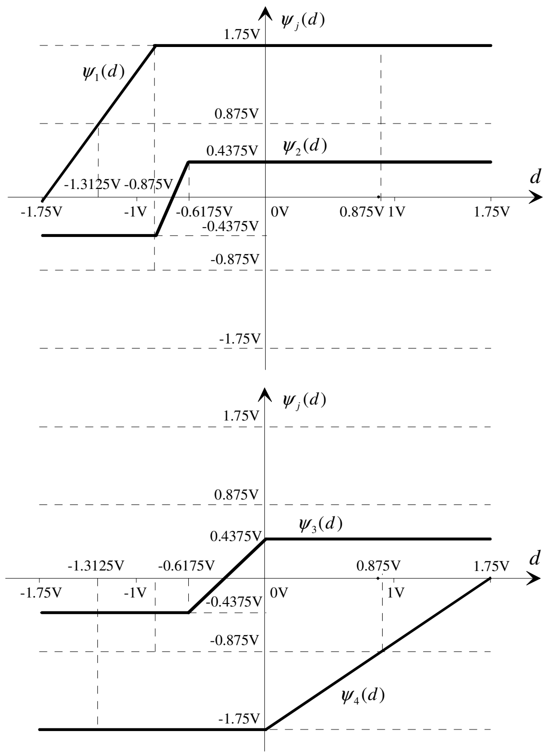

Figure 6. Component functions for the function represented in Fig.5.

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7 We do continue to assume that to the complete range of the measured distance does correspond the complete range of the supply voltage (0V ÷ 3.5V) or (-1.75V ÷ +1.75V) in the deviations from the equilibrium point of amplifier.

8 We will note that in the table 2 the linguistic variable "close" corresponds to value "log. 0" and the linguistic variable "zero" corresponds to the value "log.-1". The balance point of the amplifier input voltage corresponds to linguistic variable "medium".

The ways of forming the component functions given in Fig. 6 and the function $F_2(d)$ are shown below:

$$\begin{aligned}
&\psi_1(d) = -S(2 \cdot d + 3.5);\\
&\psi_2(d) = -0.25S(13.6 \cdot d + 10.14);\\
&\psi_3(d) = -0.25S(5.67 \cdot d + 1.75);\\
&\psi_4(d) = -S(d - 1.75);\\
&F_2(d) = S(-\psi_1(d) - \psi_2(d) - \psi_3(d) - \psi_4(d)) =\\
&S(S(2 \cdot d + 3.5) + 0.25S(13.6 \cdot d + 10.14) +\\
&0.25S(5.67 \cdot d + 1.75) + S(d - 1.75)).
\end{aligned} \qquad (2)$$

For the completion of the distance fuzzifier design there remains to determine the values of the input resistances of summing amplifiers and to conduct checking by SPICE simulation. It was done and we ensured that the fuzzifier implementation (3) is also correct.

3 Fuzzifier Implementations

A summing amplifier with saturation can be constructed on the base of any operational amplifier. One of possible summing amplifier implementations is shown in Fig.7.

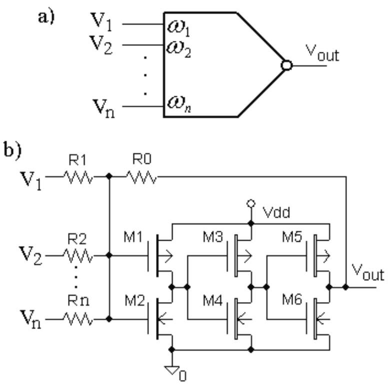

Figure 7. Summing amplifier: general structure (a); CMOS implementation using symmetrical invertors (b).

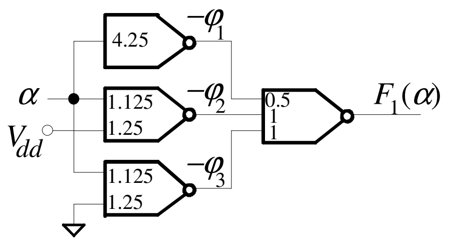

Figure 8. Fuzzifier of the variable “angle”.

Weights of input variables of this amplifier are determined as $\omega_j = R_0 / R_j$.

Fuzzification devices (fuzzifiers) built on the base of summing amplifiers and the systems of equations (1) and (2) are shown in Fig.8 and Fig.9 respectively.

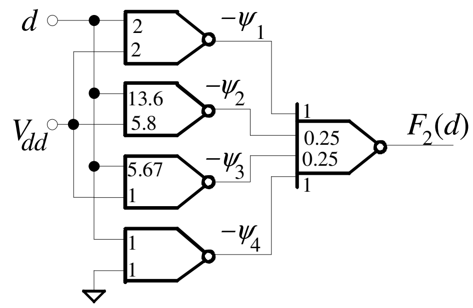

Figure 9. Fuzzifier of the variable “distance”.

Results of SPICE simulation of the fuzzifiers for variables “angle” and “distance” are shown in Fig.10 and Fig.11 respectively.

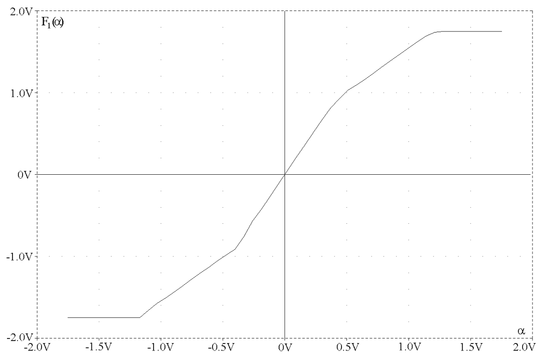

Figure 10. Output of the fuzzifier shown in Fig.8 and derived by SPICE simulation.

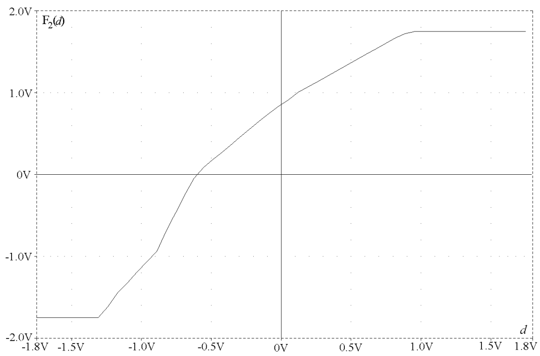

Figure 11. Output of the fuzzifier shown in Fig.9 and derived by SPICE simulation.

It is easy to see that the curves, which are show in Fig.10 and Fig.11, fully coincide with the curves represented in Fig.3 and Fig.5 that is the proof of the correctness of the fuzzifier implementations.

4 Conclusions

In the above examples of fuzzifiers, push-pull summing amplifiers are used. The summing amplifier however can be of another type, e.g. differential type or any other types of operational amplifiers.

It is obvious that the same approach can be used when output linguistic variables of a controller with their membership functions demand backward transformation evenly distributed logical levels of an output multi-valued variable to an analog variable with not evenly distributed voltages of logical levels. In this case we will speak about a defuzzification procedure that is implemented with a defuzzifier.

Thus it was shown that all parts of fuzzy controllers can be effectively implemented as analog devises on the base of summing amplifiers with saturation. Such implementation of fuzzy controllers has advantages of better response time and reliability, low power consumption, smaller die area etc.

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