

Bit Error Rate Performance of Third Generation Turbo Codes

M. A. U. BHUIYAN[†], M. S. ALAM[‡], M. S. HOSAIN^{‡‡}

[†]Department of Computer Science & Engineering

The University of Asia Pacific

Road # 3/A, Dhanmondi, Dhaka – 1209

BANGLADESH

[‡]CSE Dept., Bangladesh University of Engineering & Technology, Dhaka – 1000

^{‡‡}CSE Dept., The University of Asia Pacific, Dhaka – 1209

<http://www.uap-bd.edu/cse/faculty/akash>

<http://www.buet.ac.bd/cse/csefaculties.html>

<http://www.uap-bd.edu/cse/faculty/shazzad>

Abstract: After the invention of Turbo Codes (TCs) many coding theorist and researchers further tried to improve the performance of these codes. Now the performance of Turbo Codes is becoming closer to the Shannon's limit. The performance of turbo codes depends on its architectural component and the architecture of TCs is application dependent. So the researchers are trying to develop the standard architecture of TCs for specific application so that its performance becomes optimum. In this paper we analyze the performance, in terms of bit error rate (BER) vs. bit energy to noise spectral ratio (E_b/N_o) of Turbo Codes that are using in Third Generation Partnership Project standard (Universal Mobile Telephone Systems-UMTS) [10]. This paper focuses on the effect of the variation of the architectural components of UMTS TCs on its performance. In particular code rate, frame size i.e., length of information bits, number of decoding iteration and channel models variations of Turbo Codes are taken into account in studying the performance of Turbo Codes. These effects are shown in the performance curve (BER vs. E_b/N_o) of the UMTS standard turbo codes.

Keywords: Turbo code, Wireless communication, Shannon's limit, Bit error rate, UMTS, Code rate, Wireless channel modeling.

1 Introduction

For the last fifty years mathematicians, engineers and especially the coding theorists were vigorously searching for such error correcting codes (ECC) that can reach the Shannon limit. In 1948 Claude Shannon demonstrated in his paper [1] that proper encoding technique could reduce errors induced by a noisy channel at any desired level without sacrificing the rate of information transfer. But before 1993 all the ECCs, block or convolutional codes, can reach only within 3 dB to the Shannon limit. Shannon limit for a binary modulation scheme with rate=1/2 is $BER = 0$ (several authors take $BER = 10^{-5}$) for $E_b/N_o = 0$ dB. At 1993 Berreau et. al., in their landmark paper [2], first introduced turbo codes, which could achieve energy efficiencies within 0.7 dB of the Shannon's limit. After ten years Turbo Codes (TC) have commenced their practical services. TCs are intended to use in deep-space communication to explore solar system [3],

terrestrial digital video broadcast (DVB) [4], Bluetooth technology [5], Personal Communication System (PCS) application [6] and one of the main options for ECC in the third generation mobile standards (UMTS and cdma2000) etc [7]. In [8], performance of an analog decoder for UMTS turbo code was studied and its performance were shown. In [9], an improved UMTS turbo codes were proposed for the unequal error protection of compressed video sequences. But the performance of UMTS turbo codes for varying architectural component was not extensively studied in these literatures.

In this paper we investigate the performance of turbo codes as applications in UMTS. We show how the architecture of the turbo codes i.e., encoder structures, interleaving techniques; trellis termination techniques of UMTS TCs differ from classical turbo codes. We also present the effect of different input data sizes of the encoder, code rates [12], interleaver design

[13], channel variation (Additive White Gaussian Noise-AWGN and Rayleigh) and decoding techniques on the performance (BER vs. E_b/N_o) of UMTS TC.

The rest of the paper is organized as follows. Section 2 presents the structure of classical Turbo Codes and its performance. Different parameters of the encoder and decoder are varied to show the effect on its performance. Section 3 shows UMTS TC structure and performance. Section 4 describes the implementation issues of TCs. And finally section 5 draws a conclusion.

2 Classical Turbo Code

Forney in [16] first proposed the multilevel coding structure known as concatenated code. Then a Serially Concatenated Constituents Codes (SCCC) [17] and Parallely Concatenated Constituent Codes (PCCC) [2] were proposed. We will concentrate our analysis on the PCCC, popularly known as ‘‘Turbo Codes’’.

2.1 TC Encoder

TCs are parallel concatenation of two or more Recursive Systematic Convolutional (RSC) codes with random interleaver. Fig. 1 depicts an encoder structure and Fig. 2 shows a simple constituent RSC encoder.

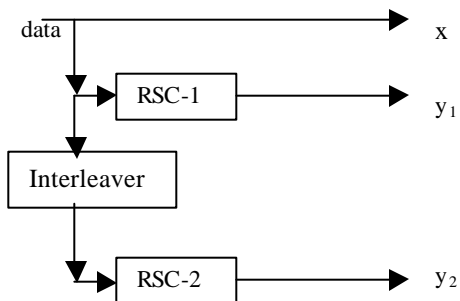


Fig.1 Basic Turbo Encoder (rate 1/3)
(x is the systematic bit, y_1 and y_2 are parity bits)

The TC consists of two identical constituent encoders, denoted as RSC Encoder-1 and RSC Encoder-2. The interleaver is the critical part of the TC. It permutes the order of the data bits in irregular manner. The input data stream and two outputs redundancy are then serialized into a single TC word. It is a rate 1/3 encoder.

2.2 Turbo Decoder

The basic concept of decoding is alternate decoding of component codes and passing the extrinsic information, which is a part of the soft output of the SISO decoder, to the next decoding stage. After transmission over a binary symmetric channel or a Gaussian or Fading channel the log-likelihood ratio of x conditioned on the output y [15] is calculated using the following equation:

$$L(u/z) = \log \frac{P(x = +1 | y)}{P(x = -1 | y)} \dots\dots\dots (1)$$

Ideally the soft-output for the information bit u can be represented by

$$L(\hat{u}) = L(u) + L_c \cdot y + L_e(\hat{u}) \dots\dots\dots (2)$$

Here $L(u)$ is the a-priori value and is produced by the preceding SISO decoder, $L_c \cdot y$ is the weighted received systematic channel value and $L_e(\hat{u})$ is the extrinsic information. This extrinsic value $L_e(\hat{u})$ is passed between the SISO decoder.

2.3 Performance of Classical TCs

Fig.2 demonstrates that TC performs well in AWGN channel model than Rayleigh Channel model. At $BER=10^{-4}$, required E_b/N_o for AWGN channel is 2 dB and that for Rayleigh channel model is 2.3 dB. So TC performance degrades 0.3 dB at that BER in Rayleigh channel model. In our experiment frame size (FS) is taken as 1000 and code rate is 1/3.

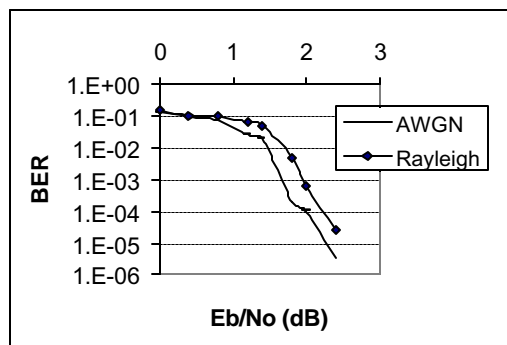


Fig.2: BER performance of TCs for AWGN and Rayleigh Channel (Frame size (FS)-1000, Rate-1/3)

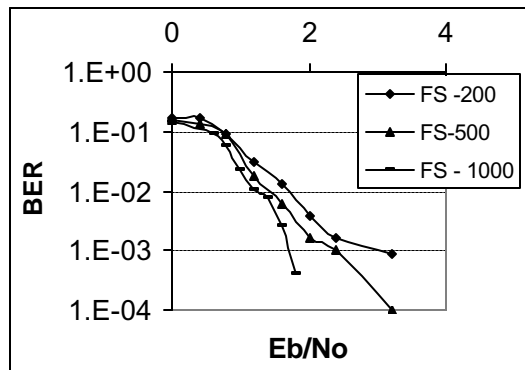


Fig.3: BER performance of TCs for different frame size (Rate-1/2, AWGN channel, iteration-5, Random inter.)

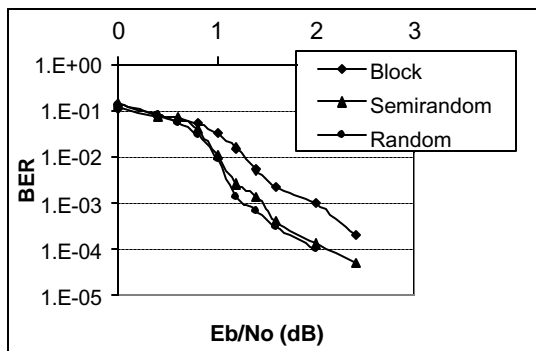


Fig.4: BER performance of TCs for different interleaver (FS-400, Rate-1/2, AWGN channel, iteration-5)

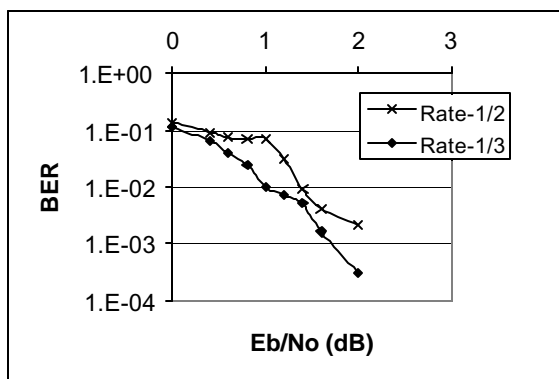


Fig.5: BER performance of the 4-state TCs for different rate (FS-400, AWGN channel and iteration-5)

Fig. 3 demonstrates the performance of TCs for different frame size. For larger block length BER

performance is improved and the code can be random if k (frame size) and n (codeword) tend to infinity. So if the frame size increases the randomness of the codeword also increases and it leads to achieve the low BER at a fixed E_b/N_0 . In Fig.4, BER performance is shown for different interleaver design. Fig.5 shows the puncturing effect on the performance of TCs. If there is no puncturing in the encoder then the code rate is 1/3. That is for one information bit, three output bits are transmitted. But in the classical TCs, there is an option that two bits will be transmitted for one information bit making the code rate 1/2. So it is likely that more errors can be corrected by decoding the 1/3 rate received bit than that of 1/2-rate bits.

3 UMTS Turbo Codes

3.1 Encoder

As a forward error correction coding scheme Third Generation Partnership Project (3GPP) uses two types of coding, convolutional coding and Turbo Coding scheme. In the convolutional coding technique constraint length is 9 and coding rates are 1/3 and 1/2. TCs uses the Parallel Concatenated Convolutional Code (PCCC). Each constituent encoder is 8-state i.e., its constraint length is 3. The code generator $G(D)$ is equal to $[1, G_1(D)/G_2(D)]$. Where $G_1(D) = 1 + D^2 + D^3$ and $G_2(D) = 1 + D + D^3$.

The initial value of the shift registers of the constituent encoder shall be all zeros when it starts to encode the input bit. The encoder is shown in Fig. 9. Here the output x_{1k} is systematic output, y_{1k} is parity output from the encoder 1 and y_{2k} is the parity output of the encoder 2. Before entering to the encoder 2 the input is scrambled by the internal interleaver. The systematic output bit x_{2k} is used at the time of trellis termination.

3.2 Trellis Termination Technique

UMTS TCs differs from the classical TCs in the trellis termination technique. The classical TCs terminate one of its encoder's trellises keeping the other encoder unterminated. Whereas in UMTS TCs the first three tail bits will be used to terminate the first constituent encoder while keeping the 2nd encoder disabled. The last three bits will be used to terminate the second constituent encoder keeping the first encoder disabled.

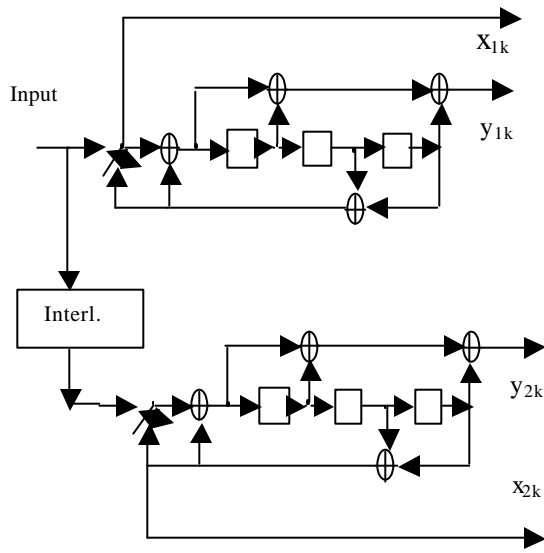


Fig.6. UMTS Turbo Encoder

3.3 TC Internal Interleaver

TCs interleaver scramble the input data by following three steps, bits-input to a rectangular matrix with padding, intra-row and inter-row permutations of the rectangular matrix with pruning. The bit input to the TC internal interleaver is recommended to be between 40 and 5114.

3.3.1 Bits input to a rectangular matrix

1. Determine number of rows (R) of the rectangular matrix:

$$R = \begin{cases} 5, & \text{if } (40 \leq K \leq 5114) \\ 10, & \text{if } (160 \leq K \leq 200) \text{ or } (481 \leq K \leq 530) \\ 20, & \text{if } (K = \text{other_value}) \end{cases}$$

2. Determine the prime number and the number of columns (C) of the rectangular matrix:

If ($481 \leq K \leq 530$)

$$p = 53 \text{ and } C = p$$

Else

$$K \leq R \times (p + 1)$$

$$C = \begin{cases} p - 1, & \text{if } K \leq R \times (p - 1) \\ p, & \text{if } R \times (p - 1) < K \leq R \times p \\ p + 1, & \text{if } R \times p < K \end{cases}$$

End if

3. Now input bits are written into the $R \times C$ rectangular matrix row by row. If $R \times C > K$, dummy bits are padded.

3.3.2 Intra-row and Inter-row Operation

1. Select a primitive root v from table of [8]
2. Construct base sequence $s(j) = (v \times s(j-1)) \bmod p, j = 1, 2, 3, \dots, (p-2)$
3. Construct the sequence $\langle q_i \rangle_{i \in \{0, 1, 2, \dots, R-1\}}$, given that $q_0 = 1$, q_i is the least integer such that $\text{g.c.d.}(q_0, p-1) = 1$, $q_i > 6$, $q_i > q_{i-1}$
4. $r_{T(i)} = q_i$, where the $T(i)$ is the inter-row permutation pattern defined in a table of [8].
5. Algorithm of intra-row operation is as follows:

If ($C = p$) then

$$U_i(j) = s((j \times r_i) \bmod (p-1)), j=0, 1, \dots, (p-2),$$

and $U_{i(p-1)} = 0$

Where U_i is the original bit position of j -th permuted bit of i -th row.

End if

If ($C = p + 1$)

$$U_i(j) = s((j \times r_i) \bmod (p-1)), \text{ where}$$

$$j=0, 1, \dots, (p-2), \text{ and } U_{i(p-1)} = 0, \text{ and } U_i(p) = p$$

If ($K = R \times C$) then

$$\text{Exchange } U_{(R-1)}(p) \text{ with } U_{(R-1)}(0)$$

End if

End if

If ($C = p - 1$) then

$$U_i(j) = s((j \times r_i) \bmod (p-1)) - 1, j=0, 1, \dots, (p-2),$$

End if

6. Perform the inter-row permutation for the rectangular matrix based on the pattern

$$\langle T(i) \rangle_{i \in \{0, 1, 2, \dots, R-1\}}$$

3.3.3 Bits-output From Matrix

The output of the TC internal interleaver is the bit sequence read out column-by-column and pruning the dummy bits.

3.4 Performance of the UMTS TCs

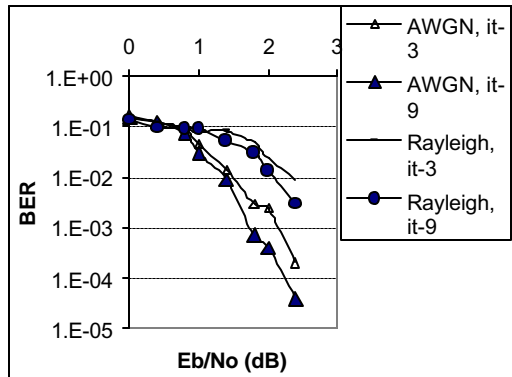


Fig. 7. BER performance of UMTS TCs for different channel (FS-800, rate-1/2)

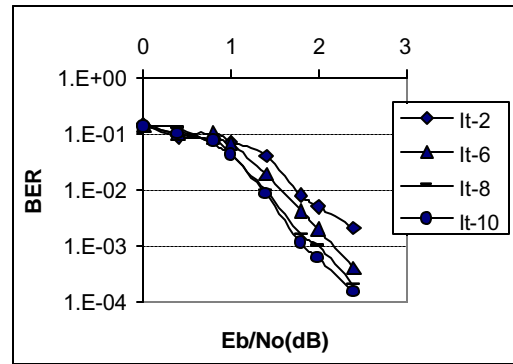


Fig.9 BER performance of UMTS TCs for different decoding iterations (FS-800, rate-1/2, AWGN channel).

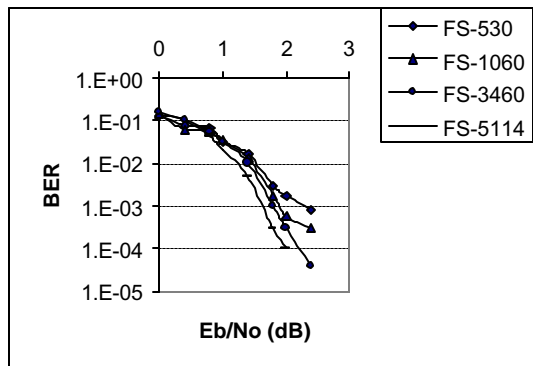


Fig.8 BER performance of UMTS TCs for different frame size (It-3, AWGN, rate-1/2)

UMTS TCs are simulated for Additive White Gaussian Noise (AWGN) channel and Rayleigh channel. In AWGN channel model as only the additive and white noise are considered, UMTS TCs are supposed to correct more errors than in the Rayleigh channel model, which incorporate the multi-path fading and motion of the receiver. Fig.7 shows that at a fixed BER, AWGN channel requires less dB less than that required for the Rayleigh channel. Fig.8 shows the performance of TCs for different frame size. If the frame size is increased from 530 to 5114, at BER of 10⁻³, required Eb/N₀ is reduced from 2.5 dB to 1.5 dB. So increase of interleaver size increases the coding gain. In Fig.9, the effect of number of decoding iterations on the performance of UMTS TCs is shown. At BER level 10⁻³ if the number of iteration is increased from 6 to 10, the required bit energy to noise ratio is reduce from 2.2 dB to 1.7dB.

4 Implementation issues

The migration of wireless networks from voice-centric services to multimedia services poses many technological challenges. First, multimedia services require very high instantaneous throughput in comparison to a voice-only service. Secondly multimedia services require much better quality of services (QoS), which may in turn require high transmission power. Again high throughput requires high spectral efficiency. And to achieve the high spectral efficiency, data transmission requires lower power within an acceptable BER range. This can be achieved by using TC, as it is capable to achieve the Shannon limit power efficiency. Since high-speed data in third generation wireless system are transmitted, large block length can be taken in the Turbo encoder. As in Fig.9 if we increase the frame size of the Turbo encoder, it needs low power to achieve a specific BER.

The performance of TC increases with the increase of interleaver size. Let assume that 64 Kbits interleaver is used and the interleaver in the encoder and decoder must inevitably store these bits for any given time. So there is latency and this in turn implies a delay through the encoder/decoder combination. Thus for an information rate of 8 kbps there is a delay of more than 8s. It is quite unacceptable in a telephone system. The delay limitation in voice service is taken as 40 ms, which corresponds to a latency of 320. But the decrease in latency increases the required dB as shown in Fig. 10. It shows that by decreasing the frame size from 5114 to 530, the required dB increases by 0.8 dB at constant BER. So there should be a trade off between latency and power when it is implemented [19].

5 Conclusions

This paper have assessed the performance of two types of TCs, classical TCs and UMTS TCs. We have shown the performance of classical TCs by varying different components of encoder and decoder in the form of BER vs. E_b/N_0 curve. It is also shown how the encoder structure, interleaving technique and trellis termination technique of 3G TCs differs from classical TCs. This paper focuses on 3G TCs performance. By Matlab simulation it is shown that its performance (BER vs. E_b/N_0) varies if we change the channel model, frame size and number of decoding iteration of UMTS TCs. TCs performs well for AWGN channel model than Rayleigh model. If the interleaver size and the decoding iteration are increased, TCs performance is also improved. Following these performance curves one can find easily the optimum architecture of UMTS TCs for specific application.

References:

- [1] C. E. Shannon, A Mathematical Theory of Communication, *Bell Systems Tech Journal*, Vol. 27, 1948.
- [2] C. Berrou, A. Glavieux and P. Thitimajshima, Near Shannon Limit Error Correcting Coding and Decoding: Turbo-codes, *Proc. of ICC '93*, Geneva, 1993, pp. 1064-1070.
- [3] D. Divsalar, F. Pollara, Low-rate Turbo Codes for Deep-space Communications, *Proc. of IEEE International Symposium on Information Theory*, 1995.
- [4] C. S. Lee, T. Keller and L. Hanzo, Turbo-Coded Digital Video Broadcasting for Mobile Environments, *proceeding of ECMCS'99, Poland*, 1999.
- [5] M. C. Valenti, M. Robert, Improving the QoS of Bluetooth Through Turbo Codes, *Proc. IEEE Military Communication Conference (MILCOM)*, Los Angeles, CA, 2002, pp. 1057-1061.
- [6] D. Divsalar, F. Pollara, Turbo Codes for PCS Application, *ICC'95*, 1995, pp. 54-59.
- [7] A. Burr, Turbo codes: The Ultimate Error Control Codes? *Electronic & Communication Engineering Journal*, Aug. 2001, pp. 155-165.
- [8] A. G. Amat, G. Montorsi, S. Benedetto, D. Vogrig, A. Neviani, A. Gerosa, An analog turbo decoder for the UMTS standard, *International Symposium on Information Theory, ISIT*, 2004, pp. 296.
- [9] F. Marx, J. Farah, Improved turbo-coded UMTS systems with Unequal Error Protection of compressed video sequences transmitted over frequency-selective channels, *International Conference on Communications*, vol. 5, June 2004, pp. 3091 - 3095.
- [10] European Telecommunication Standards Institute (ETSI), Universal Mobile Telecommunication Systems (UMTS): Multiplexing and Channel Coding (FDD), *3GPP TS 125.212 version 3.4.0*, 2000, pp. 14-20
- [11] Third Generation Partnership Project 2 (3Gpp2), Physical Layer Standard for cdma2000 Spread Spectrum Systems, *Release C, 3GPP2 C.S0002-C, version 1.0*, 2002, pp. 115-122.
- [12] Fred Daneshgram, Paolo Mulassano, The Rate Allocation Problem for Turbo Codes, *IEEE transactions on communication*, Vol. 52, No. 6, 2004, pp. 861-865.
- [13] Hamid R. Sadjadpour, Neil J. A. Sloane, Masoud Salehi, Interleaver Design for Turbo Codes, *IEEE journal on selected area in communication*, Vol. 19, No. 5, 2001, pp. 831-837.
- [14] M. Alouini, A. J. Goldsmith, Capacity of Rayleigh Fading Channels Under Different Adaptive Transmission and Diversity-combining Techniques, *IEEE trans. on vehicular technology*, Vol. 48, No. 4, July 1999, pp. 1165-1181.
- [15] Z. Wang, K. K. Pahari, High Performance, High Throughput Turbo/SOVA Decoder Design, *IEEE transactions on communication*, Vol. 51, No. 4, 2003, pp. 570-579
- [16] G. D. Forney, Jr., *Concatenated Codes*, MIT Press, Cambridge, 1966
- [17] S. Benedetto, D. Divsalar, D. Montorsi, F. Pollara, Serially Concatenation of Interleaved Codes: Performance Analysis, Design, and Iterative Decoding, *IEEE Transaction on Information Theory*, Vol. 44, No. 33, 1998, pp. 909-926
- [18] J. Hagenauer, Source-controlled Channel Decoding, *IEEE Transaction on Communication*, Vol. 53, No. 8, 1995, pp. 2449-2457.
- [19] Lin-Nan Lee, A. Roger Hammons, Jr., Feng-Wen Sun, and Mustafa Eroz, Application and Standardization of Turbo Codes in Third-Generation High-Speed Wireless Data Services, *IEEE transactions on vehicular technology*, Vol. 49, No. 6, 2000, pp. 2189-2207.