A New Single-phase Static PFC Inverter Using Pre-calculated Switching Angles

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Abstract – This paper presents a new method for static power factor correction (SPFC). A static system is used to compensate four-quadrant reactive power using only small component (capacitance) values. As such, SPFC is able to deal with inductive and/or capacitive load characteristics.

The proposed SPFC method combines a passive filter and an inverter with pre-calculated pulse-width modulation (PWM) switching angles. The obtained system produces low harmonic distortion thus providing reliable and long lasting system components.

Simulation results show that the system is able to provide variable positive and negative true and reactive powers while keeping the passive system components values constant.

Key-Words: power factor correction, single-phase inverter, static compensator, harmonic distortion, switching angles

1 Introduction

Power factor correction (PFC) has been the focus of attention for many years. Several methods have been developed based on well known circuits (Buck-boost, half-bridge, full-bridge), in an attempt to solve inherent PFC problems as caused by harmonic currents and voltages.

Harmonic distortion impairs actuators and switches as it increases eddy currents, hysteresis losses, and reduces the life time of the machine winding insulators [1, 2]. Pre-calculated pulse width modulation method is used to determine the switching angles to minimize the harmonic distortion [3].

The proposed solutions as in [4, 5, 6, 7], to name but a few, can be divided into two broad classes: dynamic PFC correction scheme through the use of a synchronous wind rotor machine (synchronous condenser), and static PFC compensation scheme consisting of switched banks of very bulky capacitors. Alternative solutions have been proposed as in [8, 9, 10] to enhance performance and/or reduce capacitors size.

It is true that the above methods have brought substantial improvement of the PFC, nevertheless, their main problem is that PFC compensation can only be performed for either inductive or capacitive load while they fail to compensate PFC for the already existing network reactive power. In this paper, we propose a novel static PFC approach that is capable of operating in the four-quadrant true-power (hereafter denoted P) –reactive power (hereafter denoted Q) ‘PQ’ plane as shown in Fig. 1. Consequently, SPFC compensates power factor for any type of reactive power.

The remaining of the paper is organized as follows: system modeling and analysis are presented in section II; section III describes the new SPFC method using pre-calculated switching angles. Simulation results are presented in section IV. At last, a conclusion is given in section V.

Fig. 1 Four-quadrant PQ plane

2 System Modeling and Analysis

Figure 2 shows the basic structure of a single-phase PFC inverter having E as dc voltage, V as ac output voltage. An LC circuit is used to filter the inverter output. The inverter filtered output voltage is taken across the capacitor C (between points a and b). R represents the inductor internal resistance. Tᵢ and T′ᵢ (i=1, 2) are the semiconductor switches.
It can be shown that the circuit of Fig. 2 can be transformed to an equivalent circuit represented in Fig. 3 as seen by the load, where $E_{Th}$ is the Thevenin’s equivalent generator voltage.

From Fig. 3, one can obtain $E_{Th}$ as given by eq. (1).

$$E_{Th} = \frac{V}{1 - LC\omega^2 + jRC\omega}$$

This leads to the transfer function $\bar{T}$ given by (2).

$$\bar{T} = \frac{E_{Th}}{V} = \frac{1}{1 - LC\omega^2 + jRC\omega}$$

It can be shown that the maximum of $T$ ($T_{max}$), is given by (3)

$$T_{max} = \frac{1}{\sqrt{1 - \left(\frac{1}{LC} - \frac{R^2}{2L^2}\right)^2 + \frac{R^2C^2}{LC} \left(\frac{1}{LC} - \frac{R^2}{2L^2}\right)}}$$

that can be reduced to

$$T_{max} = \frac{L}{\sqrt{L^2C^2 - \frac{1}{4}}}$$

From (4), it can be easily shown that $T_{max}$ is always greater than 1. Consequently, the harmonic filter behaves as an amplifying circuit for the fundamental as well. In the next section we describe the novel PFC scheme using pre-calculated switching angles.

### 3 New Static PFC Compensation Using Pre-calculated Switching Angles

#### 3.1 Pre-calculated Switching Angles

In order to achieve single-phase reactive power compensation, a pre-calculated switching angles determination method similar to that developed in [3] has been used.

The objective is to determine directly the switching angles so as to obtain the best possible match between the inverter output voltage $V$ and the desired ac voltage $V_d$.

For this purpose, we propose to compare their respective harmonics. A perfect matching between $V$ and $V_d$ is achieved only when an infinite number of harmonics is considered. Practically, the number of harmonics $N$ that can be identical is finite. This number, to be maximized, depends on the number of switching times per period.

Figure 4 gives the algorithm used to determine the switching angles $\alpha_i$ from the nonlinear set of equations $a_k = d_k$, where $NP$ is the number of parameters (switching angles).

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#### Determine the desired voltage $V_d$ in $[0, T]$

#### Compute the desired voltage harmonics

$$d_k = \frac{1}{\pi} \int_0^\pi V_d(\alpha) \cos(k\alpha) \, d\alpha$$

#### Determine the output voltage harmonics in terms of the switching angles $\alpha_i$

$$\alpha_k = -\frac{4E}{k\pi} \sum_{i=1}^{NP} (-1)^i \sin(k\alpha_i)$$

#### Compute the switching angles $\alpha_i$ from the nonlinear system of equations

$$\alpha_k = d_k \quad 1 \leq k \leq NP$$

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**Fig. 4 Single-phase switching angles determination algorithm**
3.2 Proposed SPFC

Figure 5 shows the load voltage polarity and current flow direction. One can notice that voltage and current are chosen to be of opposite direction.

![Diagram of single phase load](image)

**Fig. 5 Load voltage polarity and current flow direction**

Depending on the sign of the instantaneous power \( p = v_{out} i \), two cases may arise

- Case 1: \( p \) is positive; the power is absorbed by the load, as shown in Fig. 6(a).
- Case 2: \( p \) is negative; the power is delivered by the load as shown in Fig. 6(b).

\[
\begin{align*}
\text{dc supply} & \xrightarrow{\text{inverter}} \text{ac load} \\
\text{p>0} \\
\text{dc supply} & \xleftarrow{\text{rectifier}} \text{ac load} \\
\text{p<0}
\end{align*}
\]

**Fig. 6 Load behavior depending on the instantaneous power sign**

The active and reactive load powers are given by (5).

\[
\begin{align*}
P &= V_{out} I \cos(\phi) \\
Q &= V_{out} I \sin(\phi)
\end{align*}
\]

(5)

where \( \phi \) is the phase shift angle between the load current and voltage, \( \phi = \left( \overline{I}, \overline{V}_{out} \right) \).

From Fig. 5, the generator voltage \( E_{Th} \) can be expressed as

\[
E_{Th} = \overline{V}_{out} + \frac{j \overline{I} - 1}{j C \omega + R + j L \omega} \]

(6)

This is represented under phasor diagram form in Fig. 7 with \( \delta \) being the phase shift angle between \( \overline{V}_{out} \) and \( E_{Th} \).

![E_{Th} phasor diagram](image)

**Fig. 7 \( E_{Th} \) phasor diagram**

Ultimately, (5) can be rewritten as

\[
\begin{align*}
P &= V_{out} \left[ \frac{E_{Th} \cos(\delta) - V_{out} \cos(\alpha)}{X} \right] + \frac{E_{Th} \sin(\delta) \sin(\alpha)}{X} \\
Q &= V_{out} \left[ \frac{E_{Th} \sin(\delta) \cos(\alpha) - \cos(\delta) - V_{out} \sin(\alpha)}{X} \right]
\end{align*}
\]

(9)

It is worth to remind that our main objective is to perform power factor correction meaning that the delivered power to the load must be zero.

\( P_{\text{delivered}} = 0 \), and \( Q_{\text{delivered}} + Q_{\text{load}} = 0 \).

Consequently, (9) yields

\[
\begin{align*}
E_{Th} &= V_{out} \frac{\cos(\alpha)}{\cos(\delta - \alpha)} \\
Q &= V_{out} \frac{\sin(\alpha)}{\sin(\alpha_2)} \frac{R C \left[ \cos(\alpha) \sin(\delta - \alpha) + \sin(\alpha) \right]}{L}
\end{align*}
\]

(10)
\[
\begin{align*}
E_{Th_{ref}} &= \frac{V_{out}}{R} \cos(\delta) \\
Q &= V_{out}^2 \frac{RC}{L} \tan(\delta_{ref}) = -V_{out} I_l \sin(\phi_l)
\end{align*}
\]  
(11)

where \(\phi_l = (I_l, V_{out})\) and \(I_l\) being the load current as illustrated in Fig. 8.

Equation 11 shows that the delivered reactive power \(Q\) depends on both the capacitor \(C\) and the phase shift angle \(\delta_{ref} = (V_{out}, E_{Th})\). By varying the \(\delta_{ref}\) from \(-\frac{\pi}{2}\) to \(\frac{\pi}{2}\), the reactive power can be varied in the interval \([-\pi, \pi]\). This allows our SPFC scheme to compensate large reactive power using only small capacitance values. However, in the neighborhood of \(\pm \frac{\pi}{2}\), the amplitude of the generator voltage will increase.

Rearranging (11), we obtain the following expressions for the reference generator voltage and phase shift angle \(\delta_{ref} = (V_{out}, E_{Th})\).

\[
\begin{align*}
E_{Th_{ref}} &= V_{out} \sqrt{1 + \left(\frac{I_l \sin(\phi_l)}{V_{out} \frac{RC}{L}}\right)^2} \\
\delta_{ref} &= -\arctg\left(\frac{I_l \sin(\phi_l)}{V_{out} \frac{RC}{L}}\right)
\end{align*}
\]  
(12)

4 Simulation Results

Simulation is carried out using the parameters given in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Compensated reactive power</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E_{Th}=1000 V_{out})</td>
<td>(E_{Th}=1000 V_{out})</td>
</tr>
<tr>
<td>Conventional PFC</td>
<td>1.6 KVAR</td>
</tr>
<tr>
<td>Proposed SPFC</td>
<td>529 KVAR</td>
</tr>
</tbody>
</table>

Taking \(E_{Th} = 1000 V_{out}\) and \(\delta = 1.56\) radians, the obtained results show that, SPFC is able to compensate a reactive power of 529 KVAR. For the sake of comparison, a conventional PFC scheme will be able to compensate only 1.6 KVAR as illustrated in Table 2.

Figure 9 shows the reference voltage \(E_{Th}\) variation with respect to load current \(I_l\) and load phase angle \(\phi_l\). One can notice that \(E_{Th}\) increases when \(\phi_l\) approaches \(\pm \frac{\pi}{2}\). This increase is the cost to keep the value of capacitor \(C\) constant.

Figure 10 shows the phase shift angle \(\delta_{ref} = (V_{out}, E_{Th})\) with respect to load phase angle.
\( \phi \) and load current \((I_l)\) variation. \( \delta_{\text{ref}} \) is proportional to \( \phi \) and \( I_l \).

Fig. 10 \( \delta_{\text{ref}} \) with respect to load current variation

Figures 11 and 12 show other 3-D views of the variations of the reference voltage \( E_{Th} \) and phase shift angle \( \delta_{\text{ref}} \) respectively using another set of simulation parameters given in Table 3.

Table 3 Other simulation parameters

<table>
<thead>
<tr>
<th>R</th>
<th>L</th>
<th>C</th>
<th>( V_{\text{out}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1 ( \Omega )</td>
<td>100 ( \mu )H</td>
<td>200 ( \mu )F</td>
<td>230 V</td>
</tr>
</tbody>
</table>

Fig. 11 Reference voltage–load phase variation

Fig. 12 \( \delta_{\text{ref}} \) with respect to load phase variation

5 Conclusion

A new static PFC method based on the pre-calculated switching angles approach has been presented. The switching angles are pre-calculated by resolving a nonlinear system of equations. Compared with other methods, the proposed SPFC method is able to balance four-quadrant reactive power (load independent) using small component (mainly capacitance) values, and provides more reliable and long lasting system components due to low harmonic distortion. It can represent a cost effective method for implementing four-quadrant PFC compensation for power systems.

References:


