New Approach of Parametric Spectral Analysis Used in the Diagnosis of Induction Motors
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Abstract: Today, in response to the industrial requirements, the diagnosis and monitoring of the electric motors, turning at constant and/or variable speed, are much requested. Several works show that the faults were often studied by the analysis of the supply current. Many schemes have been proposed, all based on the Fourier analysis. However, this analysis is badly adapted to these applications. Indeed, the signals used are strongly no stationary. New tools are thus necessary. In this article, a technique for the faults detection in the induction motors is proposed. It’s based on spectral estimate using the least squares approach. The stator current is a harmonic signal. It is thus necessary to know the various amplitudes, frequencies and phases of its spectral components.

Key words: diagnosis, induction motor, spectral analysis, faults detection, least squares.

1. Introduction
In this paper, a new approach of parametric spectral analysis is proposed. This technique is used for the faults detection in the induction motors. The fault detection in these motors is based on the analysis of the stator currents. The analysis of these current uses the method of least squares. This, leads to the extraction of parameters vector characterizing the behavior of the machine.
The purpose of this spectral analysis is to identify the various harmonics present in the stator currents. The identification is not limited to the only frequencies of these harmonics but also their respective amplitudes and their phases. To determine the amplitudes of various harmonics leads to solve a system of linear equations. But for the frequencies and especially the phases the equations to solve will be nonlinear.
The goal of the approach proposed is the fault detection of a bar broken of motor rotor. Indeed, one or more broken bars of the rotor have a direct influence on the evolution of the stator currents.

2. Problem position
Digital signal processing involves techniques that improve our understanding of information contained in recorded statoric currents. Normally, these signals are measured and observed in the time domain. When the harmonics contained in these signals are of interest, it’s necessary to study it in the frequency domain (spectral analysis). The spectral analysis of the statoric currents can be used as a processing technique to permit, then, fault detection, as example: bar Broken in rotors. Generally, the fault detection in the induction motors is done directly in the temporal field (observation of the signal evolution) or by a classic spectral analysis based on the FFT algorithm. However, the no stationary character of this type of signals and also the possible presence of very close frequencies make unusable the techniques of spectral analysis based on the FFT.
The purpose of this study is to propose a new approach of the spectral analysis able to estimate the frequencies, the amplitudes and the phases of all harmonics present in the statoric currents. The statoric currents are supposed to be formed by a sum of sinuosoids or generally sum exponential complexes. Certain frequencies can be revealing presence of one or more faults. To detect and then isolate these frequencies can lead to a fine and precise diagnosis.

3. Problem solution
The signal to be analyzed can take the following complex form:
\[ s(n) = \sum_{k=1}^{M} A_k e^{j2\pi f_k n} \] (1)

Where \( \{A_k\} \) indicate a succession of M complex amplitudes and \( \{f_k\} \) a succession of M different closed frequencies [1]. The signal is sampled at \( T_e=1/Fe \). M is supposed known a priori. The goal is to estimate the values of \( A_k \), \( f_k \) and the variance of additive noise.
This is carried out starting from the observation: \( x(t) \), recorded on N samples ( \( N>M \))
\[ x(t)=s(t)+b(t) \] (2)

This measurement noise is supposed: Gaussian, centred and white. The adopted approach must to estimate, at the same time, the values of \( A_k \) and \( f_k \), which
minimize the Mean Square Error (MSE) [2], between the observed values : $x(n)$ and the desired values : $s(n)$.

$$J(a_1, \ldots, a_M, f_1, \ldots, f_M) = \sum_{n=0}^{N-1} \left( \sum_{l=1}^{M} a_l e^{j2\pi f_l n/F_e} \right)^2$$

This expression is linear according to amplitudes’ $\{a_k\}$, whereas it is non-linear according to frequencies’ $\{f_k\}$. Thus, the minimization of the MSE starts with, in first, by the calculus of amplitudes: $a = [a_1, \ldots, a_M]^T$.

With:

$$E_{f_1, \ldots, f_M} = \begin{bmatrix}
\frac{1}{\sqrt{Fe}} & 1 & 1 & \cdots & 1 \\
\frac{e^{j2\pi f_1 n}}{\sqrt{Fe}} & e^{j2\pi f_2 n} & e^{j2\pi f_3 n} & \cdots & e^{j2\pi f_M n} \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
\frac{e^{j2\pi f_1 (N-1)}}{\sqrt{Fe}} & e^{j2\pi f_2 (N-1)} & e^{j2\pi f_3 (N-1)} & \cdots & e^{j2\pi f_M (N-1)}
\end{bmatrix}$$

and $x = [x(0), \ldots, x(N-1)]^T$.

What gives then:

$$J(a_1, \ldots, a_M, f_1, \ldots, f_M) = |x - E(f_k)\hat{a}|^2$$

The expression of $\hat{a}$, corresponding at minimum, is:

$$\hat{a} = (E^H E)^{-1} E^H x$$

And this minimum is:

$$\text{Min of } J = x^H x - x^H E (E^H E)^{-1} E^H x$$

The M frequencies $\{f_k\}$ are supposed different. Thus, the “vectors columns” of $E$ are independent. Therefore square matrix $E^H E$ is invertible.

As for the estimate of the frequencies, $f_k$ they must be calculated of kind to minimize the expression of the MSE [3, 4].

The first term of the MSE is independent of $f_k$. This, it’s necessary to maximize its second term:

$$\Gamma(f_1, \ldots, f_M) = x^H E (E^H E)^{-1} E^H x$$

The method proposed calculates the $M$ maximums of the pseudo spectrum or of the periodogram, expressed by the spectral distribution: $\hat{a}(f_1, \ldots, f_M)$.

If $M=1$, the matrix $E$ becomes a column vector and $E^H E = N$ is the number of recorded samples.

The estimated frequency $\hat{f}_k$ maximizes the Criterion:

$$\Gamma(f_k) = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi f_k n/F_e}$$

To estimate, within the meaning of least squares, the frequential characteristics of a sinusoid with a white additive noise, it’s necessary to evaluate its periodogram and to locate the frequency of its maximum [5]. Its amplitude is then the surface under the peaks and the phase is the resulting angle. Therefore, the problem of multivariable maximization became a problem of monovariable maximization.

4. Results and discussion

The proposed method is now applied to the stator currents to detect possible faults. The machine used has the following characteristics:

- **Power** = 3kW
- Rotors with 28 bars
- Rotate speed $\Omega = 2823.5$ rpm.

But the measured characteristics of this motor are:

- $I_{ch} = 5.9$ A (rms value),
- Rotate speed $\Omega = 2823.5$ rpm.

The acquisition parameters used in the experiment are:

$$F_e = 10kHz \ ; \ F_{max} = 100Hz \ ; \ LF = 400 \ ; \ \Delta F = (2.5) \times 10^{-5} ;$$
$$F_b = 5Hz \ ; \ M = 2 \ \text{raies} \ ; \ d = 19 \ \text{points} \ ; \ L = 8 \ \text{points} .$$

With the terminology techniques used

- $F_e$ : The sampling rate
- $F_{max}$: the maximum frequency
- $LF$ : is the number of frequencies belonging to the range $[0, F_{max}]$
- $\Delta F = \frac{F_{max}}{LF}$ (Hz) : frequency resolution
- $F_b$(Hz) : represents the instantaneous frequency deviation between two close frequencies
- $F_q = \left[ 0 : \frac{\Delta F}{F_e} \right]$ (Hz) : the instantaneous deviation of frequency
- $M$: the number of spectral components of the observed signal
- $d = \frac{F_b}{F_e \Delta F} - 1$ : distance between two adjacent peaks
- $L = 2 \frac{LF}{F_{max}}$ : The width of the principal lobe for each peak
- $A = \left| E^H x \right|$ : pseudo spectrum evaluated
- $\hat{a}_k = 2 \frac{1}{L} \sum_{l=1}^{L} A(k + l)$ : The surface under each peak
- $\phi_k = \text{arg}(A)$ : The estimated phase
- $\hat{x}(n) = \sum_{k=1}^{M} \hat{a}_k e^{j2\pi f_k n + \phi_k}$ : The restored signal
- $e(n) = x(n) - \hat{x}(n)$ : The error of reconstructing
4.1 Motor without faults.
The results of the estimate, obtained by the proposed approach, are:

<table>
<thead>
<tr>
<th>( \hat{a}_k ) [A]</th>
<th>( \hat{f}_k ) [Hz]</th>
<th>( \hat{\phi}_k ) [rad]</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.6028</td>
<td>50.0000</td>
<td>0.0568</td>
</tr>
<tr>
<td>0.0066</td>
<td>44.5000</td>
<td>1.9941</td>
</tr>
</tbody>
</table>

Table 1: the experiment results for motor without faults

The estimated variance of noise: \( \hat{\sigma}^2 = 0.029878 \)

It is noted that the maximum amplitude corresponds to an additive noise whose variance is estimated (figure 1).

The spectrum of the error signal (figure 4) shows that an electric pollution was reinjected in the power supply network. It results in several fluctuations with frequencies that are multiples of 50Hz and odd ranks (figures 3 and 4).

4.2 Motor with faults.
The method proposed was used to analyze the current of the machine, previously described, with a broken bar in rotor (figure 5).

The nominal current is \( I_{ch} = 5.9A \) (rms value), \( \Omega_r = 2835.1 \) rpm and \( M = 5 \) (harmonics). The results are represented on the following table.

<table>
<thead>
<tr>
<th>( \hat{I}_{ch} )</th>
<th>( \hat{\Omega}_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{8.0667}{\sqrt{2}} )</td>
<td>( \frac{50 - 44.5}{50} ) (3000) = 2835 (tr / mn)</td>
</tr>
</tbody>
</table>

Thus, the stator current [6, 7, 8], of the machine without faults, is made up primarily of two sinusoids drowned in an additive noise whose variance is estimated (figure 1).

The spectrum of the error signal (figure 4) shows that an electric pollution was reinjected in the power supply network. It results in several fluctuations with frequencies that are multiples of 50Hz and odd ranks (figures 3 and 4).

The variance estimated of noise is: \( \hat{\sigma}^2 = 0.037906 \)

In the spectra (figures 7 and 8) obtained and more particularly that of the error, the presence of the frequencies of odd ranks can be the best revealing of defaults [6, 7, 8, 9, 10].

5. Conclusion:
The fault detection in the electric motors generally uses a spectral analysis, based on the FFT. But this analysis is conditioned by the stationnarity of the signal. In this paper the identification of the various frequencies, but also of their respective amplitudes and their respective phases, present in a signal was developed. It is based on the principle of least squares. From this approach it is possible to detect, with certainty, very close frequencies. This makes possible the recognition of faults having generated such frequencies.

This method was tested on simulated signals in order to evaluate its performances. Then it was used successfully to detect faults of broken bar in the rotor of an induction motor.

References
Figure 1: currents of an electric motor without faults.
Figure 2: reconstructing error.
Figure 3: The amplitude spectrum, in dB, of the current.
Figure 4: The amplitude spectrum, in Ampere, of the error.
Figure 5: currents of an electric motor with faults (broken bar in rotor)
Figure 6: reconstructing error

Figure 7: The amplitude spectrum, in dB, of the current
Figure 8: The amplitude spectrum, in Ampere, of the error