A Robust Spatio-Temporal Processing for Multipath Environments with an Antenna Array

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Abstract: In this paper, a novel algorithm of spatial and temporal equalization for multiuser detection with an antenna array is demonstrated. Our proposed approach enables us to reduce a high-level noise from noisy array signals with irrelevant signal measurements. Our robust approach includes three procedures. In the first procedure, a robust subspace technique is utilized to reduce a high-power of additive noise and a correlation among channels. In this procedure, we propose a scale-free method of cochannel interference reduction. Moreover, a cross-validation technique is applied to estimate the number of signals. In the second procedure, the independent component analysis (ICA) is applied to decompose independent components. In the third procedure, a self-adaptive multipath identification is presented. A self-adaptive algorithm is a practical technique for estimating the changing multipath in order to identify the dynamical channel. Applying our proposed approach to the experimental data, we demonstrate the effectiveness of our method.

Key-Words: Spatial and temporal equalization, blind equalization, multiuser detection, principal-factor method, scale-free algorithm, on-line adaptive algorithms, independent component analysis

1. Introduction

In the field of the wireless communication, the distortion of source signals including the cochannel interference, the intersymbol interference, and additive noises is one of the main problems. In order to reduce these interferences, various techniques on the spatial and temporal methods of signal processing associated with standard algorithms such as LMS, CMA, MUSIC, and neural network approach have been applied. Moreover, a combination of the spatial and temporal equalization is studied for upgrading the performances in the wireless communication [1]-[4].

Recently, some researchers have applied the blind source separation (BSS) approach for mobile communications environment [5]-[9]. When applying the BSS approach, not only the additive noise but also the difference of scale or power of each observation may seriously affect the estimation of sources. Since the scales of measurement in the variables are sometimes arbitrary or irrelevant, it is important to apply an appropriate approach, which yields a scale-free solution. This means that the solution does not depend on the scale of observations.

In this paper, a novel algorithm for multiuser detection with an antenna array is proposed. Our proposed approach enables us to reduce a high-level noise from noisy array signals with irrelevant signal measurements.

Our proposed approach includes following three procedures.

At first step, a subspace method including reduction of cochannel interference and additive noises is proposed. In this step, the principal-factor method (PFM) [10], [11] is proposed to reduce high-level noises. Moreover, to reduce the cochannel interference of the noisy data with irrelevant signal measurements, the scale-free principal-factor method [12] is proposed. The scale-free method (SFM) is an efficient technique for reducing the cochannel interference of noisy data in case the units of measurement are arbitrary or irrelevant. Furthermore, a novel criterion using cross-validation technique is proposed to determine the number of signals. By applying the MUSIC, PFM, and SFM to the experimental data, we demonstrate the effectiveness of our proposed method.

At second procedure, the independent component analysis (ICA) is applied to decompose independent components. In this study, the JADE algorithm [13] and the Fast-ICA algorithm [14] are applied.

At third procedure, considering a practical case when the coefficients of multipath are changing in time, a self-adaptive algorithm in which the step-size is updated automatically is developed [15]-[17]. This technique can be adopted to identify the time-varying channel either blindly or nonblindly.
2. Subspace Method for Spatial Filtering

In this section, a novel subspace method for reducing the cochannel interference with a high-level noise reduction is presented. The basic problem of array signal estimation can be defined as

\[ y(t) = As(t) + \varepsilon(t), \quad y, \varepsilon \in \mathbb{R}^m, s \in \mathbb{R}^n, A \in \mathbb{R}^{m \times n}, \]  

where \( y(t) \) represents the transpose of \( m \) channel observations at time \( t \). Each observation \( y_i \) contains several common sources in vector \( s(t) \) and a unique noise in vector \( \varepsilon(t) \). The number of observations \( m \) is larger than that of signals \( n \). The column of \( A \) represents the array steering.

Let us rewrite Eq. (1) in a data matrix form as

\[ Y_{(m \times N)} = A_{(m \times n)}S_{(n \times N)} + \Xi_{(m \times N)}, \]  

where \( N \) denotes data samples. When the sample size \( N \) is sufficiently large, the covariance matrix of the observed data in the mixing model \( \Sigma \) can be written as

\[ \Sigma = AA^T + \Psi, \quad \Psi = \Xi \Xi^T / N. \]  

For convenience, we assume that \( \Psi \) has been divided by \( \sqrt{N} \) so that the covariance matrix of the observation recorded by sensors can be given by \( C = YY^T \).

2.1 Reduction of Cochannel Interference and Additive Noises

In this subsection, we describe the principal-factor method (PFM), which is extended the principal component analysis (PCA) to reduce cochannel interference with high-level noise reduction. In the procedure of the PFM, we fit \( AA^T \) to \( C - \Psi \) using the eigenvalue decomposition (EVD) approach. A reasonable criterion for fitting the model to the data would be to minimize \( L(A, \Psi) = \text{tr}[AA^T - (C - \Psi)]^2 \). That is, the columns of \( A \) are calculated by eigenvectors of \( C - \Psi \) corresponding to the \( n \) largest eigenvalues. The estimate \( \hat{A} \) can be obtained as

\[ \hat{A} = U_\hat{h} \Lambda_\hat{h}^{1/2}, \]  

where \( \Lambda_\hat{h} \) is a diagonal matrix whose elements are eigenvalues of \( C - \Psi \), the columns of \( U_\hat{h} \) are the corresponding eigenvectors and \( \hat{h} \) is the estimated number of sources.

The act of selecting \( \Psi \) is equivalent to choosing the communalities, as the diagonal elements of \( C - \Psi \) are estimates of the communalities. The most widely used method has been that of choosing the communality of each variable in relation to the squared multiple correlation coefficient (SMC) of the variable with all other variables [11]. This can be shown to amount to choosing

\[ \hat{\Psi} = (\text{diag}(C^{-1}))^{-1}. \]  

With this choice of \( \Psi \), one can then proceed to estimate \( A \). It has been suggested that this method can be improved upon by iteration in the following way. Once the estimate \( A \) has been determined, we can select a new, and hopefully better, estimate of noise variance \( \Psi \) as

\[ \hat{\Psi} = \text{diag}(C - \hat{A}\hat{A}^T). \]  

It should be noted that both matrices \( A \) and \( \Psi \) must be estimated together from the data. \( A \) is estimated by using the PCA. \( \Psi \) is estimated by using the so-called unweighted least-squares (ULS) method which is one of the estimation methods used in the factor analysis.

Once the estimates \( \hat{A} \) and \( \hat{\Psi} \) converge to stable values, we need to finally compute the score matrix, the pseudo-inverse matrix. Since the solution for a pseudo-inverse matrix is not unique, we employ the Bartlett method which is an unbiased model and the noise variance \( \Psi \) is taken into the calculation, that is

\[ Q = (\hat{A}^T \hat{\Psi}^{-1} \hat{A})^{-1} \hat{A}^T \hat{\Psi}^{-1}, \quad Q \in \mathbb{R}^{n \times m}. \]  

Using the above result, the new set of data transformed from the observations can be obtained by \( f = Qy \). Note that the covariance matrix is \( E\{ff^T\} = I_n + Q\Psi Q^T \), which implies that the source signals in a subspace are de-correlated.

2.2 Scale-Free Principal-Factor Method

The PFM does not yield a scale-free solution, which means that the solution depends on the scale of covariance matrices. This is one of the difficulties of the estimation in this approach. The scale-free method (SFM) [12] is based on the fact that, if \( C \) is analyzed to give \( A \) and \( \Psi \), then an analysis of \( DCD \) also gives estimates \( DA \) and \( D^2\Psi \). Here, \( D \) is a diagonal matrix of scale factors. Since the scales or figures of measurement in the variables are sometimes arbitrary or irrelevant, this is a desirable property in factor analysis.

The SFM of estimating \( A \) and \( \Psi \) is developed as follows. In case \( \Psi \) is known, we have approximately that

\[ C - \Psi \approx AA^T. \]  

Pre- and post-multiplying this by \( \Psi^{-1/2} \),

\[ \Psi^{-1/2}C\Psi^{-1/2} - I \approx \Psi^{-1/2}AA^T\Psi^{-1/2} \]  

is obtained. Therefore, with \( \Psi \) known, we can compute \( \Psi^{-1/2}C\Psi^{-1/2} - I \) and fit \( \Lambda^* = \Psi^{-1/2} \hat{A}^\ast \) to this, where \( \Lambda^* = \Psi^{-1/2}A^\ast \). The estimate \( \hat{A}^\ast \) is obtained from the eigenvalues and eigenvectors of \( \Psi^{-1/2}CP\Psi^{-1/2} - I \), and then \( \hat{A}^\ast \) is computed as \( \Psi^{1/2}\hat{A}^\ast \). With the choice of \( \Psi \) as in Eq. (5), this amounts to computing the eigenvalues and eigenvectors of

\[ (\text{diag}(C^{-1}))^{1/2}C(\text{diag}(C^{-1}))^{1/2} - I. \]  

This matrix can be proved to be invariant under transformations of scale in the variables.

In case the estimate \( \Psi \) is systematically too large, Eq. (5) can be replaced by

\[ \hat{\Psi} = \theta(\text{diag}(C^{-1}))^{-1}. \]
where \( \theta \leq 1 \) is an unknown scalar, estimated from the data. Let \( \lambda_1, \ldots, \lambda_p \) be the eigenvalues of
\[
C^* = (\text{diag} C^{-1})^{1/2} C (\text{diag} C^{-1})^{1/2},
\]
then the least-squares estimate of \( \theta \) becomes
\[
\hat{\theta} = \frac{1}{p-k} \sum_{m=k+1}^{p} \lambda_m,
\]
that is, the average of the \( p-k \) smallest eigenvalues of \( C^* \). Furthermore, let \( U_k \) be the matrix of order \( p \) by \( k \), the columns of which are the orthonormal eigenvectors of \( C^* \), corresponding to the \( k \) largest eigenvalues, and let \( \Lambda_k \) be the diagonal matrix of these eigenvalues. Then the least-squares estimate of \( A \) is given by
\[
\hat{A} = (\text{diag} C^{-1})^{-1/2} U_k (\Lambda_k - \hat{\theta} I)^{1/2}.
\]

This amounts to first scaling the \( j \)-th columns of \( U_k \) by \( (\lambda_j - \hat{\theta})^{1/2} \), so that the sum of squares of the \( j \)-th column equals \( \lambda_j - \hat{\theta} \), and then scaling the \( i \)-th row by \( 1/\sqrt{\lambda_i} \), where \( \sqrt{\lambda_i} \) is the \( i \)-th diagonal element of \( C^{-1} \).

Some scale-free algorithms for factor analysis like generalized least squares (GLS) and maximum-likelihood (ML) method have been proposed. GLS minimizes \( \text{tr}(C^{-1} \Sigma - I) \) and ML minimizes \( \log |\Sigma| + \text{tr}(C^{-1} \Sigma^{-1}) + m \log 2 \pi \), where \( \Sigma = AA^T + \Psi \) and \( m \) is the number of observations. In these methods, the multivariate normal distribution must be assumed.

### 2.3 Detection of the number of signals

Detection of the number of sources impinging on the array is a key step in most of the superresolution DOA estimation techniques. Akaike information criteria (AIC) and Minimum Descriptive Length (MDL) criteria are applied to determine the number of sources [4]. This paper presents a criteria based on the cross-validatory technique.

The cross-validatory techniques have been wildly applied in multivariate statistics. It usually divides the data into two groups, and uses one group to determine some characteristics of the data, and then uses the other groups to verify the characteristics. Extending this concept, we propose a criteria for determining the estimation of the source number \( \hat{n} \) by using the error of estimating the noise variance.

Let us first divides the data matrix \( Y \) into several disjoint groups such as \( Y_i \in \mathbb{R}^{N \times K} \), where \( N \) is data samples and the group number \( i = 1, \ldots, K \). Next, we use each group data to compute one estimate of the noise variance \( \text{diag}(\hat{\Psi}_i) \) and use remaining data to compute another estimate of the noise variance \( \text{diag}(\hat{\Psi}_j) \) where \( j \neq i \). In general, when the estimate of source number \( \hat{n} \) has not been matched to its true value, a larger error will arise between the noise variance and its estimate. Based on this property, we define the criteria for each conjectured source number \( \hat{n} \) as,
\[
\text{Error}(\hat{n}) = \frac{1}{K} \sum_{i=1}^{K} \text{tr}[\text{diag}(\hat{\Psi}_i^{(\hat{n})}) - \text{diag}(\hat{\Psi}_j^{(\hat{n})})]^2.
\]

It should be noted that we are not necessary to compute all of the estimates of the source number such as from \( \hat{n} = 1 \) to \( \hat{n} = m \) (\( m \) denotes the number of sensors) when applying a sufficient condition as \( n \leq \frac{1}{2}(2m+1 - \sqrt{8m+1}) \). Under this condition, we know that the result for determining the number of sources is reliable.

### 3. Independent Component Analysis

It should be noted that the decorrelation procedure is needed to reduce the power of additive noises and the number of parameters, but it is insufficient to obtain the independent components since an orthogonal matrix in general contains additional degrees of freedom. Therefore, the remaining parameters must be further estimated by using an ICA algorithm. After the decorrelation and ICA, the decomposed independent sources \( z \in \mathbb{R}^n \) can be obtained from a linear transformation as
\[
z(t) = Wf(t) = WQy(t)
\]
where \( W \in \mathbb{R}^{n \times n} \) is termed the demixing matrix which can be computed by using an ICA algorithm. In this study, we applied the JADE algorithm [13] and the Fast-ICA algorithm [14].

#### 3.1 JADE algorithm

The joint approximate diagonalization of eigenmatrices (JADE) has been proposed in [13]. The JADE algorithm has two procedures termed orthogonalization in the PCA and rotation. We apply the rotation procedure in the JADE algorithm, described below, but instead of the orthogonalization in the PCA, we apply the PFM and the scale-free version of PFM described in Section 2.1 and 2.2.

The rotation procedure in the JADE uses matrices \( F(M) \) formulated by a fourth-order cumulant tensor of the outputs with an arbitrary matrix \( M \) as
\[
F(M) = \sum_{k=1}^{K} \sum_{l=1}^{L} \text{Cum}(f_i, f_j, f_k, f_l)m_{lk}
\]
where \( \text{Cum}(\cdot) \) denotes a standard cumulant and \( m_{lk} \) is the \( (l,k) \)-th element of matrix \( M \). The correct rotation matrix \( W \) can be obtained by diagonalizing the matrix \( F(M) \), namely, \( WF(M)W^T \) approximates a diagonal matrix.

#### 3.2 Fast-ICA algorithm

The Fast-ICA algorithm has been proposed in [14]. This algorithm is based on a fixed-point method and is represented by
\[
w^+ = w(t) - \eta \frac{E[fg(w(t)^T f)] - \beta w(t)}{E[|g[w(t)^T f]| - \beta]} \quad \beta \quad (18)
\]
\[
w(t+1) = \frac{w^+}{\|w^+\|} \quad (19)
\]
where \( g(z) = z^3 \) or \( g(z) = \tanh(z) \).

It should be noted that the fixed-point algorithm requires a preliminary sphering of the data. In this study, instead of sphering in the PCA, we apply the PFM and the scale-free version of PFM described in Section 2.1 and 2.2.

4. Self-Adaptive Algorithm for Multipath Estimation

By applying the subspace method of spatial filtering to the array signals, the cochannel interference and the power of noises have been reduced. Since the decomposed signals still contain convoluted components, the remaining parameters have to be further estimated by applying a temporal filtering technique [15]-[17]. In this section, a self-adaptive algorithm for estimating the changing multipath is presented. The task of the temporal equalization is to estimate the parameters of the multipath such that recovering the original transmitted signals.

In case an arbitrary decomposed signal \( z_i(t) \) is given, the model of multipath identification can be represented as

\[
z_i(t) = \sum_{l=-\infty}^{\infty} h_l(t) x_i(t - l), \quad (20)
\]

where \( h(t) = [\cdots, h_1(t), \cdots] \) are coefficients of multipath which are changing in time. In order to estimate these coefficients, we employ an SISO equalizer expressed by

\[
\hat{x}_i(t) = \sum_{l=-\infty}^{\infty} w_l(t) z_i(t - l), \quad (21)
\]

where \( w(t) = [\cdots, w_l(t), \cdots] \) are estimates of multipath. In the \( z \)-plane, the synthesized system is given as \( G(z) = W(z)H(z) \), where \( H(z) = \sum_{l=-\infty}^{\infty} h_l z^{-l} \) and \( W(z) = \sum_{l=-\infty}^{\infty} w_l z^{-l} \) are the unknown system and the equalizer, respectively.

When the transfer function of the synthesized system converges to \( cz^{-\Delta} \), the channel equalization is achieved. It is noted that \( \hat{x}_i(t) = cz(t - \Delta) \), where \( c \) and \( \Delta \) are arbitrary scaling and time delay factors, respectively. Using the training sequence, the on-line LMS algorithm can be applied for the unknown channel estimation as

\[
w(t + 1) = w(t) + \eta(t) z_i(t) e(t), \quad (22)
\]

where \( w = [w_0, \cdots, w_p] \), \( z_i(t) = [z_1(t), \cdots, z_{p+1}(t)] \) and \( \rho \) is the order of an FIR filter. The error signal is \( e(t) = d(t) - \hat{x}(t) \), where \( d(t) \) can be obtained from pilot signal. The step size \( \eta(t) > 0 \) is self-adaptive for catching the time-vary channel.

The step size self-adaptive algorithm for the time-varying channel is presented by a combination of the linear low-pass and nonlinear low-pass filtering technique [16] as

\[
\Delta \eta(t) = -\alpha \eta^2(t) + \alpha \beta \eta(t)|v(t + 1)|, \quad (23)
\]

\[
\Delta v(t) = \rho g(t + 1) - v(t), \quad (24)
\]

\[
g(t) = x(t) \text{sgn}[e(t)] |e(t)|. \quad (25)
\]

If the gradient term \( g(t) \) or the local average of its mean value is not zero, the corresponding step size \( \eta \) will increase to a specified value determined by the local mean value of the gradient component and the parameter \( \beta \). If the mean value of the gradient component is zero, or it oscillates around zero, then the step size parameter will decrease optimally to zero according to \( \eta(t) \sim c/t \).
This implies that the system is with stability in parameter estimation. When some parameters change in time then the absolute mean value will increase. This increase causes the step size parameter automatically and rapidly increase to adapt the new environment.

5. Computer Simulation

5.1 Results of Source Detection

In order to demonstrate the effectiveness of our proposed approach, we show the computer simulation on the problem of wireless communication. In this simulation, we used five channel signals obtained by two i.i.d. binary signals with $|s(t)| = 1$ (Fig. 1(a)). In order to compare the power of source to that of noise, the SNR was defined as

$$\text{SNR} = 10 \log \frac{E[s^2]}{E[\varepsilon^2]}$$  \hspace{1cm} (26)

The noisy array signals, in cases the SNR = $-16.0346$ dB at receiver $y_4$, are shown in Fig. 1(b).

The source number was estimated by using Eq. (15). The results of source number estimation in case the SNR = $-6.4922$, $-16.0346$ and $-21.2634$ dB at receiver $y_4$ are shown in Fig. 2. As seen from Fig. 2, when the estimated source number was $\hat{n} = 2$, we had the smallest error.

The results of recovered codes, in case applying the SFM with ICA algorithm to the array signals shown in Fig. 1(b), are shown in Fig. 1(c). This result indicates that the sources were accurately estimated. In order to evaluate the results of source estimations, we defined the error of source estimation (error ratio) as

$$\text{Error} = \frac{1}{2N} \sum_{i=1}^{N} |s(t) - \hat{s}(t)|,$$  \hspace{1cm} (27)

where $\hat{s}(t)$ denotes the estimated source and $N$ is the number of samples. The error ratios in case applying the MUSIC, PFM and SFM with ICA (JADE and FastICA) are shown in Fig. 3. This results indicate that the smallest value of the error was obtained in case applying the SFM. Given these results, we confirm that the SFM is the robust approach for the high-level noises reduction.

5.2 Estimation of Changing Multipath

In order to confirm the validity and the performance of developed self-adaptive algorithm, we demonstrate the simulation of time-varying channel identification.

In this simulation, an arbitrary decomposed signal $z_i(t)$ ($t = 1, \cdots, N$) with samples $N = 12000$ was given and the impulse responses $h(T)$ were switched for each duration as shown in Table 1. Initial conditions are also shown in Table 1. The parameters of the impulses responses and the results of estimated inverse system $w(T)$ for each time period are shown in Fig. 4. In order to evaluate the performance of channel equalization, the formula as $w(T) * h(T)$ was employed, where $*$ is a convolution operator (see Fig. 4). These results indi-

<table>
<thead>
<tr>
<th>condition</th>
<th>sample</th>
<th>$h(T)$</th>
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<tbody>
<tr>
<td>1</td>
<td>$1 - 3000$</td>
<td>0.3 0.7 0.1</td>
</tr>
<tr>
<td>2</td>
<td>$3001 - 6000$</td>
<td>-0.2 0.6 0.4</td>
</tr>
<tr>
<td>3</td>
<td>$6001 - 9000$</td>
<td>0.1 -0.8 0.2</td>
</tr>
<tr>
<td>4</td>
<td>$9001 - 12000$</td>
<td>-0.2 0.7 0.3</td>
</tr>
</tbody>
</table>

$$w = [0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0]$$

$\alpha = 0.0002, \ \beta = 10, \ \rho = 0.01$
cates the proposed method for estimating the multipath works very well.

In order to demonstrate the performance of the dynamic behavior, the intersymbol interference (ISI) was defined as

\[ ISI = \frac{\sum_i |g_i|^2 - \max |g_i|^2}{\max |g_i|^2}. \] (28)

Here \( g_i \) is the i-th coefficient of the synthesized vector \( g = \sum_{i=1}^m h_i \ast w_i \) and \( m \) denotes the number of channels (in this case \( m = 1 \)). A small ISI indicates the proximity to the desired response. The dynamic behavior of the system identification using Eq. (28) is shown in Fig. 5. This result indicates that, even if the impulse responses are changing in time, the developed self-adaptive algorithms are able to estimate these coefficients with a good performance.

6. Conclusions

In this paper, a novel algorithm applying the principal-factor method and the scale-free method with ICA for the spatial and temporal equalization was presented. The main advantages of our proposed algorithm are that which enable us to reduce cochannel interference as well as high-power noises from the noisy data with irrelevant signal measurements. The number of signals can be further estimated by proposed criterion. For the time-varying channel identification, the self-adaptive algorithm was proposed for updating the step-size coefficient automatically. Computer simulation results were presented to illustrate the effectiveness and performance of our proposed method.

Acknowledgments

This research is partially supported by the Research Fellowships of the Japan Society for the Promotion of Science (JSPS) for Young Scientists.

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